Math 22 A: Homework 3

1. (a) Find two 3×3 matrices A and B such that $A \cdot B \neq B \cdot A$

   (b) Find a 3×3 matrix A that is not the zero matrix or the identity matrix but such that

   $$A \cdot B = B \cdot A$$

   for all 3×3 matrices B.

2. Show that if a matrix A has an inverse A^{-1}, then in fact A^{-1} has an inverse as well.

3. (a) Show that if A, B, C are matrices such that A has an inverse and such that

   $$A \cdot B = A \cdot C$$

   then

   $$B = C$$

   (b) The situation is different if A does not have an inverse: Consider

   $$A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

   Show that A has no inverse and find two matrices B and C with $A \cdot B = A \cdot C$ but with $B \neq C$.

4. Suppose A is an $n \times n$ matrix such that A^{2018} has no inverse. Show that A has no inverse.

5. Solve the system of equations

   $$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 0 \\ -6 & 1 & -1 \end{bmatrix} \cdot \overline{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

   in two ways:

   (a) Use row operations to reduce the augmented matrix to an augmented matrix with upper triangular coefficient matrix.

   (b) Use our algorithm from class (using the "very augmented matrix") to calculate the inverse matrix A^{-1} of the coefficient matrix A and calculate

   $$A^{-1} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

6.

(a) In class we showed that the matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

has an inverse precisely when $ad - bc \neq 0$. Suppose now that indeed $ad - bc \neq 0$. Use our algorithm to calculate A^{-1}.

(b) Suppose now that A has no inverse, meaning $ad - bc = 0$. Show that this implies that one of the following two vectors is a multiple of the other:

$$\begin{bmatrix} a \\ c \end{bmatrix}, \begin{bmatrix} b \\ d \end{bmatrix}$$

7. Consider

$$B = \begin{bmatrix} 1 & b & 0 \\ -1 & 1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

Find all values of b such that B is has no inverse.