MAT 149A University of California Winter 2006

Homework 4

due Wednesday February 8 in class

1. Biggs 20.2 # 2 page 262

There are eight symmetry transformations of a square. List them, and draw up the group table (as we did for the symmetries of the equilateral triangle).

2. Biggs 20.3 # 3 page 264

Suppose G is a group with the property that $g^2 = 1$ for all $g \in G$. Prove that G is a commutative group.

3. Biggs 20.3 # 5 page 265

Show that the following latin square of order 5 is not a group table.

$$\begin{matrix} 1 & a & b & c & d \\ a & b & 1 & d & c \\ b & c & d & a & 1 \\ c & d & a & 1 & b \\ d & 1 & c & b & a \end{matrix}$$

4. Biggs 20.5 # 2 page 267

By analysing the possible group tables show that, if isomorphic groups are regarded as the same, then

(1) there is just one group of order 2;
(2) there is just one group of order 3;
(3) there are just two groups of order 4.

5. Let a and b be elements of a group G. Show that a and bab^{-1} have the same order. Give an example when a and bab have different orders.

6. Let $SL(2)$ be the group of 2×2 matrices with determinant 1.

(1) Show that $SL(2)$ is an infinite group (hint: produce infinitely many 2×2 matrices with determinant one).
(2) Find two matrices in $SL(2)$ that do not commute.