Numerical computation of jetting impacts

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Abstract: Numerical computations based on a high-order Godunov method for multiple phases are presented. Computations of asymmetric oblique impacts reveal an instability that may reduce the jet velocity below predicted values. At small impact angles the peak pressure experienced by jetted materials is substantially less than that given by the standard model. Calculated temperatures and the fraction of melt and vapor created by a jetting collision must be reduced commensurately.

Key words: Godunov methods, Shock waves, Jetting

1. Introduction

We have developed an Eulerian high-order Godunov method for shock-capturing computations with multiple condensed phases. The thermodynamic model is based on a Mie-Grüneisen equation of state and a linear Hugoniot. This numerical method includes a volume-of-fluid algorithm for tracking solid-solid and solid-vacuum interfaces and an adaptive mesh refinement algorithm for locally resolving important features of the flow field in space and time. Details are presented in Miller & Puckett (1996). We have used this computational model to study shock refraction in experimental assemblies (Miller & Puckett 1994). Here, we present new computational results aimed at understanding the fluid-dynamic regime in asymmetric collisions that lead to jetting.

2. Jetting

The theory that provides jet mass and momentum is based on the fully developed flow resulting from the collision of thin plates (Birkhoff et al. 1948, Walsh et al. 1953, Harlow and Pracht 1966). In that theory, a length scale is provided by the plate thicknesses. In problems involving the collision of spheres (e.g., Melosh & Sonett 1986, Vickery 1993) the relevant length scale is not at all obvious, and in fact fully developed flow of the sort described in the thin-plate theory never occurs. Likewise the relevant scale governing the short-time behavior of plate impacts (or equivalently the impact of plates that are thick compared to their lengths) is not known. There is some experimental work addressing these issues. Yang et al. (1992) have shown good agreement between theory and jet properties in experiments with plates that are thin compared to their lengths, while Yang & Ahrens (1995) found a striking disagreement between thick-plate experimental results and the steady-state thin-plate theory. To reconcile the theory with experiment it is necessary to understand how the jet develops in the self-similar regime, and its subsequent transition to steady state. Preliminary work aimed at addressing that question is described here.

The inset to Fig. 1a shows the initial geometry of our computation — an aluminum (Al) plate strikes a 50° Al ramp at $V_{\mathrm{imp}} = 2$ km/s. The bottom boundary is a reflecting wall. The problem domain is 10 cm by 5 cm. Figure 1a shows the configuration of the material interfaces 900 ns after impact. A jet has formed, and the slip plane joining the impacted materials has undergone a Kelvin-Helmholtz instability.

Figure 1b shows the pressure contours at 900 ns. It is apparent from this figure that the material flowing from the colliding materials into the jet is not compressed by a single shock or collection of shocks, but rather by a complex and dispersed compressional fan.

The vortices we observe are similar to those observed in shock welding in the irregular regime (e.g.,Bahrani et al. 1967, Godunov et al. 1971). Theories for the origin of these features derive the vortex length scale from the plate thicknesses (e.g., Hunt 1968, Godunov et al. 1970, Bazdenkov et al. 1985, Gupta & Kainth 1990). It seems likely that the vortex formation evident in our computations is responsible for the wave formation observed in shock welding. If so, we believe the theories for wave formation that draw on analogy to von Karman vortex streets are incorrect since we find only a single vortex sheet in our results. Godunov et al. (1971) reached this same conclusion using an argument based on the Reynolds and Strouhal numbers.

3. Theoretical considerations

The model (Kieffer 1977) that is commonly used to calculate the pressure at the stagnation point is incorrect, and hence in many cases will give inaccurate values for the stagnation pressure. The calculation is based on transforming the problem to a reference frame in which the point of collision is stationary and then using Bernoulli's law

$$I + |\vec{U}|^2/2 = \text{const.} \qquad \text{(along streamlines in stationary, isentropic flow)} \qquad (1)$$

to calculate the enthalpy $I = E + PV$ at the stagnation point. The model assumes the streamline connecting the stagnation point $\vec{x}_{sp}$ to some upstream point $\vec{x}_0$ crosses a single shock. Consequently, (1) together with a form of (1)

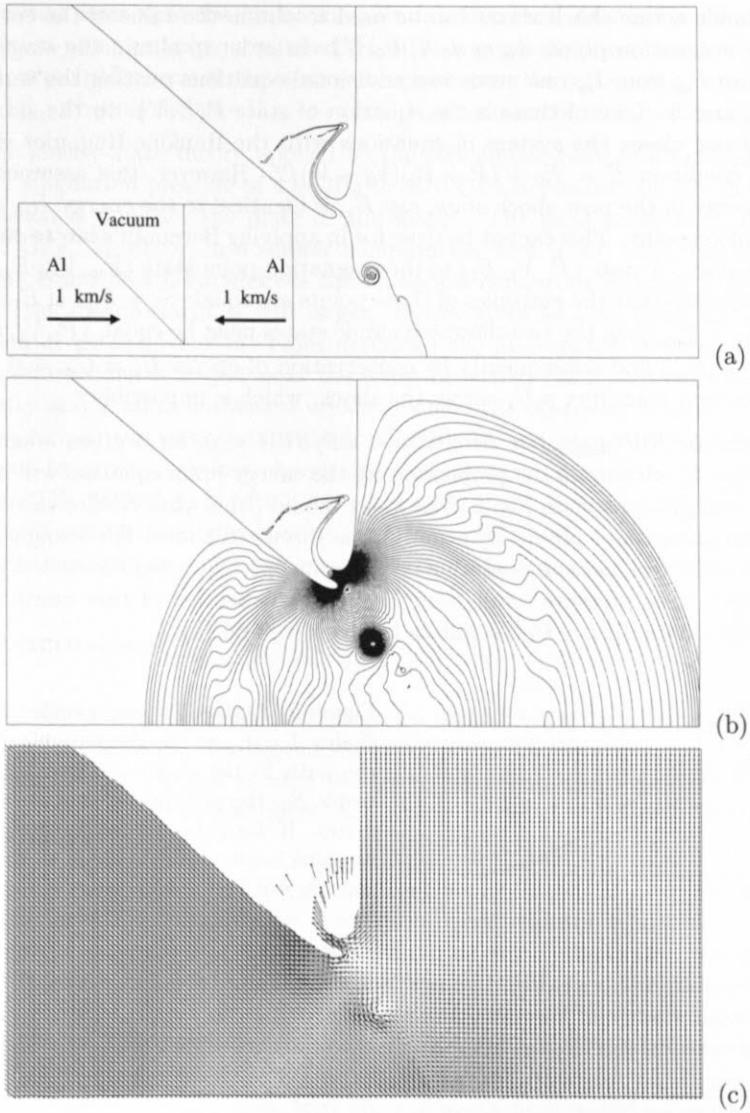

Figure 1. (a) Material interfaces at 900 ns after impact and at 0 ns (inset); (b) pressure contours from 0.2 to 10.2 GPa in steps of 0.2 GPa, and superimposed solid-vacuum interface boundary; (c) velocity vectors relative to the stagnation point.

that holds across shock waves can be used to obtain the value of the enthalpy at the stagnation point: $I_{sp} = I_0 + |\vec{U}_0|^2/2$. In order to obtain the stagnation pressure P_{sp} from I_{sp} one needs two additional equations relating the variables P, V, and E. One of these is the equation of state $P(E, V)$. In the standard model one closes the system of equations with the Rankine-Hugoniot energy jump condition $E = E_0 + (P + P_0)(V_0 - V)/2$. However, this assumes that the energy in the post-shock state, say E_1, is identical to the energy E_{sp} at the stagnation point. This cannot be true, for in applying Bernoulli's law to connect the post-shock state (P_1, V_1, E_1) to the stagnation-point state (P_{sp}, V_{sp}, E_{sp}) one must assume that the entropies of these points are equal: $S_1 = S_{sp}$. If $E_1 = E_{sp}$ and $S_1 = S_{sp}$, then the two thermodynamic states must be equal, $(P_1, V_1, E_1) = (P_{sp}, V_{sp}, E_{sp})$ and consequently by conservation of energy $\vec{U}_1 = \vec{U}_{sp} = 0$. This implies zero mass flux $\rho_1 U_1$ across the shock, which is impossible.

Since $\partial P/\partial V|_{\text{Hugoniot}} < \partial P/\partial V|_S < \partial P/\partial V|_I < 0$, for a given stagnation enthalpy I_{sp}, closure of the problem using the energy-jump equation will underestimate P_{sp} as shown in Fig. 2. The lower bound (b) is what Kieffer calculated, but this point is not physically attainable as discussed above. For a single-shock process, the true pressure must lie on an isentrope ($S_1 > S_0$) connected to the Hugoniot $\mathcal{H}$ at pressure lower than P_b. The intersection of this isentrope S_1 with the isenthalp $I = {U_0}^2/2$ will lie between (a) and (b).

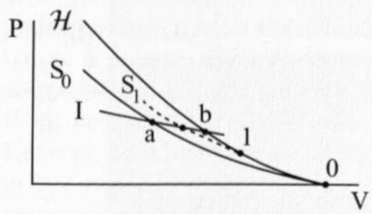

Figure 2. Bernoulli's law specifies the enthalpy $I = I_{sp}$ at the stagnation point. If the path to the stagnation point is isentropic S_0, the pressure P_a will be a maximum. If dissipation is maximized, which we think implies a single shock $\mathcal{H}$, the pressure P_b will be a minimum.

We also note that this pressure calculation is not well posed for asymmetric oblique collisions (e.g., Fig. 1) since in the stationary reference frame the initial kinetic energy of the plate and wedge are different, and so the stagnation pressures calculated from them using Bernoulli's law differ.

The computations shown in Fig. 1 are nearly self-similar (Jones et al. 1951). Under these circumstances the flow is not stationary and (1) must be modified. Instead,

$$I_{sp} + \frac{1}{2}|x_{sp}/t|^2 = I_0 + \frac{1}{2}|\vec{U}_0'|^2 - \int_{s_0}^{s_{sp}} |\vec{U}'|^2 ds \tag{2}$$

where $\vec{U}' = \vec{U} - \vec{x}/t$, and ∇' is the gradient operator with respect to $\vec{\xi} = \vec{x}/t$. The integral is taken along the integral curve defined by $d\vec{\xi}/ds = \vec{U}'(\vec{\xi}(s))$ from some initial point s_0 to the stagnation point s_{sp} (Fig. 1c). At small angles α the stagnation-point enthalpy computed from (2) is generally less than that given by (1), principally because the upstream velocities in the self-similar regime are 0 and V_{imp} whereas they are $V_{\text{imp}} \cot \alpha$ and $V_{\text{imp}} \csc \alpha$ in the steady-state regime.

The steady-state theory applied to the self-similar computations (Fig. 1) gives a stagnation pressure of 4.0 GPa or 10.0 GPa depending on whether the calculation is done for the plate or the wedge. The peak pressure in Fig. 1 is 10.2 GPa. However, in a similar computation with a 20° angle the theory gives 48.4 and 55.4 GPa whereas the numerical computation gives 27.5 GPa. At 15° the disagreement is still larger: theory gives 94.1 or 101.4 and computation gives 35.9. For the cases described the discrepancy between theory and computations is largest near the onset of jetting (13.2° with the present geometry and 2 km/s horizontal velocity) where the upstream velocity U_0 differs most from the steady-state value. Geological and planetary applications that seek to explain tektites, chondrules, and the formation of the Moon (e.g., Kieffer 1975, Melosh & Sonett 1986, McKinnon 1989, Vickery 1993) are more nearly self-similar than steady-state, and it is under such conditions that our computations suggest the commonly used theory fails most egregiously.

4. Conclusions

Oblique, asymmetric collisions lead to vortex sheets that in turn lead to Kelvin-Helmholtz instabilities. Jetting models based on the idealized symmetric geometry presented in Birkhoff et al. (1948), i.e., assuming locally smooth, linear, and stationary material interfaces, are wrong in detail and possibly lead to inaccurate conclusions.

A conclusion that is independent of our computations is that the model commonly used to compute the stagnation-point pressure is incorrect. Our computations support this result by revealing more complex wave interactions than commonly supposed, and by demonstrating poor agreement between the theoretical and the computed peak pressures. Because of this we suggest that calculations of the thermodynamic state of jetted materials by the commonly used model require reevaluation. Our numerical results suggest the discrepancy is largest at small jetting angles, with the commonly used model overestimating the stagnation pressure by as much as a factor of three in the examples given.

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