Homework 3: due Wednesday, May 26 2010

Problem 1: Same setup as for Problem 3 in Homework 2. Compare the performance of Basis Pursuit to a greedy algorithm of your choice (e.g. OMP, Subspace Pursuit, CoSAMP,...) with respect to maximal sparsity that can be recovered. You may use a public-domain Matlab implementations of the greedy algorithm of your choice for your comparison (but make sure that you know, what the algorithm is doing). Do the comparison for two different (random) choices of real-valued x: (a) x the signs of x and the amplitudes of x are random; (b) the signs of x are all equal, (all positive or all negative - but neither algorithm does not make use of this information) and only the amplitudes of x are random. How does the greedy algorithm compare to Basis Pursuit?

Problem 2: Let A be a Gaussian random matrix of dimension 100×400 and consider the recovery of x from $Ax = b$. Assume we know that x has only non-negative entries. Determine via numerical simulations the range of s for which we are able to recover such an s-sparse vector x from $y = Ax$ by considering

$$\min_x \|x\|_1 \quad \text{subject to} \quad Ax = b, \text{ and } x \geq 0.$$

(The positivity constraint is easy to include in CVX). Compare your findings to the results from your experiments of Problem 3 from Homework 2 and try to quantify the difference.

Problem 3: Prove the following statement:
Let A and B be matrices of the same dimensions. If the row and column spaces of A and B are orthogonal then

$$\|A + B\|_* = \|A\|_* + \|B\|_*,$$

where $\|.\|_*$ denotes the nuclear (or Schatten-1) norm.

Problem 4: Prove the following statement:
Suppose that the linear map $\mathcal{A}$ satisfies the matrix-RIP with $\delta_{2r} < 1$ for some integer $r \geq 1$ and let X_0 be a matrix of rank r. Set $b = \mathcal{A}(X_0)$. Then X_0 is the only matrix of rank at most r satisfying $\mathcal{A}(X) = b$.