Practice Final

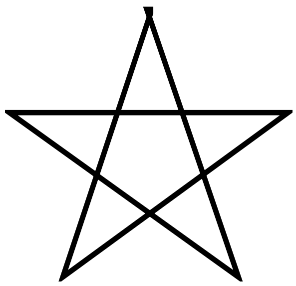
Here is a practice final exam. The real final will have a similar format. Note that **one of the homework problems** will be on the real final exam. I encourage you to work through the posted homework solutions!!!

The solutions to this practice exam will be discussed in class on March 15.

1.
(a) Take the permutation π of S_9 that maps i to $9 - i$ and 9 remains fixed. How many permutations have the same type as π? Express π as the product of transpositions. Is π even or odd?
(b) Let $\alpha = (1, 3, 6, 2, 4)(5, 8, 7)(9)$ and $\beta = (1, 5, 8, 6, 2)(3, 9, 4)(7)$. Are they conjugate?

2.
(a) The game of Domino is quite popular all over the world. The pieces consist of rectangles divided in half. Each half-rectangle is labeled by a number from 0 to 6. Use Polya's theorem to determine how many pieces there are.
(b) Consider the following three-dimensional domino game. Suppose the pieces are not rectangles, but cubes such that each face is labeled by a number from 0 to n. How many pieces are there?

3. What is the cardinality of the group of automorphisms of the following graph?



Write down a subgroup of this group and explain how many left cosets there are. How many orbits of the vertices are there? In how many different ways can you color the vertices using only three colors?

4. A high-tech toy company needs to manufacture 300 distinct (up to rotational symmetry) cubes with colored faces. What is the smallest number of colors that you need to achieve this task?

5. Decide whether the following statements are true or false:
(a) Let G be the group of permutations of the finite set X. Suppose that for all x and y in X there is a $g \in G$ such that $y = gx$. Then $|X|$ divides $|G|$.
(b) There is a subgroup of order 5 inside $C_6 \times C_2$.
(c) If G is a group of order 77 and $g \in G$, then $x^{77} = 1$.