MAT 149B University of California Spring 2006

Homework 8

due June 2, 2006 in class

1. Biggs 24.7.8 (pg. 356)

2. Biggs 24.7.12 (pg. 356)

3. Biggs 24.7.18 (pg. 356)

4. Biggs 25.3.5 (pg. 367)

5. Biggs 26.1.3 (pg. 378)

6. For integers $0 \le k \le n$, the q-binomial coefficient is defined as

$$\binom{n}{k}_q = \frac{(q)_n}{(q)_k (q)_{n-k}}$$

where $(q)_m = (1-q)(1-q^2)\cdots(1-q^m)$. Show that $\binom{n}{k}_q$ is the generating function of partitions with at most k parts and no part exceeding $n-k$.
[Hint: Show that

$$\binom{n}{k}_q = q^k \binom{n-1}{k}_q + \binom{n-1}{k-1}_q$$

and use induction.]