MAT 149B University of California Spring 2006

Practice Problems

May 5, 2006

There won't be a homework set this week. Instead here are a bunch of practice problems for the midterm (to be held May 10) not to be handed in. Wong will discuss the solutions in the discussion session on May 5.

Let's start with some questions that should be easy to answer knowing the definitions and theorems discussed in class:

(1) Find the gcd of $a(x) = x^3 + x^2 + x + 1$ and $b(x) = x^2 + 2$ in $\mathbb{Z}_3[x]$. Find polynomials $\lambda(x)$ and $\mu(x) \in \mathbb{Z}_3[x]$ such that $\lambda(x)a(x) + \mu(x)b(x) = \gcd(a(x), b(x))$.
(2) True or false? Let $f \in R[x]$ for R a ring. Then f can always be factored in an essentially unique way as a product of irreducible polynomials.
(3) Is $x^3 + 2x + 2$ irreducible in $\mathbb{Z}_3[x]$?
(4) How many roots can $x^2 + x + 1$ at most have?
(5) How many irreducible monic quadratic polynomials are there in $\mathbb{Z}_p[x]$?
(6) Is there a field of order 9? How would you construct it?
(7) Show that $L(i, j) = i + j$ for $i, j \in \mathbb{Z}_m$ is a Latin square.
(8) True or false? There is a 1-design with parameters $(9, 6, 4)$.
(9) True or false? All irreducible polynomials are primitive.
(10) Write down the definition of a difference set. What do they have to do with 2-designs?
(11) What kind of 2-design does the affine plane yield?
(12) What kind of 2-design does the projective plane yield?

Some further problems:

1. Show that for every prime integer p there are finite fields of order p^2 and p^3.

2. Which members of F_9 have square roots in F_9?

3. Let $\mathbb{B}$ be a 2-design with parameters (v, k, r_2) obtained from a difference set in $\mathbb{Z}_v$. Prove that any two distinct blocks of $\mathbb{B}$ have exactly r_2 common members.