TELECOMMUTING IN AN ECONOMY WITH TRANSPORT NETWORKS

QINGLAN XIA AND SHAOFENG XU

Abstract. This paper examines the implications of transport networks for the spatial allocation of telecommuting. We develop a tractable monocentric model that explicitly incorporates the network structure. Route prices for on-site workers depend on the geometry and utilization of the transport network. The resulting coordination mechanism can yield multiple equilibria when the network exhibits economies of scale. We show that the efficiency of an equilibrium hinges on network pricing rules and achieving it requires uneven subsidies across communities. These results are robust in an extended setting with multiple employment centers.

1. Introduction

In the aftermath of the COVID-19 pandemic, return-to-office mandates have become increasingly common as firms seek to restore in-person work arrangements. These mandates, however, have faced substantial resistance from workers bearing renewed commuting burdens, often leading to contentious labor market negotiations over workplace flexibility. The debates highlight the importance of examining how commuting costs shape telecommuting decisions, particularly when such costs are determined not only by geographic distance but also by the structure of the transport network. Characterizing telecommuting equilibria in a spatial economy with transport networks would advance our understanding of work-location choices and offer insights for policy design.

To this end, we develop a tractable spatial equilibrium model that explicitly incorporates the structure of the transport network. In the economy, households reside in a finite number of locations and they can work from home or in office. They also consume a numeraire good, enjoy amenities associated with remote work and dislike commuting, with their preferences represented by a quasilinear utility function. The representative firm hires on-site and remote labor to produce the numeraire good. Production takes place in a single employment center and residential communities are connected to the center through a transport network. This paper models the network as a weighted directed graph, where each community is connected to the center by a unique route. Every route consists of a sequence of edges and community routes may overlap on edges near the center, so they carry larger passenger flows than their peripheral counterparts. Operating the transport network is costly, where the cost of each edge increases with its length linearly and its flow nonlinearly. The nonlinearity reflects the level of efficiency in mass transportation or transport economies of scale and paves the way for interactions between on-site workers. The network operator charges route-dependent prices on commuters and any network surplus or deficit is rebated or financed through lump-sum transfers. Overall, work-mode choices across space are linked not only through labor markets but also through the geometry and utilization of the commuting network.

Key words and phrases. Telecommuting, transport network, transport economies of scale, remote work. *JEL classification*: E2, J2, R3, R4.

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We start our analysis by deriving a potential-based representation of a spatial equilibrium, upon which model characterization is conducted. The main takeaways are three-fold. First, we prove the existence of an equilibrium and establish the possibility of multiple equilibria. The multiplicity emerges due to an interesting coordination mechanism. In the model, both on-site wage premium and remote-work valuation decrease with office attendance, so technology and preferences form a joint force against multiple equilibria. Their effects, however, can be offset or even reversed when the cost function is concave in flow, i.e., the transport system exhibits economies of scale. In this case, for two communities whose routes overlap, higher office attendance by one community raises traffic flows on edges shared with the other and thus lowers its route price. Essentially, an on-site worker confers a positive network-participation benefit on all other communities using the same edge. Low office attendance can thus be self-reinforcing as thin edges are expensive per user, whereas high attendance encourages downtown traffic as costs are shared with a large number of commuters.

Second, we construct a measure of the strength of the coordination mechanism based on a weighted route-overlap matrix. Its off-diagonal entries show that the coordination between communities is stronger when their shared edges are greater in length and smaller in flow. Third, a comparative statics analysis sheds light on the implications of economic primitives and network geometry for telecommuting. Two findings are noteworthy. Neither remote-work productivity nor elasticity of substitution between on-site and remote labor exerts a definite impact on office attendance, as changes in either parameter trigger conflicting substitution and productivity effects. Meanwhile, varying the curvature of the cost function redistributes the cost burden across routes rather than moving commuting costs in the same direction.

To assess the efficiency of an equilibrium, we solve and analyze the social planner's problem. Two lessons emerge. First, the network pricing rules matter for the efficiency. Under marginal-cost pricing, private and social first-order conditions coincide so an equilibrium is socially optimal. The two sets of conditions, however, diverge under both average-cost and external-finance regimes. When the network operator adopts the average-cost pricing rule, communities pay more than marginal physical network cost, so the planner obtains a positive marginal return from increasing office attendance implied by an equilibrium. As shown, the corrective policy is to subsidize on-site labor. By contrast, when the network operation is financed externally, taxing commuters helps decentralize the optimum. Second, the equilibrium-optimum wedge is not uniformly distributed across communities, with the wedge wider on routes that are longer or lower in traffic. Greater subsidies should be offered to communities that lie farther from the employment center or closer to the peripheral of the transport network. This finding suggests that a uniform return-to-office mandate might not be in the best interest of firms and spatial heterogeneity should be an important consideration in developing these policies. Our results are robust in an extended setting with multiple employment centers.

This paper naturally contributes to the nascent literature that examines the ramifications of telecommuting. Examples include [4], [1], [2], [5], [7], [8], [18], [9], and [15]. Our study advances the literature by developing a tractable spatial equilibrium framework that highlights the role of transport networks in shaping work-mode choices. Contemporaneous work by [15] is the study most closely related to ours, because both identify a coordination mechanism that helps underpin multiple equilibria.[1] Our paper complements theirs along two dimensions. First, their coordination result is derived from the assumption that the value of working in office depends

[1]The authors document that trips in the largest U.S. cities have stabilized at levels that are only about 60 percent of pre-pandemic levels, while those in smaller cities have largely returned to pre-pandemic normalcy. They attribute this city heterogeneity to a coordination game, where people prefer to work in office only if others do and prefer to work from home otherwise.

on proximity to other workers. By contrast, our result is associated with the assumption that route prices paid by on-site workers depend on the usage of the commuting network. Second, the authors estimate the welfare costs for cities with persistent losses in downtown traffic, whereas we analyze the efficiency of an equilibrium allocation and propose corrective policies that emphasize spatial heterogeneity.

The current study also relates to the wealth of literature that applies the theory of optimal transportation to economic problems, such as [10], [13], [12], [11], and [17]. Our modeling strategy of the transport network is based on the work of ramified optimal transportation as exemplified in [16] and [6], where transport economies of scale help justify branching structures in many natural and engineered systems. Up to our best knowledge, this paper presents the first theoretical attempt to analyze the effects of network structure on telecommuting.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 characterizes the multiplicity of equilibria and identifies a coordination mechanism. Section 4 conducts a comparative statics analysis. Section 5 analyzes the social planner's problem. Section 6 extends the model into a multi-center setting. Section 7 concludes.

2. The Model

This section lays out a tractable spatial equilibrium model of remote work that puts the role of transport network at the center. Households reside in a number of locations and they can work from home or in office. Production takes place in a single employment center (downtown).[2] On-site workers travel through a transport network with commuting costs determined by the geometry and utilization of the network.

2.1. **Communities and downtown.** There are ℓ residential communities indexed by $j = 1, \ldots, \ell$. Community j contains n_j identical workers and is located at $y_j \in X$, where X is a spatial domain. Let $q_j \in [0, n_j]$ be the mass of workers in community j who work on site in the downtown $O \in X$. The remaining $n_j - q_j$ workers work remotely. The aggregate supply of on-site and remote labor equals

$$L_o = \sum_{j=1}^{\ell} q_j, \; L_r = N - L_o, \tag{1}$$

where $N = \sum_{j=1}^{\ell} n_j$ denotes the total workforce.

2.2. **Production.** In the economy, labor is the only factor input for production. A representative firm produces a numeraire good using on-site labor Q_o and remote labor Q_r via the following technology

$$Y = F(L) = L^{\beta}, \tag{2}$$

where $0 < \beta < 1$ and

$$L = L(Q_o, Q_r) = \left[\frac{1}{2} Q_o^{\frac{\sigma-1}{\sigma}} + \frac{1}{2} (\chi Q_r)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

represents the effective labor in production. The parameter $\sigma > 1$ governs the elasticity of substitution between the two types of labor and $\chi > 0$ measures the productivity of remote labor relative to on-site labor.

[2] For analytical and expositional tractability, we adopt a monocentric setting to develop results. The main insights are robust to an extension with multiple employment centers, with details presented later in the paper.

2.3. **Preferences.** Communities derive utility from the consumption of the numeraire good, enjoyment of amenities associated with remote work, and distaste for commuting. Preferences of community j are represented by a quasilinear utility function

$$U_j(c_j, q_j) = c_j + n_j H_j(s_j) - \tau_j d_j q_j, \tag{4}$$

where c_j denotes the consumption of the numeraire good, s_j the share of remote workers

$$s_j = 1 - \frac{q_j}{n_j}, \tag{5}$$

and d_j the commute distance, with coefficient $\tau_j \geq 0$ measuring the private disutility per unit of commute distance.[3] Working from home imparts workers better work-life balance but also increased distractions from household tasks. The function H_j captures net benefits of remote work and takes the form of

$$H_j(s) = \eta_j s - \frac{\kappa_j}{2} s^2, \tag{6}$$

where η_j and κ_j are positive parameters. A household's consumption is financed by its labor income and lump-sum transfer, with its budget constraint written as

$$c_j + p_j^{G,\zeta}(q) q_j = W_o q_j + W_r(n_j - q_j) + T_j. \tag{7}$$

In (7), W_o and W_r denote, respectively, wages paid to on-site and remote labor, whereas $p_j^{G,\zeta}(q)$ and T_j represent in order route price and lump-sum transfer to be defined shortly.

2.4. **Transport network and pricing rules.** Residential communities and the employment center are connected by a transport network as illustrated in Figure 1. The network is represented by a weighted directed graph $G = (V, E)$ in space X, which consists of vertex set V and edge set E. By construction, $\{y_1, \ldots, y_\ell, O\} \subseteq V$. Every community y_j is connected to center O by a unique directed path γ_j, oriented from the community toward downtown. Each edge $e \in E$ has length ℓ_e. The commuting distance of community j then equals

$$d_j = \sum_{e \in \gamma_j} \ell_e. \tag{8}$$

For edge $e \in E$, its flow of commuters amounts to the total office attendance of communities whose routes contain that edge and is expressed as

$$w_e(q) = \sum_{j : e \in \gamma_j} q_j. \tag{9}$$

Near the center, several community routes may share the same path, so those edges carry larger passenger flows. By contrast, peripheral edges are used by fewer communities and generally associated with a smaller traffic.

[3]The assumption of quasilinear utility is taken for the benefit of analytical tractability. It eliminates income effects from the work-mode decision and allows lump-sum transfers and profit distributions to be separated from the choice between remote and on-site work.

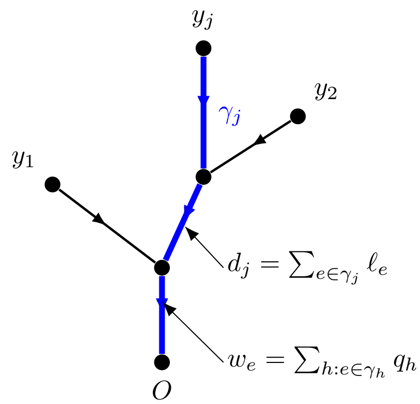

FIGURE 1. A monocentric transport network. The blue arrows show the directed route γ_j from community y_j to employment center O. Its total length is d_j. Each edge e carries the combined on-site flow of all downstream communities whose routes use that edge.

Operating the transport network is costly. We assume that the operation cost, measured in the numeraire good, increases with flow and length of each edge and takes the following form:

$$C_G(q) = \lambda \sum_{e\in E} w_e(q)^\alpha \ell_e, \tag{10}$$

where λ and α are positive parameters.[4] The latter is particularly noteworthy as it regulates the concavity of the cost function C_G and thus the responses of marginal and average costs to increasing network ridership. The value of α can be interpreted as a result of competition between two opposite forces, congestion (cost-increasing) and efficiency (cost-decreasing), in mass transportation. When $\alpha < 1$, edge cost is concave in flow, so additional users lower average and marginal costs on a shared edge, manifesting the dominance of economies of scale in the network. Conversely, when $\alpha > 1$, additional attendance inflates per-user route cost, suggesting that congestion outweighs efficiency as the primary cost factor. As will be evident later, the parameter α would profoundly shape the configuration of equilibria and its role will be stressed repeatedly in the subsequent analysis.

To finance the operation cost, the network operator implements a pricing scheme to collect commuting fees. For a pricing rule ζ, denote $p_e^\zeta(w_e)$ as the per-commuter charge on edge e and

$$p_j^{G,\zeta}(q) = \sum_{e\in\gamma_j} p_e^\zeta(w_e(q)) \tag{11}$$

as the route price for community j. We consider three alternative pricing rules

$$p_e^\zeta(w_e) = \zeta\lambda w_e^{\alpha-1}\ell_e,\ \zeta \in \{0, \alpha, 1\} \tag{12}$$

where cases $\zeta = 0$, $\zeta = \alpha$, and $\zeta = 1$ correspond, respectively, to external finance, marginal-cost pricing, and average-cost pricing.[5] Ceteris paribus, commuters pay most under average-cost (marginal-cost) pricing if $\alpha < (>)\ 1$ as

$$p_e^1(w_e) - p_e^\alpha(w_e) = \lambda(1-\alpha)w_e^{\alpha-1}\ell_e. \tag{13}$$

[4]This functional specification is standard in the literature of optimal ramified transportation, such as [16] and [6]. We invite the reader to regard α as a structural parameter for a given transport network. Our study, however, does not confine the parameter to be less than one as typically assumed in the literature.

[5]Detailed analysis of transport pricing can be found in studies such as [14] and [3].

Total network revenue amounts to

$$R_G^{\zeta}(q) = \sum_{e\in E} w_e(q) p_e^{\zeta}(w_e(q)) = \zeta C_G(q). \tag{14}$$

Therefore, average-cost pricing exactly recovers the operation cost, while marginal-cost pricing recoups only the fraction α. The model assumes that any network surplus or deficit is rebated or financed through lump-sum transfers.

2.5. **Competitive equilibrium.**

Definition 1 (Competitive equilibrium)**.** *For a given transport network G and pricing rule $\zeta \in \{0, \alpha, 1\}$, a competitive equilibrium is a collection $\left(\bar{q}, \bar{c}, \bar{Q}_o, \bar{Q}_r, \bar{W}_o, \bar{W}_r, \bar{\Pi}, \bar{T}\right)$ satisfying the following conditions:*

(i) Utility maximization. *Given wages, route prices and lump-sum transfer, community j chooses c_j and q_j to maximize*

$$U_j = c_j + n_j H_j\left(1 - \frac{q_j}{n_j}\right) - \tau_j d_j q_j$$

subject to the budget constraint (7).

(ii) Profit maximization. *Given wages, the representative firm chooses Q_o and Q_r to maximize*

$$\Pi(Q_o, Q_r) = F(L(Q_o, Q_r)) - \bar{W}_o Q_o - \bar{W}_r Q_r. \tag{15}$$

(iii) All markets clear. *The labor market clears when*

$$\bar{Q}_o = \sum_{j=1}^{\ell} \bar{q}_j, \;\; \bar{Q}_r = \sum_{j=1}^{\ell} (n_j - \bar{q}_j), \tag{16}$$

and the good market clears when

$$\sum_{j=1}^{\ell} \bar{c}_j + C_G(\bar{q}) = F(L(\bar{Q}_o, \bar{Q}_r)). \tag{17}$$

(iv) Transfer consistency. *Lump-sum transfers distribute firm profits and the network operator's net revenue as*

$$\sum_{j=1}^{\ell} \bar{T}_j = \bar{\Pi} + R_G^{\zeta}(\bar{q}) - C_G(\bar{q}). \tag{18}$$

For an interior household choice, it holds that

$$W_o - W_r = H_j'(s_j) + \tau_j d_j + p_j^{G,\zeta}(q). \tag{19}$$

In the case of boundary solutions, the difference between left- and right-hand sides is weakly negative at $q_j = 0$ and weakly positive at $q_j = n_j$. An interior firm solution necessitates

$$W_o = F'(L)\frac{\partial L}{\partial Q_o}(Q_o, Q_r), \;\; W_r = F'(L)\frac{\partial L}{\partial Q_r}(Q_o, Q_r). \tag{20}$$

For expositional convenience, formulate output as a function of aggregate on-site labor

$$P(Q_o) = \left[\frac{1}{2}Q_o^{\rho} + \frac{1}{2}\big(\chi(N - Q_o)\big)^{\rho}\right]^{\frac{\beta}{\rho}}, \quad \rho = \frac{\sigma - 1}{\sigma} \in (0, 1). \tag{21}$$

The on-site wage premium then satisfies

$$P'(Q_o) = W_o - W_r. \tag{22}$$

Merging optimality conditions (19) and (22) with labor-market clearing condition (16) yields

$$P'(\bar{Q}_o) = H_j'(\bar{s}_j) + \tau_j d_j + p_j^{G,\zeta}(\bar{q}) \tag{23}$$

for every community with an interior work-mode choice.

3. Equilibrium Allocation

This section proposes a tractable representation of an equilibrium and proves its existence. The model features multiple equilibria with varying remote work intensity and the multiplicity stems from a coordination mechanism rooted in the transport network.

3.1. **Existence and multiplicity.** The following proposition proposes a potential-based representation of an equilibrium work-mode allocation $\bar{q}$, upon which the existence is affirmed. The potential yields one equilibrium condition for each community. Once allocation is known, other equilibrium variables, such as wages and transfers, can be easily pinned down by optimality and market-clearing conditions. In light of condition (23), define an excess function

$$\mathcal{E}_j(q) = P'\left(\sum_{h=1}^{\ell} q_h\right) - H_j'\left(1 - \frac{q_j}{n_j}\right) - \tau_j d_j - p_j^{G,\zeta}(q), \tag{24}$$

which can be interpreted as net marginal benefit from supplying an additional unit of remote work by community j. The action earns the community the on-site wage premium (P') at the expense of disutility in remote work (H_j'), commuting ($\tau_j d_j$) and per-user route price ($p_j^{G,\zeta}$). If the excess function $\mathcal{E}_j(q)$ is positive, the community gains from increasing on-site work; if it is negative, the community is better off with more telecommuting.

Proposition 1 (Existence of an equilibrium). *An allocation $\bar{q} \in K = \prod_j [0, n_j]$ is an equilibrium work-mode allocation if and only if, for every j,*

$$\mathcal{E}_j(\bar{q}) \begin{cases} \leq 0, & \bar{q}_j = 0, \\ = 0, & 0 < \bar{q}_j < n_j, \\ \geq 0, & \bar{q}_j = n_j. \end{cases} \tag{25}$$

Define a potential

$$\mathcal{V}_\zeta(q) = P\left(\sum_{j=1}^{\ell} q_j\right) + \sum_{j=1}^{\ell} n_j H_j\left(1 - \frac{q_j}{n_j}\right) - \sum_{j=1}^{\ell} \tau_j d_j q_j - \frac{\zeta\lambda}{\alpha} \sum_{e \in E} w_e(q)^\alpha \ell_e. \tag{26}$$

Then, it holds that

$$\frac{\partial \mathcal{V}_\zeta}{\partial q_j}(q) = \mathcal{E}_j(q) \tag{27}$$

whenever relevant route flows are positive. Hence, every maximizer of $\mathcal{V}_\zeta$ on K is an equilibrium allocation and an equilibrium exists.

Answers to the question of uniqueness, however, are far from definite. The complication can be seen from the curvature of the potential $\mathcal{V}_\zeta(q)$. All four terms on the right-hand side of (26) are concave except for the last one. The outlier represents the negative route price for community j and its concavity depends on whether α is greater or smaller than one. When the parameter lies sufficiently below one, i.e., economies of scale in mass transportation are very potent, the curvature of $\mathcal{V}_\zeta(q)$ becomes dominated by that of $-\frac{\zeta\lambda}{\alpha}\sum_{e\in E} w_e(q)^\alpha \ell_e$, making the potential a convex function. Thus, maximizers of $\mathcal{V}_\zeta(q)$ may not be unique, so do equilibria.

Developing clean results on the multiplicity is nevertheless a daunting task for a general setting. To best illuminate the mechanism, we confine our analysis to a symmetric economy constructed as follows. Assume that all communities are identical in the sense that

$$n_j = n,\ H_j = H,\ \tau_j d_j = \tau d, \quad j = 1, ..., \ell$$

and the transport network is symmetric under permutations of the communities. The total workforce then equals $N = \ell n$. For every edge $e \in \gamma_j$, define

$$a_e = \#\{h : e \in \gamma_h\},$$

as the number of communities whose routes use edge e and assume that the multiset of pairs (a_e, ℓ_e) encountered along γ_j is the same for every community j. Thus, all communities face the same route price under a symmetric office-attendance choice. For a given $x \in [0, n]$, let $q(x) = (x, ..., x)$ denote the work-mode allocation vector. Then, the aggregate on-site labor is

$$Q(x) = \ell x,$$

and the flow on edge $e \in \gamma_j$ is

$$w_e(q(x)) = a_e x.$$

By symmetry, the excess function $\mathcal{E}_j(q(x))$ is independent of j and we denote this common value by $\mathcal{E}(x)$. The next lemma makes the shape of the function explicit.

Lemma 1. *Denote $b_0 = \frac{1}{2}\chi^\rho N^\rho$ and $c_0 = \frac{\beta}{2} b_0^{\beta/\rho - 1}$. For $x \in (0, n)$, it holds that*

$$\mathcal{E}(x) = P'(\ell x) - \eta + \kappa\left(1 - \frac{x}{n}\right) - \tau d - \zeta\lambda\Gamma_\alpha x^{\alpha-1}, \tag{28}$$

and

$$\mathcal{E}'(x) = \ell P''(\ell x) - \frac{\kappa}{n} + \zeta\lambda(1-\alpha)\Gamma_\alpha x^{\alpha-2}, \tag{29}$$

where

$$\Gamma_\alpha = \sum_{e\in\gamma_j} a_e^{\alpha-1}\ell_e. \tag{30}$$

Define

$$\begin{aligned} A(Q) &= \frac{1}{2}\left[Q^\rho + \chi^\rho (N-Q)^\rho\right], \\ B(Q) &= \frac{1}{2}\left[Q^{\rho-1} - \chi^\rho (N-Q)^{\rho-1}\right], \\ C(Q) &= \frac{1}{2}\left[Q^{\rho-2} + \chi^\rho (N-Q)^{\rho-2}\right]. \end{aligned}$$

In (28) and (29), derivatives P' and P'' are

$$P'(Q) = \beta A(Q)^{\beta/\rho-1} B(Q), \tag{31}$$

$$P''(Q) = -\beta A(Q)^{\beta/\rho-2}\left[(1-\beta)B(Q)^2 + (1-\rho)\big(A(Q)C(Q) - B(Q)^2\big)\right] < 0, \tag{32}$$

and they satisfy

$$P'(Q) = c_0 Q^{\rho-1}\big(1 + o(1)\big),\ P''(Q) = -(1-\rho)c_0 Q^{\rho-2}\big(1 + o(1)\big),\ \text{as } Q \downarrow 0 \tag{33}$$

and

$$P'(Q) \to -\infty,\ P''(Q) \to -\infty,\ \text{as } Q \uparrow N. \tag{34}$$

Consequently,

$$\mathcal{E}(x) \to -\infty,\ \mathcal{E}'(x) \to -\infty,\ \text{as } x \uparrow n. \tag{35}$$

The lemma derives an explicit formula for the derivative $\mathcal{E}'(x)$ in (29). As shown, its first two components are always negative, whereas the third changes sign at $\alpha = 1$. Thus, the monotonicity of $\mathcal{E}(x)$, which regulates the number of its zeros, depends critically on α. The next proposition showcases that the economy may exhibit multiple symmetric equilibria.

Proposition 2 (Multiple symmetric equilibria)**.** *Suppose $\zeta > 1$. It holds that*

(i) If $\alpha \geq 1$, then $\mathcal{E}'(x) < 0$ for $x \in (0, n)$ and the symmetric equilibrium, if it exists, is unique.

(ii) If $0 < \alpha < 1$, then the economy may exhibit multiple symmetric equilibria:

(a) If $0 < \alpha < \rho$, then $\mathcal{E}(x) \to -\infty$ and $\mathcal{E}'(x) \to +\infty$ as $x \downarrow 0$. There exists $\bar{\lambda} > 0$ such that whenever $0 < \lambda < \bar{\lambda}$, the economy has at least two symmetric equilibria.

(b) If $\alpha = \rho$, define

$$\lambda_0 = \frac{c_0 \ell^{\rho-1}}{\zeta \Gamma_\rho}. \tag{36}$$

If $0 < \lambda < \lambda_0$, then $\mathcal{E}(x) \to +\infty$ and $\mathcal{E}'(x) \to -\infty$ as $x \downarrow 0$; If $\lambda > \lambda_0$, then $\mathcal{E}(x) \to -\infty$ and $\mathcal{E}'(x) \to +\infty$ as $x \downarrow 0$. Define

$$\lambda_\rho^* = \frac{1}{\zeta(1-\rho)\Gamma_\rho} \inf_{0<x<n} \left[\left(\frac{\kappa}{n} - \ell P''(\ell x) \right) x^{2-\rho} \right] \leq \lambda_0. \tag{37}$$

If $\lambda < \lambda_\rho^*$, then the economy has a unique symmetric equilibrium; If $\lambda > \lambda_\rho^*$, then suitable values of $\eta + \tau d - \kappa$ may generate multiple symmetric equilibria.

(c) If $\rho < \alpha < 1$, then $\mathcal{E}(x) \to +\infty$ and $\mathcal{E}'(x) \to -\infty$ as $x \downarrow 0$. Define

$$\lambda_\alpha^* = \frac{1}{\zeta(1-\alpha)\Gamma_\alpha} \min_{0<x<n} \left[\left(\frac{\kappa}{n} - \ell P''(\ell x) \right) x^{2-\alpha} \right] < \infty. \tag{38}$$

If $0 < \lambda < \lambda_\alpha^*$, then the economy has a unique symmetric equilibrium; If $\lambda > \lambda_\alpha^*$, then suitable values of $\eta + \tau d - \kappa$ may generate multiple symmetric equilibria.

Proposition 2 estabilishes the possibility of multiple equilibria, with a three-equilibria example displayed in Figure 2. It demonstrates that $\alpha < 1$ is necessary, though not sufficient, for the multiplicity. The finding is economically transparent. Equation (29) tells that both on-site wage premium ($P'' < 0$) and remote-work valuation ($-\frac{\kappa}{n} < 0$) decreases with office attendance. Hence, technology and preferences form a joint force against multiple equilibria in the model. The multiplicity is made possible entirely by the presence of the third term in (29), which represents the change in (negative) per-user route price

$$\zeta \lambda \sum_{e \in \gamma_j} a_e^{\alpha-1} \ell_e x^{\alpha-1}$$

following a universal unit increase in on-site labor supply from all communities. When $\alpha > 1$, larger passenger flow raises the route price and creates a negative network-participation complementarity. This reinforces the aforementioned impacts from technology and preferences and rules out multiplicity. When $\alpha = 1$, the route price is independent of edge flow so the complementarity disappears. When $\alpha < 1$, additional network traffic lowers the route price and creates a positive complementarity. This can help offset or even reverse the effects from technology and preferences and open the door for multiple equilibria. Evidently, the prospect of multiplicity is heightened by a larger operation-cost scale λ, a smaller α, longer shared edges ℓ_e, and greater route overlap a_e, but dampened by a bigger amenity-curvature parameter κ and stronger concavity of the reduced production production as measured by $-P''$.

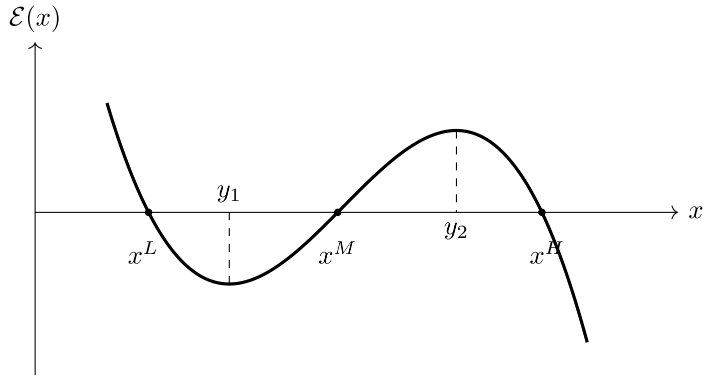

FIGURE 2. An illustration of a three-equilibrium case. The function $\mathcal{E}(x)$ in (28) has a local minimum at y_1 below zero and a local maximum at y_2 above zero and it crosses the horizontal axis three times, at $x^L < x^M < x^H$.

3.2. **The coordination mechanism.** The participation complementarity amounts to a coordination mechanism that shapes the interactions between network users. In what follows, we explicitly formulate the mechanism in the notion of weighted route-overlap matrix.

Proposition 3 (Coordination mechanism)**.** *For any two communities $j, h \in \{1, \ldots, \ell\}$, the effect of community h's on-site labor supply on community j's route price equals*

$$\frac{\partial p_j^{G,\zeta}(q)}{\partial q_h} = \zeta\lambda(\alpha - 1)\Omega_{jh}(q), \tag{39}$$

where $\Omega(q)$ is a weighted route-overlap matrix such that for any pair (j, h)

$$\Omega_{jh}(q) = \sum_{e \in \gamma_j \cap \gamma_h} w_e(q)^{\alpha-2}\ell_e. \tag{40}$$

Two lessons are noteworthy. First, the value of α dictates the nature of community interactions through a transport network. Suppose $\alpha < 1$. A rise in office attendance by community h raises traffic flows on edges shared with community j and thus lowers the per-user route price faced by the latter. In other words, an on-site worker imparts a positive participation benefit to all other communities using the same edge. Low office attendance can thus be self-reinforcing as thin edges are expensive per user. Conversely, high attendance stimulates downtown traffic as costs are shared with a large passenger flow. This is exactly the coordination mechanism underpinning the cases of multiple equilibria in Proposition 2. The mechanism is absent or reversed when either a commuter pays a flow-independent amount on every edge ($\alpha = 1$) or additional office attendance in one community elevates per-user route price for communities sharing its route ($\alpha > 1$). In both scenarios, only a unique equilibrium is viable.

Second, the intensity of the coordination between two distinct communities hinges only on the edges shared by their routes to the employment center as summarized by the weighted route-overlap matrix $\Omega(q)$. The off-diagonal entry $\Omega_{j,h}(q)$ captures how strongly communities j and h are linked through shared network infrastructure. For example, a Y-shaped transport network typically contains more positive common routes than a V-shaped network. By (40), when $\alpha < 1$, shared edges with great-length and small-flow receive especially large weight. As such, coordination is strongest among communities using thin peripheral edges. In view of this, the matrix $\Omega(q)$ summarizes important geometric information about the transport network and serves as a useful metric for the strength of the coordination mechanism.

3.3. **Examples.** This subsection presents two examples involving linear and ramified transport networks. The objective is to illustrate the computation of an equilibrium and the implications of network structure for the spatial distribution of telecommuting.

Linear network Suppose communities lie sequentially on one line:

$$0 < d_1 < d_2 < \cdots < d_\ell,$$

and let e_h denote the segment between the $(h-1)$-st and h-th branching points and ℓ_h its length. If communities are indexed from the center outward, then the flow on edge e_h is

$$w_h = \sum_{j=h}^{\ell} q_j, \quad h = 1, ..., \ell$$

and community j uses precisely edges $e_1, \ldots, e_j$. By (11), the community faces route price

$$p_j^{G,\zeta}(q) = \zeta\lambda \sum_{h=1}^{j} \left(\sum_{k=h}^{\ell} q_k \right)^{\alpha-1} \ell_h. \tag{41}$$

In light of Proposition 1, the equilibrium system becomes

$$P'\left(\sum_{k=1}^{\ell} q_k\right) - H_j'\left(1 - \frac{q_j}{n_j}\right) - \tau_j d_j - \zeta\lambda \sum_{h=1}^{j} \left(\sum_{k=h}^{\ell} q_k\right)^{\alpha-1} \ell_h = 0, \quad j = 1, \ldots, \ell. \tag{42}$$

whose solution $\bar{q}$ informs the equilibrium office attendance across communities. To sharpen insights, assume that $n_j = n$, $H_j(s) = \eta s - \frac{\kappa}{2}s^2$, $\tau_j = \tau$ for all j. Subtracting the equilibrium condition (42) for community j from that for community $j+1$ gives

$$\bar{s}_{j+1} - \bar{s}_j = \frac{\tau\ell_{j+1} + \zeta\lambda w_{j+1}(\bar{q})^{\alpha-1}\ell_{j+1}}{\kappa} > 0, \quad j = 1, \ldots, \ell - 1. \tag{43}$$

which predicts that remote-work shares increase strictly with distance from the employment center. The prediction is based on two reinforcing effects of spatial location on remote work. One is the ordinary distance effect $(\tau\ell_{j+1})$ and the other is a network-periphery effect $(\zeta\lambda w_{j+1}(\bar{q})^{\alpha-1}\ell_{j+1})$ generated by the additional low-flow edge used by community $j+1$. This example crystallizes the impact of route overlap on telecommuting: a community located farther from the center not only incurs a longer route, but also uses more shared segments, so its office attendance decision both affects and is affected by a larger set of communities.

Ramified network We next consider two transport networks with Y- and V-shaped structures as depicted in Figure 3. Both networks have the same total route length for each community but different overlap. The subsequent numerical exercise imposes a community symmetry with $n_j = n$, $H_j(s) = \eta s - \frac{\kappa}{2}s^2$, and $\tau_j d_j = \tau d$, and adopts parameter values $n = 1$, $\eta = 0.8$, $\kappa = 1.5$, $\tau d = 0.1$, $\chi = 0.6$, $\sigma = 2.2$, $\beta = 0.72$, $\alpha = 0.55$, $\lambda = 0.15$, and $\zeta = 1$. For the Y network, let every edge length equal one, so

$$\ell_0 = \ell_1 = \ell_2 = 1,$$

while for the V network each direct edge has total length

$$\tilde{\ell}_1 = \tilde{\ell}_2 = 2.$$

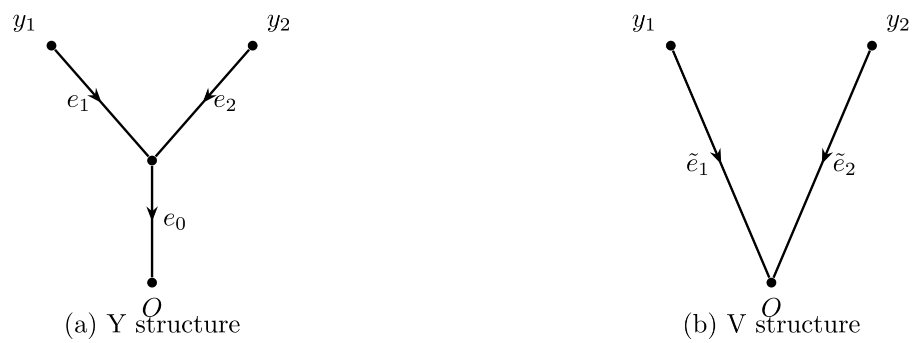

FIGURE 3. Two ramified transport networks with the same total route length for each community. Commuters share edge e_0 in the Y network, whereas routes are disjoint in the V network.

First, consider the Y-shaped network. Let q_1 and q_2 denote on-site labor supplied by the two communities. The traffic flows on involved edges are $w_0 = q_1 + q_2,\ w_1 = q_1,$ and $w_2 = q_2$. Accordingly, route prices for the two communities are

$$p_1^Y(q_1, q_2) = \lambda\left[(q_1+q_2)^{\alpha-1} + q_1^{\alpha-1}\right],\ p_2^Y(q_1, q_2) = \lambda\left[(q_1+q_2)^{\alpha-1} + q_2^{\alpha-1}\right].$$

Substituting the above formulas into conditions $\mathcal{E}_j(q) = 0$ gives the equilibrium system

$$0 = P'(q_1+q_2) - (\eta - \kappa(1-q_1)) - \tau d - p_1^Y(q_1, q_2), \tag{44}$$

$$0 = P'(q_1+q_2) - (\eta - \kappa(1-q_2)) - \tau d - p_2^Y(q_1, q_2). \tag{45}$$

Given the above parameterization, the system has three interior solutions:

$$(\bar{q}_1, \bar{q}_2) = (0.2435, 0.2435),\ (\bar{q}_1, \bar{q}_2) = (0.0395, 0.2942),\ (\bar{q}_1, \bar{q}_2) = (0.2942, 0.0395).$$

The symmetric equilibrium reflects balanced supply of remote labor, whereas the two asymmetric equilibria arise because the shared edge e_0 activates a coordination mechanism under which one community may see higher office attendance than the other.

Second, consider the V-shaped network. Since the two routes are disjoint, route prices are simply

$$p_1^V(q_1) = \lambda\,\tilde{\ell}_1\,q_1^{\alpha-1} = 0.30\,q_1^{-0.45},\ p_2^V(q_2) = \lambda\,\tilde{\ell}_2\,q_2^{\alpha-1} = 0.30\,q_2^{-0.45}.$$

The associated equilibrium system is

$$0 = P'(q_1+q_2) + 0.6 - 1.5q_1 - 0.30\,q_1^{-0.45}, \tag{46}$$

$$0 = P'(q_1+q_2) + 0.6 - 1.5q_2 - 0.30\,q_2^{-0.45}, \tag{47}$$

which has a unique solution

$$(\bar{q}_1, \bar{q}_2) = (0.1873, 0.1873).$$

Compared to its Y counterpart, the V structure eliminates route overlap and thus shuts down the coordination channel. The two communities remain linked through the common wage-premuim term P', but this linkage only works to rule out multiple equilibria.

In summary, results of this section highlight that the equilibrium allocation of telecommuting may be unique or multiple, hinging on the interaction between technology, preferences, and network structures. When $0 < \alpha < 1$, route overlap creates a genuine coordination mechanism: on-site workers on a common edge confer cost-reducing benefits on one another, so the network structure affects not only commuting costs but also equilibrium selection.

4. Comparative statics

This section conducts a comparative statics analysis for two sets of parameters. The first set relates to economic parameters that shape preferences and technology, whereas the second concerns network parameters about transportation and geometry. As the model admits multiple equilibria, results must be understood locally.

4.1. **Economic parameters.**

Throughout the section, we fix an interior equilibrium $\bar{q}$ that is a maximizer of the potential $\mathcal{V}_\zeta$ and define

$$\bar{A} = -D^2_{qq}\mathcal{V}_\zeta(\bar{q}). \tag{48}$$

Proposition 4 (Preferences and technology)**.** *For communities $j, h \in \{1, ..., \ell\}$,*

$$\frac{\partial \bar{q}_j}{\partial \eta_h} = -((\bar{A})^{-1})_{jh},\ \frac{\partial \bar{q}_j}{\partial \kappa_h} = \bar{s}_h((\bar{A})^{-1})_{jh},\ \frac{\partial \bar{q}_j}{\partial \tau_h} = -d_h((\bar{A})^{-1})_{jh}, \tag{49}$$

and

$$\frac{\partial \bar{q}_j}{\partial \eta_j} = -((\bar{A})^{-1})_{jj} < 0,\ \frac{\partial \bar{q}_j}{\partial \kappa_j} = \bar{s}_j((\bar{A})^{-1})_{jj} > 0,\ \frac{\partial \bar{q}_j}{\partial \tau_h} = -d_j((\bar{A})^{-1})_{jj} < 0. \tag{50}$$

Moreover, for $z \in \{\chi, \sigma\}$,[6]

$$\frac{\partial \bar{q}}{\partial z} = (\bar{A})^{-1}\mathbf{1}\,\frac{\partial^2 P}{\partial Q_o\,\partial z}(\bar{Q}_o; \chi, \sigma), \tag{51}$$

where $\bar{Q}_o = \sum_{j=1}^{\ell}\bar{q}_j$ is aggregate on-site labor, and

$$\frac{\partial \bar{Q}_o}{\partial z} = \mathbf{1}^\top(\bar{A})^{-1}\mathbf{1}\,\frac{\partial^2 P}{\partial Q_o\,\partial z}(\bar{Q}_o; \chi, \sigma). \tag{52}$$

so the response of aggregate office attendance to z has the same sign as the effect of z on the marginal return to on-site labor.

Proposition 4 shows that stronger preferences for telecommuting (larger η or smaller κ) or aversion to commuting (larger τ) discourage office attendance, where $(\bar{A})^{-1}$ captures the propagation of shocks through the equilibrium system. However, the proposition is silent on the sign of the impact of remote-work productivity (χ) or elasticity-of-substitution (σ). The ambiguity arises because changes in either parameter would ignite two competing forces. To make the competition transparent, recall that $P(Q_o; \chi, \sigma) = F(L(Q_o, N - Q_o; \chi, \sigma))$, where functions F and L are given in (2) and (3), respectively. Then, we have $\frac{\partial P}{\partial Q_o}(Q_o; \chi, \sigma) = \beta L^{\beta-1}\frac{\partial L}{\partial Q_o}$ and for $z \in \{\chi, \sigma\}$

$$\frac{\partial^2 P}{\partial Q_o\,\partial z} = \beta L^{\beta-1}\left[\frac{\partial^2 L}{\partial Q_o \partial z} - \frac{1-\beta}{L}\frac{\partial L}{\partial Q_o}\frac{\partial L}{\partial z}\right]. \tag{53}$$

The first term in the bracket represents how changes in the parameter z alter $\frac{\partial L}{\partial Q_o}$, the gap in marginal productivity between on-site and remote labor, and thus their relative attractiveness in production. This points to a *substitution effect.* The second term, meanwhile, captures how effective labor L and thus the common marginal productivity $\beta L^{\beta-1}$ respond to variations in z. We dub it as a *productivity effect.* As the two forces can work in opposite directions, the net impact of χ or σ on office attendance $\bar{q}$ may not have a universal sign.

[6] With a bit abuse of notation, we occasionally spell out the dependence of functions on relevant parameters.

4.2. **Network parameters.**

Proposition 5 (Transportation and geometry)**.** *For community j, define*

$$k_j(q) = \sum_{e\in\gamma_j} w_e(q)^{\alpha-1}\ell_e,\ \ g_j(q) = \sum_{e\in\gamma_j} w_e(q)^{\alpha-1}\log w_e(q)\,\ell_e,$$

and for edge f, denote $a_f \in \mathbb{R}^\ell$ as its route-incidence vector with

$$(a_f)_j = \mathbf{1}_{\{f\in\gamma_j\}}.$$

It holds that

$$\frac{\partial \bar{q}}{\partial \lambda} = -\zeta(\bar{A})^{-1}k(\bar{q}),\ \ \frac{\partial \bar{q}}{\partial \alpha} = -\zeta\lambda(\bar{A})^{-1}g(\bar{q}),\ \ \frac{\partial \bar{q}}{\partial \ell_f} = -\zeta\lambda w_f(\bar{q})^{\alpha-1}(\bar{A})^{-1}a_f. \tag{54}$$

Interpretations of Proposition 5 are as follows. A larger scale parameter λ raises every route price proportionally, dampening the desire for working in office across communities. Likewise, increasing the length of edge f inflates commuting costs for communities whose routes use that edge and the exposure is identified by the incidence vector a_f. By comparison, the impact of α is more distributional. Given $\frac{\partial}{\partial\alpha}w_e^{\alpha-1} = w_e^{\alpha-1}\log w_e$, an increase in α raises (lowers) the route price on edges with $w_e > (<)1$, so the way α shapes office attendance hinges on the scale of edge flows relative to the unit used in the model. Intuitively, the parameter α governs the magnitude of transport economies of scale or how strongly a transport network rewards concentration of flow: a smaller α creates stronger economies of scale, making high-flow and low-flow segments of the network respond differently. Thus, unlike λ or an individual edge length, a change in α can redistribute the relative cost burden across routes rather than shifting all commuting costs in the same direction.

5. Socially Optimal Allocation

This section analyzes the social planner's problem and identifies a wedge between equilibrium and optimal levels of telecommuting. The wedge originates from pricing rules of the network operator and community-dependent subsidy policies are proposed to close the gap.

5.1. **Social optimum.**

Definition 2 (Social planner's problem)**.** *A social optimum allocation (q^*, c^*) solves*

$$\max_{\substack{0\le q_j\le n_j\\ c_j\ge 0}} \sum_{j=1}^{\ell}\left[c_j + n_jH_j\left(1-\frac{q_j}{n_j}\right) - \tau_j d_j q_j\right] \quad \textit{subject to} \quad \sum_{j=1}^{\ell} c_j + C_G(q) = P\left(\sum_{j=1}^{\ell} q_j\right), \tag{55}$$

where C_G and P are defined in (10) and (21), respectively.

Eliminating aggregate consumption yields the equivalent problem

$$\max_{q\in K} \mathcal{W}(q), \tag{56}$$

where $K = \prod_{j=1}^{\ell}[0, n_j]$ and

$$\mathcal{W}(q) = P\left(\sum_{j=1}^{\ell} q_j\right) + \sum_{j=1}^{\ell} n_jH_j\left(1-\frac{q_j}{n_j}\right) - \sum_{j=1}^{\ell}\tau_j d_j q_j - \lambda\sum_{e\in E} w_e(q)^\alpha \ell_e. \tag{57}$$

The function $\mathcal{W}$ is the optimum counterpart of the equilibrium potential $\mathcal{V}_\zeta$ in (26) such that

$$\mathcal{V}_\zeta(q) = \mathcal{W}(q) - \left(\frac{\zeta}{\alpha} - 1\right)\lambda \sum_{e\in E} w_e(q)^\alpha \ell_e.$$

Parallel to (24), define

$$\mathcal{S}_j(q) = \frac{\partial \mathcal{W}}{\partial q_j}(q) = P'\left(\sum_{h=1}^{\ell} q_h\right) - H_j'\left(1 - \frac{q_j}{n_j}\right) - \tau_j d_j - \lambda\alpha \sum_{e\in\gamma_j} w_e(q)^{\alpha-1}\ell_e. \tag{58}$$

Proposition 6 (Existence and multiplicity of social optima)**.** *A socially optimal allocation exists. Any optimum $q^* \in K$ satisfies*

$$\mathcal{S}_j(q^*) \begin{cases} \le 0, & q_j^* = 0, \\ = 0, & 0 < q_j^* < n_j, \\ \ge 0, & q_j^* = n_j. \end{cases} \tag{59}$$

If $\alpha \ge 1$, the optimum is unique; If $0 < \alpha < 1$, multiple optima may exist.

An interior optimum satisfies

$$P'\left(\sum_{h=1}^{\ell} q_h\right) = H_j'\left(1 - \frac{q_j}{n_j}\right) + \tau_j d_j + \lambda\alpha \sum_{e\in\gamma_j} w_e(q)^{\alpha-1}\ell_e. \tag{60}$$

The next proposition points to a wedge between optimum and equilibrium allocations and proposes a subsidy system to decentralize the optimum.

Proposition 7 (Equilibrium–optimum wedge and corrective policy)**.** *For a pricing rule $\zeta \in \{0, \alpha, 1\}$, define*

$$\Xi_j^\zeta(q) = (\zeta - \alpha)\lambda \sum_{e\in\gamma_j} w_e(q)^{\alpha-1}\ell_e. \tag{61}$$

At any interior equilibrium $\bar{q}$,

$$\frac{\partial \mathcal{W}}{\partial q_j}(\bar{q}) = \Xi_j^\zeta(\bar{q}). \tag{62}$$

Suppose that $\bar{q}$ is close to an interior optimum q^* and define $A^* = -D_{qq}^2\mathcal{W}(q^*)$. Then, it holds that

$$\bar{q} - q^* = -(A^*)^{-1}\Xi^\zeta(\bar{q}) + o(\|\bar{q} - q^*\|)\,. \tag{63}$$

Moreover, the per-unit subsidy that locally decentralizes allocation q^* is

$$z^\zeta = \Xi^\zeta(q^*). \tag{64}$$

Main takeaways from Proposition 7 are two-fold. First, the sign of the equilibrium-optimum wedge, defined by $\Xi_j^\zeta(q)$ in (61), depends on the pricing rules adopted by the network operator. When marginal-cost pricing rule is adopted ($\zeta = \alpha$), $\Xi_j^\alpha(q) = 0$ so every interior equilibrium satisfies the first-order conditions for an optimum and no policy intervention is needed ($z^\alpha = 0$). Under average-cost pricing regime ($\zeta = 1$), the wedge

$$\Xi_j^1(q) = \lambda(1-\alpha) \sum_{e\in\gamma_j} w_e(q)^{\alpha-1}\ell_e. \tag{65}$$

In case of $\alpha < 1$, $\Xi_j^1(q) > 0$ so communities pay more than marginal physical network cost. Consequently, the planner's marginal return from additional office attendance is positive at the equilibrium $\bar{q}$ as predicted by (62). The corrective policy is offering subsidy to on-site labor ($z^1 > 0$). The converse is true if $\alpha > 1$. When the network operation is financed externally ($\zeta = 0$), the wedge

$$\Xi_j^0(q) = -\lambda\alpha \sum_{e\in\gamma_j} w_e(q)^{\alpha-1}\ell_e < 0. \tag{66}$$

In other words, commuters do not fully bear the marginal physical network cost so the planner's marginal return from additional attendance is negative at $\bar{q}$. This suggests that attendance at the equilibrium may be excessive and a tax is required to curb downtown traffic ($z^0 < 0$).

Second, the equilibrium-optimum wedge is not uniformly distributed across communities. For example, (65) shows that the wedge is wider on routes that are longer or lower in traffic. Greater subsidies should thus be granted to communities that lie farther from the employment center or closer to the peripheral of a transport network. This finding has direct policy implications for workplace arrangements. As the pandemic has ignited a greater demand for autonomy over work flexibility, return-to-office mandates have become a source of contentious debate between workers and managers. The above analysis pinpoints that a uniform mandate might not be in the best interest of firms and spatial heterogeneity should be an important consideration in designing these policies.

5.2. **An example.** Consider a four-source ramified transport network in Figure 4. Communities $1, 2$ share edge e_{12}, communities $3, 4$ share e_{34}, and routes of all four communities overlap at edge e_0. The following numerical exercise assumes identical workforce $n_1 = n_2 = n_3 = n_4 = 1$ but different edge length $\ell_0 = 1.00$, $\ell_{12} = 0.85$, $\ell_{34} = 1.15$, $\ell_1 = 0.75$, $\ell_2 = 1.05$, $\ell_3 = 0.90$, and $\ell_4 = 1.20$. The parameters for preferences are set at $(\eta_1, \eta_2, \eta_3, \eta_4) = (0.82, 0.76, 0.88, 0.72)$, $(\kappa_1, \kappa_2, \kappa_3, \kappa_4) = (1.45, 1.60, 1.35, 1.70)$, and $(\tau_1 d_1, \tau_2 d_2, \tau_3 d_3, \tau_4 d_4) = (0.08, 0.12, 0.10, 0.15)$, while those for technology and network are $\chi = 0.62$, $\sigma = 2.1$, $\beta = 0.72$, $\alpha = 0.45$, and $\lambda = 0.055$.

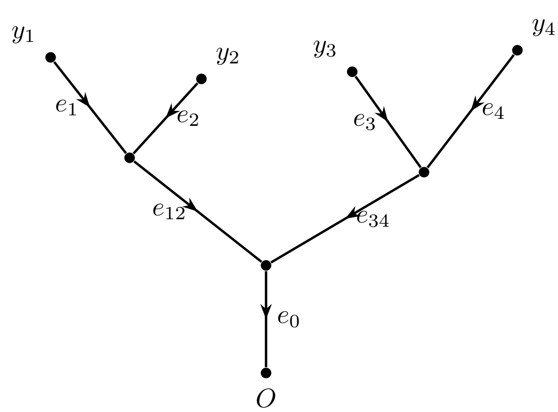

FIGURE 4. An asymmetric four-source transport network.

For a given q, define

$$\begin{aligned}
R_1(q) &= \ell_0 Q^{\alpha-1} + \ell_{12}(q_1+q_2)^{\alpha-1} + \ell_1 q_1^{\alpha-1},\\
R_2(q) &= \ell_0 Q^{\alpha-1} + \ell_{12}(q_1+q_2)^{\alpha-1} + \ell_2 q_2^{\alpha-1},\\
R_3(q) &= \ell_0 Q^{\alpha-1} + \ell_{34}(q_3+q_4)^{\alpha-1} + \ell_3 q_3^{\alpha-1},\\
R_4(q) &= \ell_0 Q^{\alpha-1} + \ell_{34}(q_3+q_4)^{\alpha-1} + \ell_4 q_4^{\alpha-1},
\end{aligned}$$

where $Q = q_1 + q_2 + q_3 + q_4$. By (60), the system of equations for an optimum allocation is

$$P'(Q) - \eta_j + \kappa_j(1 - q_j) - \tau_j d_j - \alpha\lambda R_j(q) = 0, \qquad j = 1, \dots, 4. \tag{67}$$

Under the current parameterization, the economy has a unique optimum

$$q^* = (0.4029, 0.4658, 0.2816, 0.4948). \tag{68}$$

By (64), the optimum can be decentralized by granting subsidies

$$(z_1^{AC}, z_2^{AC}, z_3^{AC}, z_4^{AC}) \approx (0.0882, 0.0991, 0.1176, 0.1164).$$

under average-cost pricing regime or collecting taxes

$$(z_1^0, z_2^0, z_3^0, z_4^0) \approx (-0.0722, -0.0811, -0.0963, -0.0953).$$

under external-finance regime. As expected, both sets of subsidies exhibit strong spatial heterogeneity, varying with length or usage of commuting routes.

6. An Extension

This section conducts a robustness exercise by extending the baseline model into a setting with multiple employment centers. Workers have an option to choose destination. This flexibility introduces a new source of spatial coordination because passenger flows on shared routes can affect which employment center attracts on-site workers.

6.1. **Equilibrium with multiple centers.** Let $r = 1, \dots, R$ index employment centers. For each community j, let $q_{jr} \geq 0$ denote the mass of residents working on site at center r. The total on-site and remote labor supplied by community j are

$$q_j = \sum_{r=1}^{R} q_{jr}, \; n_j s_j = n_j - q_j,$$

Each center r employs

$$Q_r = \sum_{j=1}^{\ell} q_{jr}$$

units of on-site labor and produces $P_r(Q_r)$, where P_r is a center-specific reduced production function similar to that in (21). Each origin–destination pair (j, r) uses a prescribed route γ_{jr} in a transport network G. The flow on edge e thus equals

$$w_e(q) = \sum_{j=1}^{\ell} \sum_{r=1}^{R} \mathbf{1}_{\{e \in \gamma_{jr}\}} q_{jr}. \tag{69}$$

Let d_{jr} be the origin–destination distance and τ_{jr} its per-unit private commuting disutility. Under network pricing rule $\zeta \in \{0, \alpha, 1\}$, the route price faced by a commuter from j to r is

$$p_{jr}^{G,\zeta}(q) = \zeta\lambda \sum_{e \in \gamma_{jr}} w_e(q)^{\alpha - 1} \ell_e. \tag{70}$$

Analogous to (26), define the multi-center equilibrium potential

$$\begin{aligned}\mathcal{V}_\zeta^R(q) = &\sum_{r=1}^{R} P_r(Q_r) + \sum_{j=1}^{\ell} n_j H_j \left(1 - \frac{\sum_{r=1}^{R} q_{jr}}{n_j}\right) \\ &- \sum_{j=1}^{\ell} \sum_{r=1}^{R} \tau_{jr} d_{jr} q_{jr} - \frac{\zeta\lambda}{\alpha} \sum_{e \in E} w_e(q)^\alpha \ell_e.\end{aligned} \tag{71}$$

Proposition 8 (Multi-center equilibrium)**.** *Denote the feasible set as*

$$\mathcal{K}_R = \left\{ q = (q_{jr}) : q_{jr} \geq 0,\ \sum_{r=1}^{R} q_{jr} \leq n_j \text{ for every } j \right\}. \tag{72}$$

An allocation $\bar{q} \in \mathcal{K}_R$ is an equilibrium work-mode allocation if and only if it maximizes maximizes $\mathcal{V}_\zeta^R$ over $\mathcal{K}_R$. An equilibrium exists. Moreover, an interior equilibrium satisfies[7]

$$P_r'(Q_r) = H_j'(s_j) + \tau_{jr} d_{jr} + \zeta\lambda \sum_{e \in \gamma_{jr}} w_e(q)^{\alpha-1} \ell_e. \tag{73}$$

Without doubt, the aforementioned coordination mechanism still operates in the multi-center economy and helps foster multiple equilibria. The extended setting, however, is interesting in itself as it creates another lay of coordination between network users. To see this more precisely, differentiating (70) with respect to q_{ht} gives the effect of commuters from source h to destination t on the route price of those from source j to destination r

$$\frac{\partial p_{jr}^{G,\zeta}(q)}{\partial q_{ht}} = \zeta\lambda(\alpha - 1) \sum_{e \in \gamma_{jr} \cap \gamma_{ht}} w_e(q)^{\alpha-2} \ell_e. \tag{74}$$

Suppose $\alpha < 1$. Higher office attendance by people residing in community h and working in center t depresses the route price for those traveling from community j to center r, given their routes overlap somewhere in the network. Hence, on-site workers whose routes coincide locally interact through edge flows, even if they travel to different centers. By implication, the coordination may result in an equilibrium with concentrated production, as shifting on-site workers toward a limited number of employment centers raises passenger flows on a subset of network edges and lowers route prices for all commuters. Consequently, a center can become attractive not only because it is productive or nearby, but also because other commuters make its access routes cheaper.

6.2. **Social optimum and decentralization policy.** The social planner chooses an allocation $q \in \mathcal{K}_R$ to maximize

$$\begin{aligned} \mathcal{W}^R(q) = & \sum_{r=1}^{R} P_r(Q_r) + \sum_{j=1}^{\ell} n_j H_j \left(1 - \frac{\sum_{r=1}^{R} q_{jr}}{n_j} \right) \\ & - \sum_{j=1}^{\ell} \sum_{r=1}^{R} \tau_{jr} d_{jr} q_{jr} - \lambda \sum_{e \in E} w_e(q)^{\alpha} \ell_e. \end{aligned} \tag{75}$$

An interior optimum necessitates

$$P_r'(Q_r) = H_j'(s_j) + \tau_{jr} d_{jr} + \alpha\lambda \sum_{e \in \gamma_{jr}} w_e(q)^{\alpha-1} \ell_e. \tag{76}$$

As in the baseline model, an equilibrium may not be socially optimal due to network pricing rules. The next proposition shows that a subsidy system similar in spirit to that advocated in Proposition 7 can still effectively decentralize an optimum.

Proposition 9 (Wedge and decentralization)**.** *At an interior equilibrium $\bar{q}$,*

$$\frac{\partial \mathcal{W}^R}{\partial q_{jr}}(\bar{q}) = \Xi_{jr}^{\zeta}(\bar{q}), \quad \Xi_{jr}^{\zeta}(q) = (\zeta - \alpha)\lambda \sum_{e \in \gamma_{jr}} w_e(q)^{\alpha-1} \ell_e. \tag{77}$$

[7] Proofs in this section are parallel to their baseline counterparts and thus skipped for space consideration.

Suppose that $\bar{q}$ is close to an interior optimum q^*. The per-unit subsidy that locally decentralizes allocation q^* is origin–destination specific and equals

$$z^{\zeta}_{jr} = \Xi^{\zeta}_{jr}(q^*). \tag{78}$$

As noticeable as it is, the corrective policy is now origin-destination specific. On-site workers from the same community may require different subsidies if their routes to different centers vary in lengths, overlap patterns, or passenger flows. Conversely, commuters from different communities may require similar subsidies if they use the same high-flow trunk. Thus, a uniform return-to-office subsidy may fail to solve the inefficiency in telecommuting across communities or concentration across employment centers.

7. Conclusion

We analyze the role of transport networks in shaping telecommuting decisions. A coordination mechanism emerges when the network exhibits economies of scale: a commuter raises flow on every edge of her route and thereby lowers route prices for other commuters using those edges. The mechanism raises the possibility of multiple equilibria. Depending on network pricing rules, an equilibrium may not be optimal. Efficiency can nevertheless be achieved using route-specific subsidies, implying that uniform return-to-office policies are generally inefficient.

Several directions remain open for future explorations. First, to preserve tractability and intuition, this paper takes the transport network as given. Endogenizing the network would connect work-mode choices to infrastructure design and allow the topology of the commuting system to respond to workplace arrangements. In addition, the model generates a series of predictions about telecommuting and they deserve a careful empirical evaluation. Furthermore, it would be a fruitful exercise to extend the current framework to study the design of return-to-office policies.

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Appendix A. Proofs

A.1. **Proof of Proposition 1.** The advocated representation follows directly from a community's utility maximization behavior. An interior choice requires $\mathcal{E}_j(q)$ to be zero. At $q_j = 0$, the community cannot reduce on-site work further, so an equilibrium requires $\mathcal{E}_j(q) \leq 0$. At $q_j = n_j$, it cannot supply more on-site labor, so an equilibrium requires $\mathcal{E}_j(q) \geq 0$. Differentiating the potential $\mathcal{V}_\zeta(q)$ with respect to q_j immediately yields (27), where for the last term the corresponding derivative

$$\frac{\partial}{\partial q_j}\left[-\frac{\zeta\lambda}{\alpha}\sum_e w_e^\alpha \ell_e\right] = -\zeta\lambda \sum_{e\in\gamma_j} w_e^{\alpha-1}\ell_e = -p_j^{G,\zeta}(q).$$

Because the potential $\mathcal{V}_\zeta(q)$ is continuous, it attains a maximum on the compact set K. At a maximizer, the sign of each coordinate derivative satisfies (25). Hence, any allocation that maximizes the potential is an equilibrium work-mode allocation.

A.2. **Proof of Lemma 1.** Equalities (28) and (29) follow directly from (24). By definition, we have $A'(Q) = \rho B(Q)$ and $B'(Q) = -(1-\rho)C(Q)$. Differentiating (21) once gives (31) and twice gives

$$P''(Q) = \beta A(Q)^{\beta/\rho-2}\left[(\beta-\rho)B(Q)^2 - (1-\rho)A(Q)C(Q)\right].$$

Applying the identity

$$(\beta-\rho)B^2 - (1-\rho)AC = -(1-\beta)B^2 - (1-\rho)(AC - B^2)$$

yields (32). Because $AC - B^2 > 0$ for $Q \in (0, N)$ and $\beta, \rho \in (0,1)$, we have $P''(Q) < 0$.

As $Q \downarrow 0$, $A(Q) \to b_0$ and $B(Q) = \frac{1}{2}Q^{\rho-1}\big(1 + o(1)\big)$. This proves P' in (33). Differentiating the leading term or using the displayed formula for P'' derive its first-order approximation. As $Q \uparrow N$, the term $-\frac{1}{2}\chi^\rho(N-Q)^{\rho-1}$ in $B(Q)$ diverges to $-\infty$ and the term $\frac{1}{2}\chi^\rho(N-Q)^{\rho-2}$ in $C(Q)$ diverges to $+\infty$. It follows that $P'(Q) \to -\infty$ and $P''(Q) \to -\infty$. The final assertions follow from $Q = \ell x$ and that the terms with Γ_α remain finite as $x \uparrow n$.

A.3. **Proof of Proposition 2.** If $\alpha \geq 1$, then every term in (29) is nonpositive and the first two are strictly negative so $\mathcal{E}'(x) < 0$ for every $x \in (0, n)$. As such, $\mathcal{E}(x)$ is strictly decreasing and can cross zero at most once. Thus, if a symmetric equilibrium exists, it must be unique.

Now suppose $0 < \alpha < 1$. Substituting (31) and (32) into (28) and (29) yield

$$\mathcal{E}(x) = c_0\ell^{\rho-1}x^{\rho-1} - \zeta\lambda\Gamma_\alpha x^{\alpha-1} - \eta + \kappa - \tau d + o(x^{\rho-1}), \tag{79}$$

$$\mathcal{E}'(x) = -(1-\rho)c_0\ell^{\rho-1}x^{\rho-2} + \zeta\lambda(1-\alpha)\Gamma_\alpha x^{\alpha-2} - \frac{\kappa}{n} + o(x^{\rho-2}). \tag{80}$$

For part (a) with $\alpha < \rho$. In this case, $\alpha - 1 < \rho - 1 < 0$ and $\alpha - 2 < \rho - 2 < 0$. Thus, as $x \downarrow 0$, the negative term $-\zeta\lambda\Gamma_\alpha x^{\alpha-1}$ dominates (79), while the positive term $\zeta\lambda(1-\alpha)\Gamma_\alpha x^{\alpha-2}$

dominates (80). This implies that $\mathcal{E}(x) \to -\infty$ and $\mathcal{E}'(x) \to +\infty$ as $x \downarrow 0$. In light of (35), it suffices to show that $\mathcal{E}(x)$ is positive for small enough λ. Denote

$$x_\lambda = \left[\frac{(1-\alpha)\zeta\lambda\Gamma_\alpha}{(1-\rho)c_0\ell^{\rho-1}}\right]^{1/(\rho-\alpha)}.$$

It follows that as $\lambda \downarrow 0$, $x_\lambda \downarrow 0$ and

$$c_0\ell^{\rho-1}x_\lambda^{\rho-1} - \zeta\lambda\Gamma_\alpha x_\lambda^{\alpha-1} = \frac{\rho-\alpha}{1-\alpha}c_0\ell^{\rho-1}x_\lambda^{\rho-1} \to +\infty.$$

By (79), we have $\mathcal{E}(x_\lambda) > 0$ for sufficiently small λ. Since the function $\mathcal{E}(x) \to -\infty$ at both endpoints $x = 0, n$, it has at least two interior zeros.

For part (b) with $\alpha = \rho$. Equations (79) and (80) turn into

$$\mathcal{E}(x) = \left(c_0\ell^{\rho-1} - \zeta\lambda\Gamma_\rho\right)x^{\rho-1} + O(1) + o(x^{\rho-1}),$$

$$\mathcal{E}'(x) = (1-\rho)\left(\zeta\lambda\Gamma_\rho - c_0\ell^{\rho-1}\right)x^{\rho-2} + O(1) + o(x^{\rho-2}).$$

The limits of $\mathcal{E}(x)$ and $\mathcal{E}'(x)$ as $x \downarrow 0$ result immediately from (36). By (29), $\mathcal{E}'(x) < 0$ requires

$$\lambda < R_\rho(x) = \frac{\left(\frac{\kappa}{n} - \ell P''(\ell x)\right)x^{2-\rho}}{\zeta(1-\rho)\Gamma_\rho}.$$

By Lemma 1, the continuous function $R_\rho(x)$ is positive with $\lim_{x\downarrow 0} R_\rho(x) = \lambda_0$ and $\lim_{x\uparrow n} R_\rho(x) = +\infty$. Hence, its infimum is positive and no larger than λ_0, i.e., $0 < \lambda_\rho^* \le \lambda_0$. If $\lambda < \lambda_\rho^*$, $\mathcal{E}'(x) < 0$ for $x \in (0, n)$, so $\mathcal{E}(x)$ is strictly decreasing and has a unique interior zero. If $\lambda_\rho^* < \lambda < \lambda_0$, $\mathcal{E}'(x)$ is positive somewhere. Because $\mathcal{E}'(x) \to -\infty$ as $x \downarrow 0$ and $x \uparrow n$, it has at least two interior zeros, implying that $\mathcal{E}(x)$ has at least a local minimum and a local maximum. If $\lambda > \lambda_0$, $\mathcal{E}'(x)$ has at least one zero since it approaches $+\infty$ as $x \downarrow 0$ and $-\infty$ as $x \uparrow n$. Accordingly, $\mathcal{E}(x)$ has at least one local maximizer. In either of the preceding two cases, multiple symmetric equilibria can be obtained by properly varying $\eta + \tau d - \kappa$, which would shift the function $\mathcal{E}(x)$ vertically without altering its derivative.

For part (c) with $\alpha > \rho$. In this case, $\rho - 1 < \alpha - 1 < 0$ and $\rho - 2 < \alpha - 2 < 0$. Thus, as $x \downarrow 0$, the positive term $c_0\ell^{\rho-1}x^{\rho-1}$ dominates (79), while the negative term $-(1-\rho)c_0\ell^{\rho-1}x^{\rho-2}$ dominates (80). This implies that $\mathcal{E}(x) \to +\infty$ and $\mathcal{E}'(x) \to -\infty$ as $x \downarrow 0$. By (33), as $x \downarrow 0$,

$$R_\alpha(x) \sim \frac{(1-\rho)c_0\ell^{\rho-1}}{\zeta(1-\alpha)\Gamma_\alpha}x^{\rho-\alpha} \longrightarrow +\infty,$$

whereas as $x \uparrow n$, $-P''(\ell x) \to +\infty$ so $R_\alpha(x) \to +\infty$. Thus, $R_\alpha(x)$ attains a finite positive minimum in $(0, n)$, which is precisely λ_α^*. If $\lambda < \lambda_\alpha^*$, then $\lambda < R_\alpha(x)$ for every x so $\mathcal{E}'(x) < 0$ everywhere. Thus, $\mathcal{E}(x)$ is strictly decreasing and has a unique interior zero. If $\lambda > \lambda_\alpha^*$, then $\mathcal{E}'(x)$ is positive somewhere. Because $\mathcal{E}'(x) \to -\infty$ as $x \downarrow 0$ and $x \uparrow n$, it has at least two interior zeros. So $\mathcal{E}(x)$ has at least a local minimum and a local maximum. Varying $\eta + \tau d - \kappa$ properly would then make the function $\mathcal{E}(x)$ cross zero multiple times.

A.4. **Proof of Proposition 3.** For an edge $e \in \gamma_j \cap \gamma_h$, we have

$$\frac{\partial w_e(q)}{\partial q_h} = 1.$$

For edges outside $\gamma_j \cap \gamma_h$, the derivative is zero. Differentiating (11) with respect to q_h yields

$$\frac{\partial p_j^{G,\zeta}(q)}{\partial q_h} = \zeta\lambda \sum_{e\in\gamma_j\cap\gamma_h} \frac{d}{dw_e}\left(w_e^{\alpha-1}\right)\frac{\partial w_e(q)}{\partial q_h}\ell_e = \zeta\lambda(\alpha-1)\sum_{e\in\gamma_j\cap\gamma_h} w_e(q)^{\alpha-2}\ell_e.$$

A.5. **Proof of Proposition 4.** An interior equilibrium $\bar{q}$ satisfies
$$\mathcal{E}(\bar{q};\vartheta)=0,$$
where $\mathcal{E}(q;\vartheta)=D_q\mathcal{V}_\zeta(q;\vartheta)$ and ϑ represents the set of model parameters. Given $D_q\mathcal{E}(\bar{q};\vartheta)=D^2_{qq}\mathcal{V}_\zeta(\bar{q})=-\bar{A}$, the implicit function theorem yields
$$\frac{\partial\bar{q}}{\partial z}=(\bar{A})^{-1}\frac{\partial\mathcal{E}}{\partial z},\quad z\in\vartheta.$$
By (24), we have
$$\frac{\partial\mathcal{E}}{\partial\eta_h}=-e_h,\ \frac{\partial\mathcal{E}}{\partial\kappa_h}=\bar{s}_he_h,\ \frac{\partial\mathcal{E}}{\partial\tau_h}=-d_he_h,$$
where e_h is a vector that equals one at its h-th entry and zeros elsewhere. Results in (50) then follow. By definition, $\bar{A}$ is symmetric and positive definite, so is $\bar{A}^{-1}$. Thus, $((\bar{A})^{-1})_{jj}>0$, verifying the signs in (50). Because parameters χ and σ explicitly enter every equilibrium condition via the common on-site wage premium $\frac{\partial P}{\partial Q_o}(\bar{Q}_o;\chi,\sigma)$, we have
$$\frac{\partial\mathcal{E}}{\partial z}=\frac{\partial^2P}{\partial Q_o\,\partial z}(\bar{Q}_o;\chi,\sigma)\mathbf{1},\quad z\in\{\chi,\sigma\}$$
which leads to (51). Summing its components gives (52). Finally, the positive definiteness of $\bar{A}^{-1}$ implies $\mathbf{1}^\top(\bar{A})^{-1}\mathbf{1}>0$, so $\frac{\partial\bar{Q}_o}{\partial z}$ has the same sign as the effect of z on P'.

A.6. **Proof of Proposition 5.** Thanks to the proof of Proposition 4, the derivative of $\mathcal{E}$ with respect to parameter $z\in\{\lambda,\alpha,\ell\}$ equals
$$\frac{\partial\bar{q}}{\partial z}=(\bar{A})^{-1}\frac{\partial\mathcal{E}}{\partial z}.$$
By (11) and (24), we have
$$\frac{\partial\mathcal{E}_j}{\partial\lambda}=-\zeta k_j(q),\ \frac{\partial\mathcal{E}_j}{\partial\alpha}=-\zeta\lambda g_j(q),\ \frac{\partial\mathcal{E}_j}{\partial\ell_f}=-\zeta\lambda w_f(q)^{\alpha-1}(a_f)_j,$$
confirming results in (54).

A.7. **Proof of Proposition 6.** Existence is guaranteed by the continuity of $\mathcal{W}$ on the compact set K and (59) represents the associated first-order conditions. For $\alpha\geq 1$, w_e^α is convex. Together with the concavity of P and the strict concavity of H_j in q_j, $\mathcal{W}$ is a strictly concave function so its maximizer must be unique. For $0<\alpha<1$, the negative cost function is convex so the concavity of $\mathcal{W}$ may fail, leaving multiple optima a real possibility.

A.8. **Proof of Proposition 7.** By (25), an interior equilibrium satisfies
$$P'\left(\sum_{h=1}^{\ell}q_h\right)=H_j'\left(1-\frac{q_j}{n_j}\right)+\tau_jd_j+\zeta\lambda\sum_{e\in\gamma_j}w_e^{\alpha-1}\ell_e.$$
Substituting the above equality into (58) gives (62). Since $D_q\mathcal{W}(q^*)=0$, a first-order expansion around q^* results in
$$D_q\mathcal{W}(\bar{q})=-A^*(\bar{q}-q^*)+o(\|\bar{q}-q^*\|)\,.$$
As the matrix A^* is symmetric positive definite and hence nonsingular, the above expansion yields (63). Finally, if each community j receives a subsidy z_jq_j, the first-order conditions for an equilibrium fully match that for an optimum, thus confirming (64).