Practice Midterm

1. Show that from any three integers you can choose two whose squares are congruent modulo 3.

2. Compute $45^{47} \mod 16$ (without a calculator).

3. Let $G = \{a, b, c\}$ be a set with 3 elements. Define the binary operation on G via the following table

	a	b	c
a	c	a	b
b	a	b	c
c	b	c	a

Is G with this binary operation a group?

4. Decide whether the statement is true or false (give a short explanation).

(1) There are no even permutations of order 3.
(2) The order of the permutation β is 5 and the order of the permutation α is 2. Then the order of $\alpha\beta\alpha^{-1}$ is 7.
(3) The permutation $(1, 2, 3)(3, 5, 6)(4, 5, 2) \in S_6$ has order three.
(4) There are permutations of order 15 in S_8.
(5) The direct product of two cyclic groups is cyclic.