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The Anomalous Refraction of Shock Waves in Gases

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Abstract

Anomalous refraction comprises at least five refracting shock systems. All need sonic or subsonic flow downstream for their existence, which is induced by overtaking downstream disturbances, thus the limiting sonic condition determines their onset. The wave impedances are also an important factor for their existence. The theory predicts a new system (ARe) that has not yet been observed. Numerical results for all these refractions and related systems are presented and some comparison is made with experiment.

Introduction

Consider two gases, differing in composition and, or in state and meeting at a plane interface. A plane (i-) shock in the *incident* gas propagates parallel to, and towards (angle of incidence $\alpha_i = 0°$) the interface. The speed of sound in the gas is α_{0i}. The i-shock crosses the interface and enters the *receiving* gas (with speed-of sound α_{0t}) where it becomes the *transmitted* (t-) shock. In general a reflected wave which may be a shock (r-) or an expansion (e-) is sent back into the i-gas. By symmetry all the waves in the system are parallel to the interface. This phenomenon is *normal* (1-D) shock refraction.

There is (2-D) *oblique* shock refraction when $\alpha_i > 0°$ with respect to the upstream interface. If α_i is sufficiently small the refraction is *regular* i.e. the gas between any two adjacent waves has a uniform state (v, s) and speed (u), figure 1(a). Here v is specific volume and s, is entropy. If (v,s,u) are non-uniform between any wave pair, the refraction is *irregular*, figure 1(c). A regular (i, t, e) wave system (RRE) can be transformed into an irregular *anomalous refraction* (ARE) via a transitional system figure 1(b) by sufficiently increasing $\alpha_i > \alpha_i^*$. At $\alpha_i = \alpha_i^*$ the *flow* Mach number is sonic at the foot of the i -shock, $M_{i1} = 1$. For $\alpha_i > \alpha_i^*$, the shock and the e-waves are steeper and the e-waves partly overrun the shock causing attenuation of part (i′) of it, causing it to curve backwards. There is a sonic surface on the rear of i′ and a distributed band of supersonic expansions emanating downstream from it. The refraction law [3] relates the angles α_i, α_t, at the interface,

$$U_i/\sin\alpha_i = U_t/\sin\alpha_t\,, \qquad (1)$$

where U is shock speed. These angles determine the direction of flow of information. If for regular refraction $0 < \alpha_i < 90°$, and $\alpha_t > 90°$ the i-shock *arrives* at the interface and the t-shock *leaves* it [4].

If $\alpha_{0i} > \alpha_{0t}$ the refraction is *fast-slow* and vice-versa for *slow-fast*. ARE has appeared in experiments with the fast-slow combinations air / CO_2 and air / SF_6 [1] [2], figure 1(b). The results

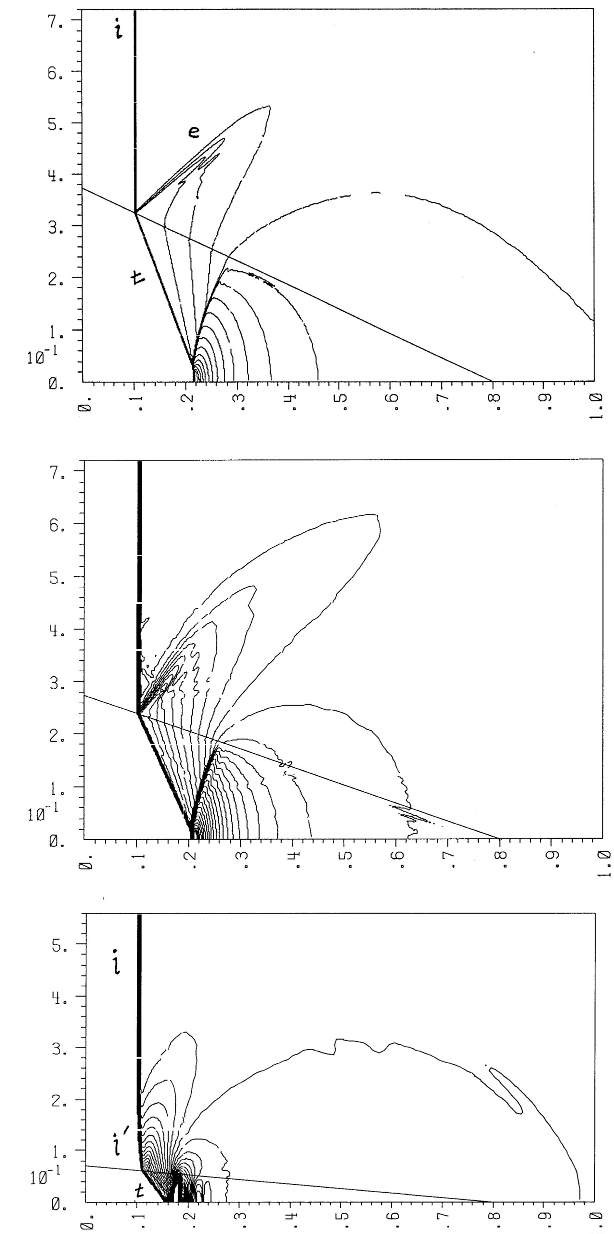

Figure 1. Computed Refracting Shock Systems in the Air-CO_2 Combination $\xi_i = 0.85$. a) Regular refraction with reflected expansion RRE; b) Transitional ; c) Anomalous refraction ARE.

of a numerical study of ARE and related refractions are reported here and in forthcoming papers [5], [6]. A second order Godonov code was used to integrate the equations of motion that included the Euler equations. Some of the ARE experiments in [1], [2], were simulated numerically.

Theory

In our earlier papers we defined the relative refractive index (η) and the incident shock impedance (Zi) for *normal* shock refraction [3], [4],

$$\eta \equiv U_i \big/ U_t \qquad (2)$$

$$Z_i \equiv \pm\left(P_1 - P_o\right)/U_{pi} \qquad (3)$$

Where $U_{pi} = u_1 - u_0$ is the shock piston speed in rest frame coordinates, P is pressure, u is particle speed, and the subscripts 0,1 refer to conditions upstream and downstream of the incident i-shock. It follows from the shock momentum equation that the impedance is also the mass flux $Z_i = \rho_0 u_0$ (ρ is density) through the shock [3]. The reflection and transmission coefficients are,

$$R \equiv \frac{\left(P_2 - P_1\right)}{\left(P_1 - P_2\right)} \equiv \frac{Z_r\left(Z_t - Z_i\right)}{Z_i\left(Z_r - Z_t\right)}, \qquad (4)$$

$$T \equiv \frac{\left(P_t - P_0\right)}{\left(P_1 - P_0\right)} \equiv \frac{Z_t\left(Z_i - Z_r\right)}{Z_i\left(Z_t - Z_r\right)} \qquad (5)$$

The R, T, expressions are also valid for oblique shock refraction provided that the impedance is defined as,

$$Z_i \equiv \pm\left(P_1 - P_0\right)/U_{pi} \cos\beta_i\,, \qquad (6)$$

where β_i is the angle between the i–shock and the *deflected* gas interface. With similar definitions for the other shocks; an e-wave requires the denominator of (6) to be replaced by an integral [3]. Regular refraction solutions can be found from the algebraic Rankine-Hugoniot equations and the equations of states (EOS's) for the gases. Solutions for irregular refractions require numerical integration of the conservation equations. For ARE the i-shock Mach number M_i is typically small $M_i \le 1.7$ so it is sufficient to use the perfect gas EOS for each gas. In the important special case when the i- and t-shocks have equal impedances $Z_i = Z_t$, corresponding to $\alpha_i = \alpha_p$, and R = 0, T = 1, there is no reflected wave and this condition is called *total transmission*. The system is reduced to a regular shock pair refraction (RSP) and both i– and t-shocks are plane. A more detailed study [6] shows that a convenient parameter space for refraction in any particular gas combination is (ξ_i, α_i) where $\xi_i \equiv P_0/P_1$ is the inverse shock strength. Each system has a unique parameter space in this plane. The domain boundaries are defined by the curves for $\alpha_i = \alpha_p$, and $\alpha_i = \alpha_i^*$; for a perfect gas the latter curve can be found as the closed form expression,

$$\sin^2\alpha_i^* = \frac{\left[(\gamma+1)+(\gamma-1)\xi_i\right]^2}{2\gamma\left[(\gamma+1)+(\gamma+1)\xi_i - 2\xi_i^2\right]} \qquad (7)$$

A map of every refraction domain for Air/CO_2 is shown in figure 2. The ARE, ARe, ARc, are anomalous refractions, the upper case E signifies supersonic/sonic flow downstream of the reflected wave, the lower case e, and c, signifies subsonic expansion and compression respectively. RRR, RRE, refer to regular refraction with either a reflected shock or expansion, while MRR is an irregular Mach reflection refraction [4]. The ARE, ARe, and RRE, systems occur only for $\alpha_i > \alpha_p$, or $Z_i > Z_t$ the ARc, RRR, and MRR, only for $\alpha_i < \alpha_p$, or $Z_i < Z_t$. It is shown elsewhere [6] that the AR boundary (7) may be used with some modification to predict the ARc ⇔ RRR transition.

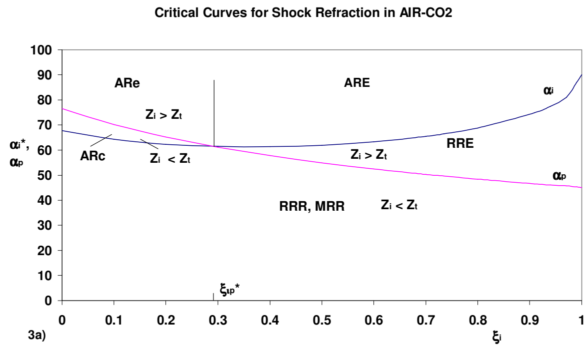

Figure 2. Critical Curves for Air-CO_2 for Anomalous Refraction and Equality of Impedance.

Numerical Results

Returning to figure 1 it shows a transition sequence RRE ⇔ ARE all for $\xi_i = 0.85$ and $\alpha_i > \alpha_p$ The RRE where $\alpha_p < 69° < \alpha_i^*$ has a reflected Prandtl-Meyer expansion. The transitional system at $\alpha_i^* = 71.116°$ has sonic flow at the foot of the i-shock, followed by the P-M expansion. For ARE where α_i = 85° > α_i* some of the P-M waves overrun the i-shock producing the attenuated and curved i'-shock. But a remnant of the P-M wave still exists, which is contrary to the Jahn model. This remnant has been detected in experiment [2]. The ARE ⇔ ARe transition occurs when the remnant vanishes, so (7) applies at the foot of the i-shock. Figure 3 shows an ARc, an RSP, and an ARe all with $\xi_i = 0.1$. The disturbed part of the i–shock in the ARc leans slightly forward near the interface as it is overtaken by subsonic compressions; $Z_i > Z_t$, $\alpha_i^* < 67.24° < \alpha_i$. The RSP where $\alpha_i = \alpha_p = 70.2°$, and $Z_i = Z_t$, is the transition system ARc ⇔ ARe. The disturbed part of the i–shock in the ARe, where $\alpha_i = 85°$, $Z_i > Z_t$, leans backwards as it is overtaken by subsonic expansions. The flow is subsonic downstream of an ARe, and sonic / supersonic for an ARE. The transition ARe ⇔ ARE is at the sonic condition ξ^* figure 2.

Anomalous Refraction at a Slow-Fast Interface

In this case a regular refraction may disintegrate during transition to an irregular system. Specifically the t-shock moves ahead of the refraction point (node) and becomes a free precursor shock. These refractions have been observed by experiment for the Air/CH_4, and CO_2/He interfaces [1], [7]. The refraction law (1) is violated, along the interface,

$$U_t/\sin\alpha_t > U_i/\sin\alpha_i \qquad (8)$$

In node fixed coordinates there is a streamline that coincides with the interface, and at this point it is deflected through an angle that exceeds an angle analogous to the detachment angle for the t-shock. This implies that a necessary condition for the onset of a t-precursor is that the flow downstream of it is either subsonic or at *most sonic* [6], which once more allows the application of the AR equation (7). Experiment show that for such a wave $\alpha_t > 90°$ so the t-wave now *arrives* at the interface and it transmits a side (s-) shock back into the initial gas which interacts with the i-shock. Numerical results agree with experiment [4]. But this means that the t-shock is undergoing *fast-slow* refraction; the t-s shock pair may be either a regular (RSP) or an irregular pair (ISP) refraction [5]. The latter paper has a detailed comparison of numerical results with experiments for the CO_2/CH_4 combination. There are no disturbances ahead of the t-s pair so they can only be generated by the downstream shattering of the shock tube diaphragm. Thus a necessary condition for the existence of a t-precursor is for its downstream flow to be subsonic or at most sonic. At the sonic limit the AR condition (7) can again be applied.

Evanescent Precursor Waves

The α_t data from experiment shows that it is possible for a precursor t-shock to change into a distributed (evanescent) band of compressions. This also forces the s-wave to be also evanescent. It is assumed that every wavelet moves at the local speed of sound. Then for the leading wavelet $U_t = a_{0t}$ *and* $\alpha_t = \nu_t$, where ν_t is the Mach angle, and so by (8),

$$a_t/\sin\nu_t \geq U_i/\sin\alpha_i, \Rightarrow a_t \geq U_i \sin\nu_t/\sin\alpha_i. \quad (9)$$

Thus an acoustic wave in the receiving gas travels at an equal or greater speed along the interface than the i–shock does in the incident gas. The equality defines the boundary between a shock and an evanescent wave,

$$\sin\alpha^*_{sc} \equiv U_i \sin\nu_t/a_t. \quad (10)$$

This relation is more general than others given in literature [6]. The refraction law and the equality of impedance condition for the leading t-s pair gives respectively, $a_t/\sin\nu_t = a_i/\sin\nu_i$ and $\rho_t a_t/\cos\nu_t = \rho_i a_i/\cos\nu_s$.

They are valid for any wavelet pair in the t-s band. Figure 4 shows a computed evanescent precursor t-s wave system in a CO_2-CH_4 gas combination with $a_i = 60°$, $\xi_i = 0.78$.

Conclusions

Anomalous refraction occurs for both the slow-fast and fast-slow gas combinations. It is necessary for AR that $\alpha_i > \alpha_i^*$. There are three AR systems for the fast-slow combination, ARE, ARc, ARe. The parameter spaces for their existence are shown in figure 2 for Air/CO_2. ARE and ARc and they have been observed in the cited experiments, but ARe is a new system predicted by the theory that has not yet been observed. Numerical evidence for all three systems in the corresponding parameter spaces of figure 2 are presented in figures 1, 3. The model proposed by Jahn becomes correct by the addition of a centered reflected expansion wave. There are two AR systems for the slow-fast combination, one with a precursor t-shock and the other where the precursor t-wave is evanescent.

The onset of all five AR systems is caused by disturbances arising downstream and overrunning a pre-existing system. The AR criterion (7) corresponds to sonic flow downstream, but if the flow becomes subsonic then a pre-existing system can be overrun.

The numerical data supports the theory and we found no counter examples to the theory either in the numerical or experimental results.

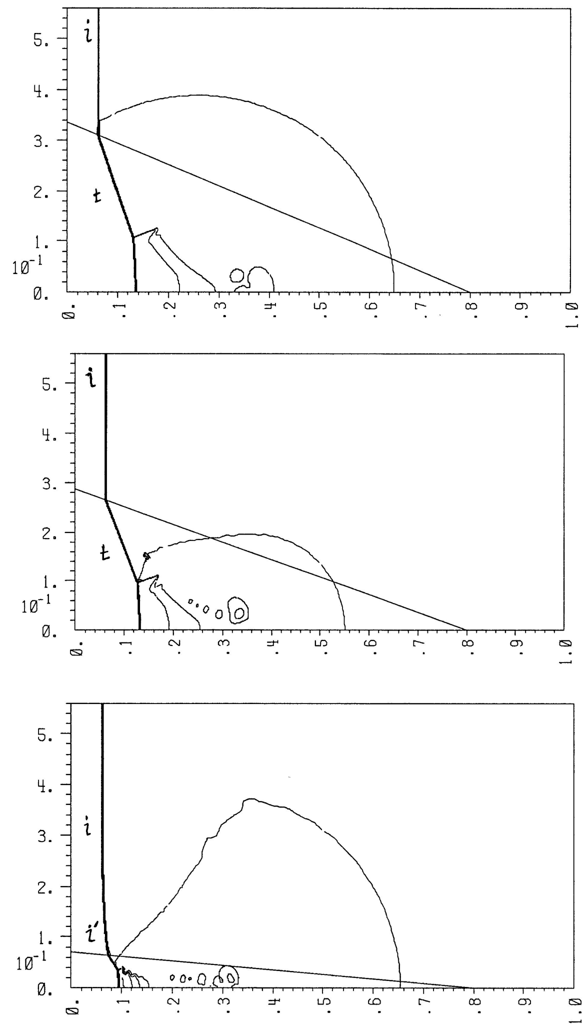

Figure 3. Computed Refracting Shock Systems in the Air-CO_2 Combination $\xi_i = 0.1$. a) ARc; b) RSP ; c) Are

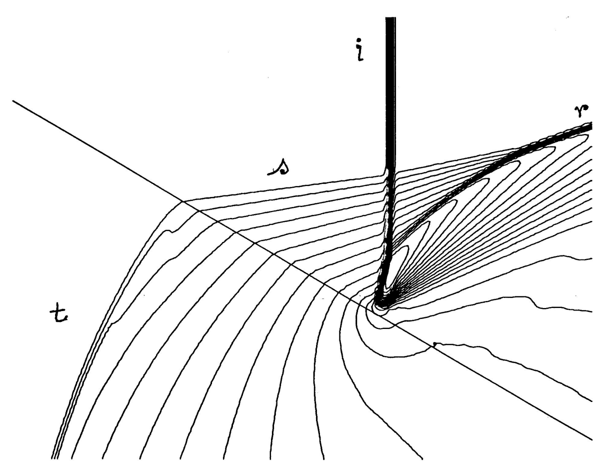

Figure 4. Free Precursor Irregular Refraction in the CO_2-CH_4 Combination $\xi_i = 0.78$. a) Regular refraction with reflected expansion RRE; b) Transitional; c) Anomalous refraction ARE.

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