Homework 3

due April 27, 2005 in class

(1) Artin 11.3.1 (pg. 443)
(2) Artin 11.5.2 (pg. 444)
(3) Artin 11.5.5 (pg. 444)
(4) Artin 11.5.6 (pg. 445)
(5) For the proof of Theorem 3.8 of Artin Chapter 11 we assumed that factorization exists in the polynomial ring $\mathbb{Z}[x]$. Explain why this is true.
(6) Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 \in \mathbb{Z}[x]$ and let $p \in \mathbb{Z}$ be prime. Suppose that the coefficients of f satisfy the following conditions:
   (a) p does not divide a_n;
   (b) p divides $a_{n-1}, \cdots, a_0$;
   (c) p^2 does not divide a_0.
   Show that $f(x)$ is irreducible in $\mathbb{Q}[x]$. If f is primitive, it is irreducible in $\mathbb{Z}[x]$.
(7) Use Problem 6 to show that $x^4 + 10x + 5$ is irreducible in $\mathbb{Z}[x]$. Show that $x^n - p$ is irreducible in $\mathbb{Z}[x]$ for $n \geq 2$ and p a prime integer. Is it possible to use Problem 6 to show that $x^4 + 1$ is irreducible? (Hint: Combine Problem 6 with Problem 1 with $a = b = 1$).