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A Local Discontinuous Galerkin Method for Stable Advection of non-Diffusive Fields in Computational Geodynamics

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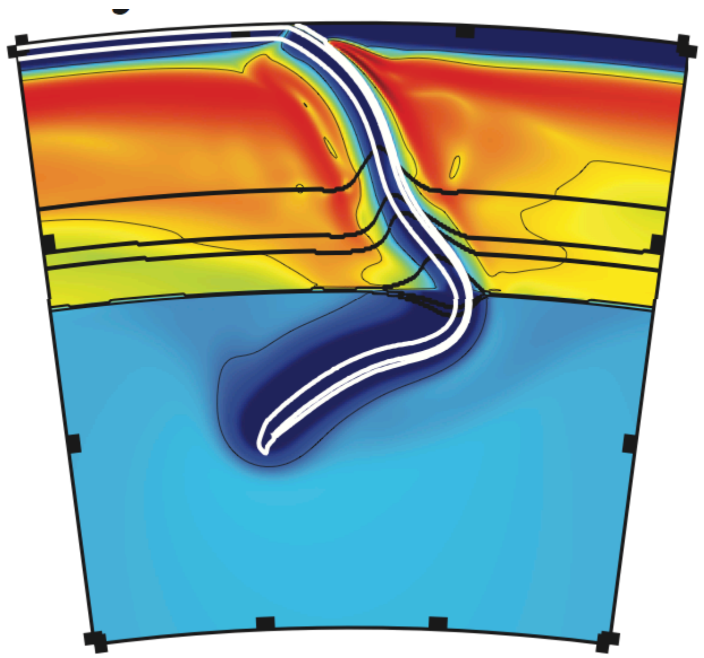
Dec. 16th, 2015 AGU Fall Meeting @ San Francisco, CA

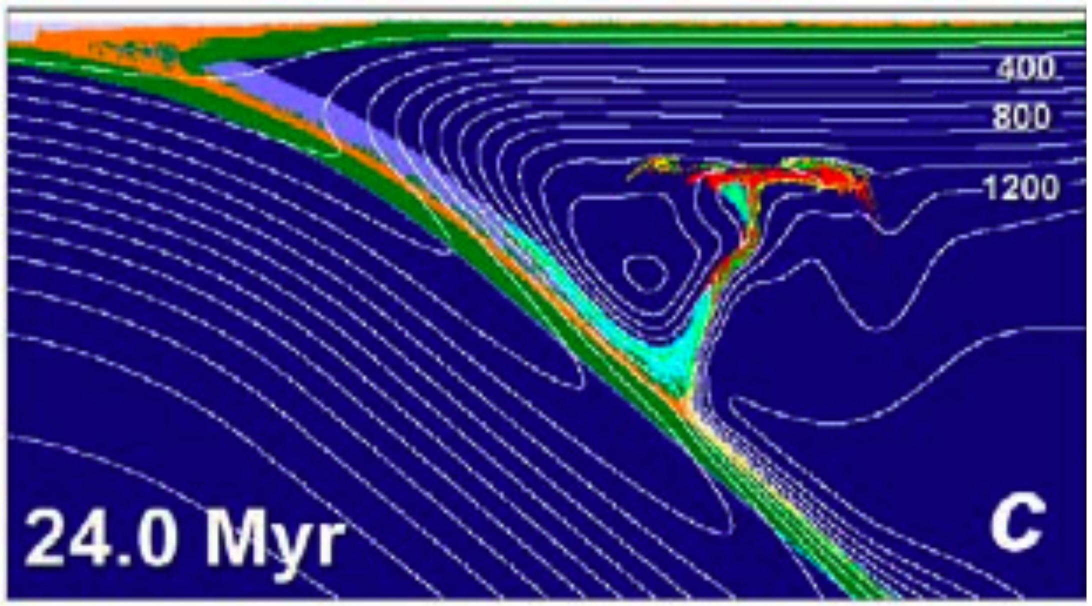
Geodynamics Simulations Need to Track Compositions

Crustal layers for slab dynamics



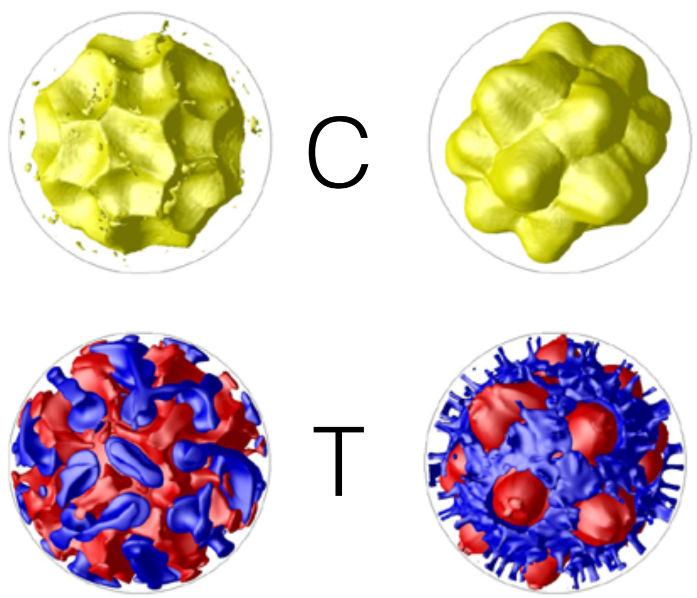
Arredondo & Billen, J. Geodyn. submitted

Sediments, water, melts
for mantle wedge dynamics



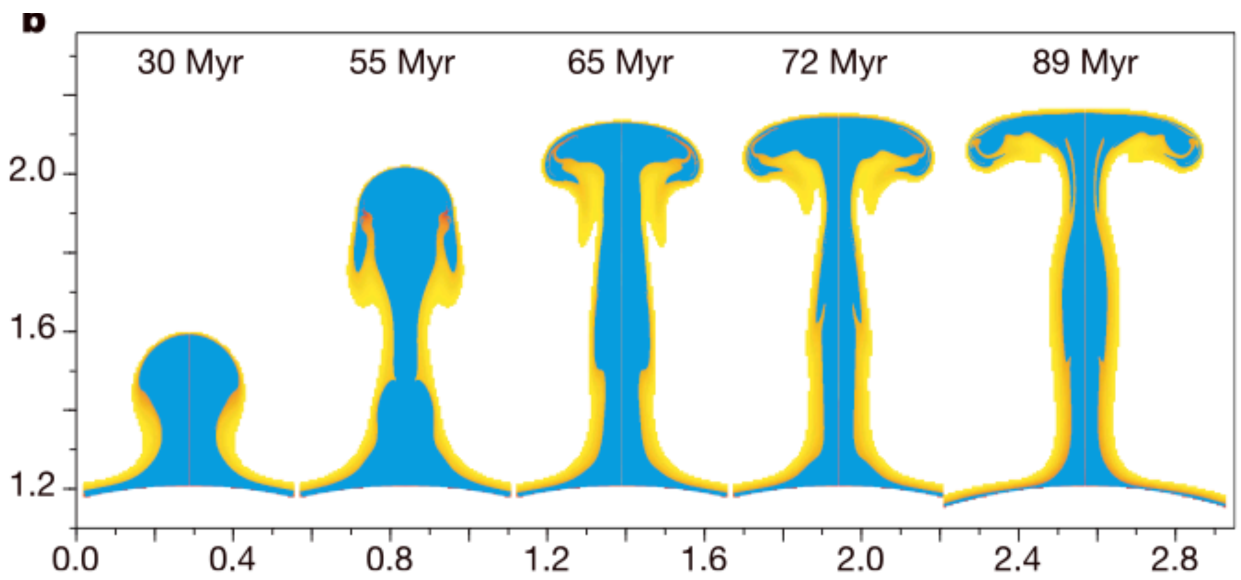
Gerya & Yuen, EPSL., 2003

Chemical piles for deep mantle dynamics



McNamara & Zhong, JGR, 2004

Chemical entrainment for plume dynamics



Lin & van Keken, Nature, 2005

Mathematical Modeling

$$-\nabla \cdot (2\eta(C)\epsilon(\mathbf{u})) + \nabla \mathbf{p} = \rho(C)\mathbf{g}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{C} = 0, \quad (3)$$

Numerical Challenges

Multi-scale problem

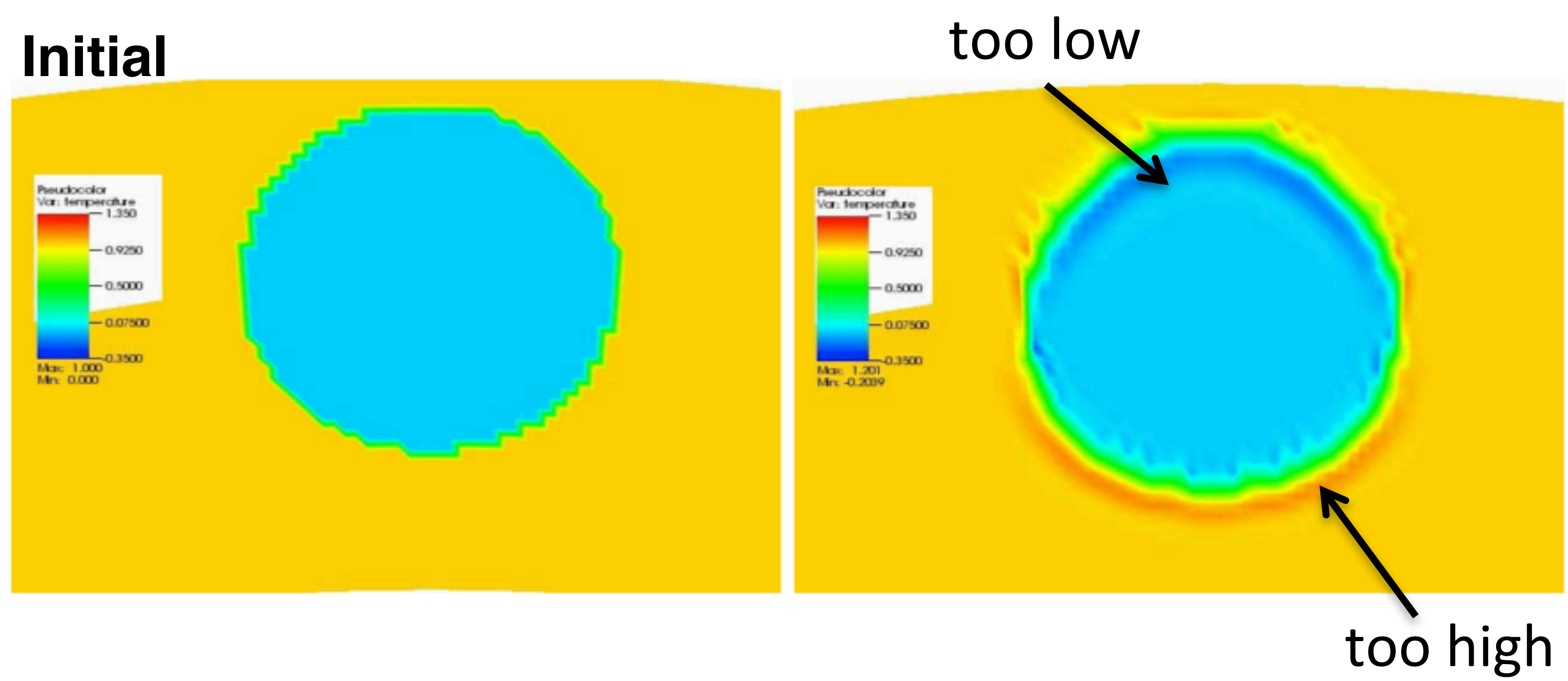
Nonlinear system

Strong discontinuity: composition field, viscosity and density

Overshoot/undershoot: bound preserving, numerical instability

Various Methods: Each Has Pros & Cons

- Tracers
  - Pros: separate from solver
  - Cons: expensive (lots of tracers)
- Field Method
  - Pros: cheaper
  - Cons: needs solver – usually get either overshoots or excessive smoothing.



Example of temperature overshoot for falling sphere from CitcomS

Overview of this Study

Goal:

Develop a stable, accurate, and efficient mixed CG-DG finite element method for composition advection equation problems of mantle dynamics

Approach:

Apply Local Discontinuous Galerkin (LDG) Method to composition advection equation with a bound preserving limiter (Zhang & Shu 2010, 2013).

Apply CG Finite Element Method (FEM) to Stokes equation for the velocity field.

Use Adaptive Mesh Refinement technique.

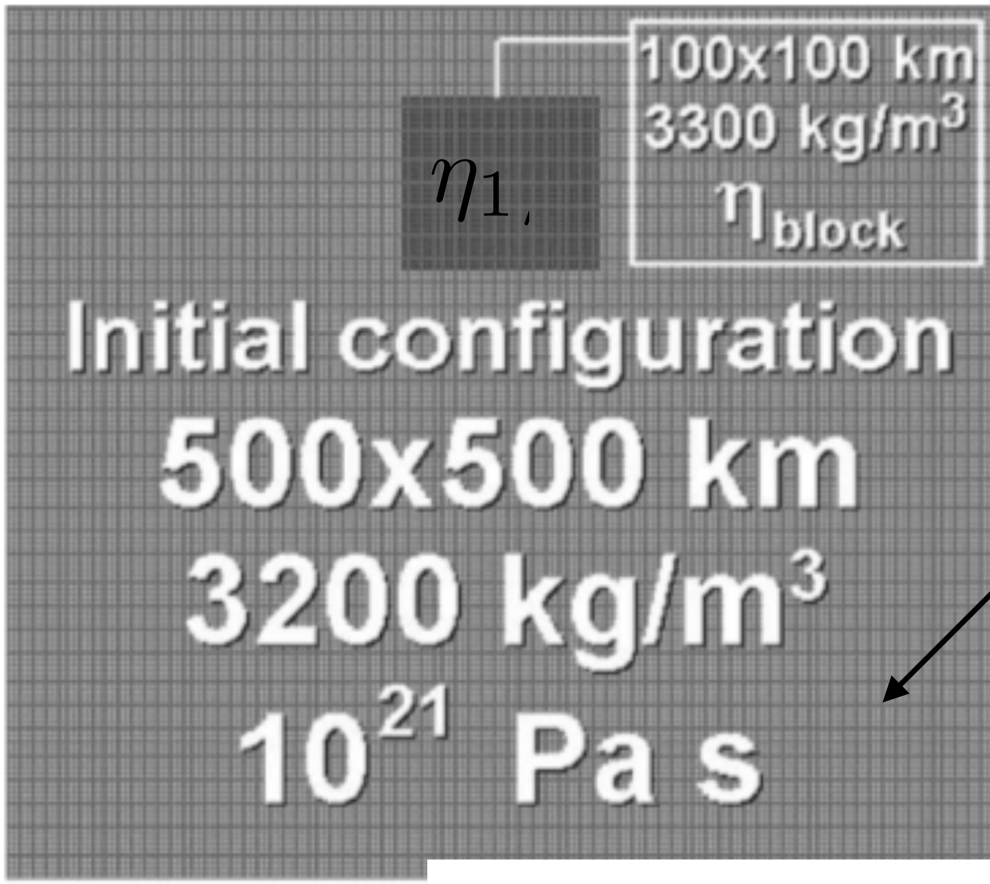
Codes implementation in deal.ii.

Test case:

Use falling box benchmark and compare to the FEM with Entropy Viscosity Stabilization (EVS) (J-L Guermond 2011). Tests run in ASPECT and deal.ii.

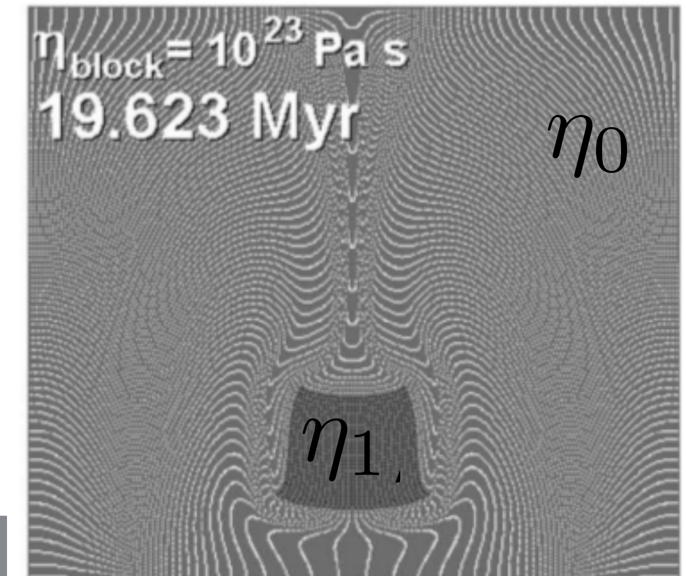
Test example: sinking hard box problem

T. V. Gerya, D. A. Yuen 2003

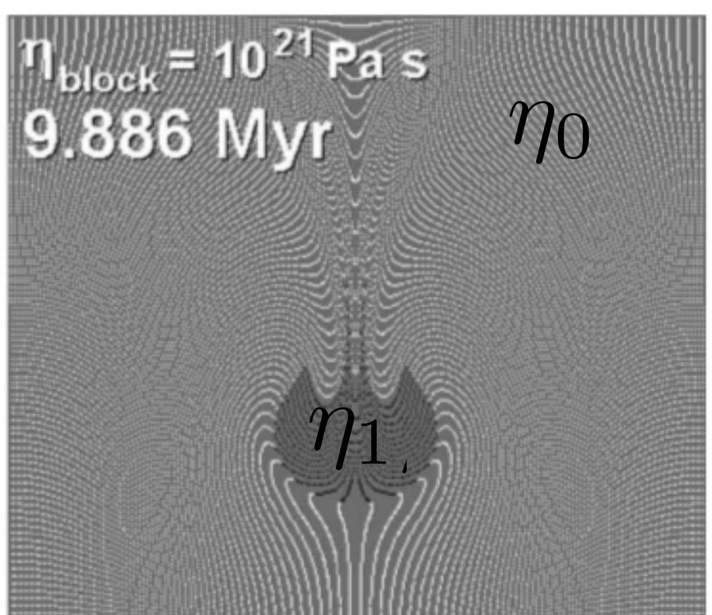


$$\eta_0 = 10^{21} Pa \cdot s$$

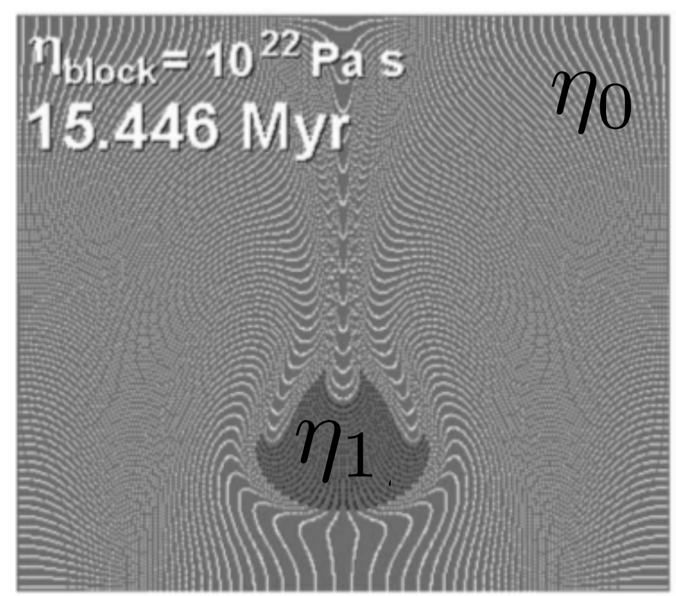
Markers' position only



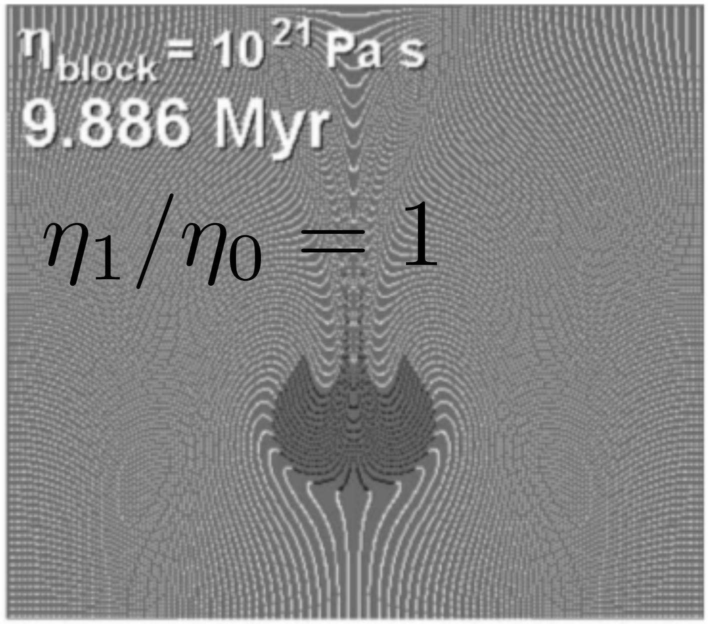
$\eta_1/\eta_0 = 100$



$\eta_1/\eta_0 = 1$



$\eta_1/\eta_0 = 10$

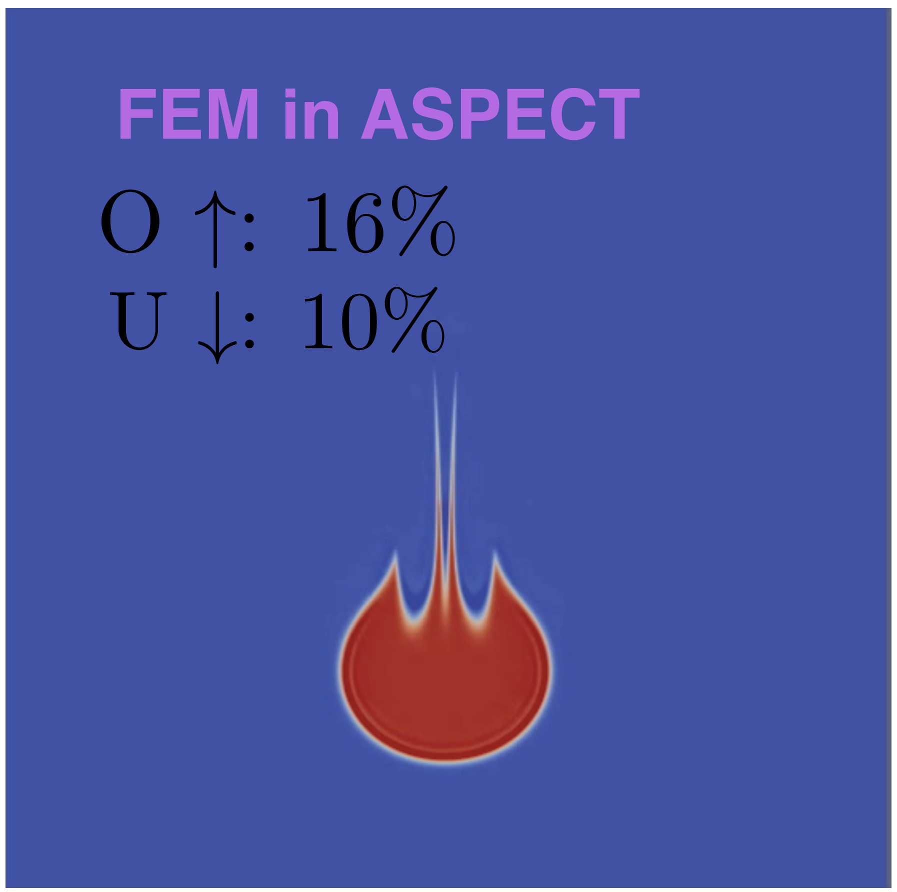


FEM in ASPECT with default entropy viscosity parameters

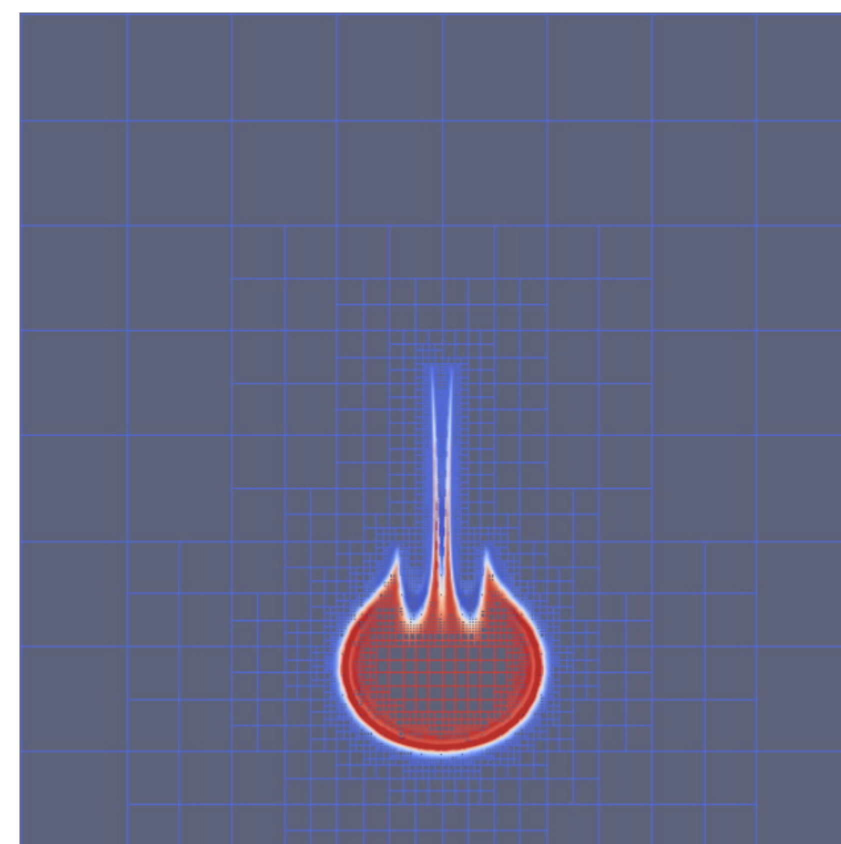
Large overshoot 16% & undershoot 10%

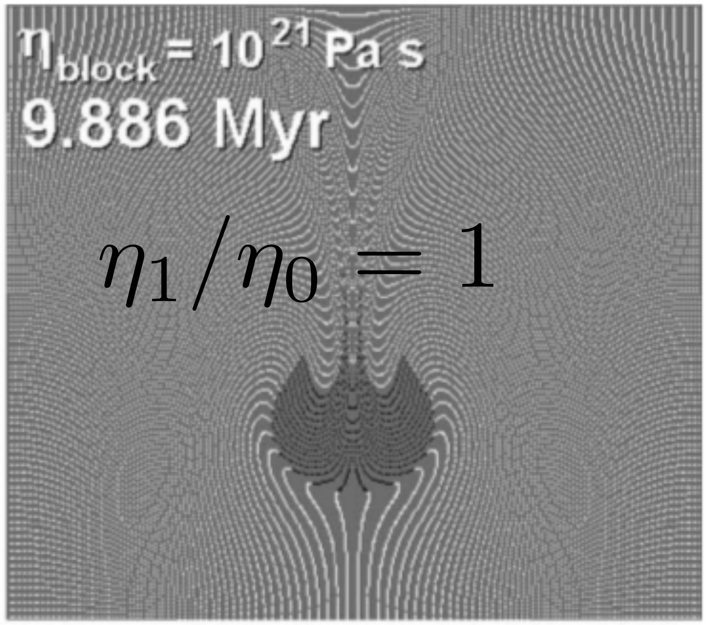
#cells: 9k
#DoFs(C): 39k
#DoFs(u+p): 88k

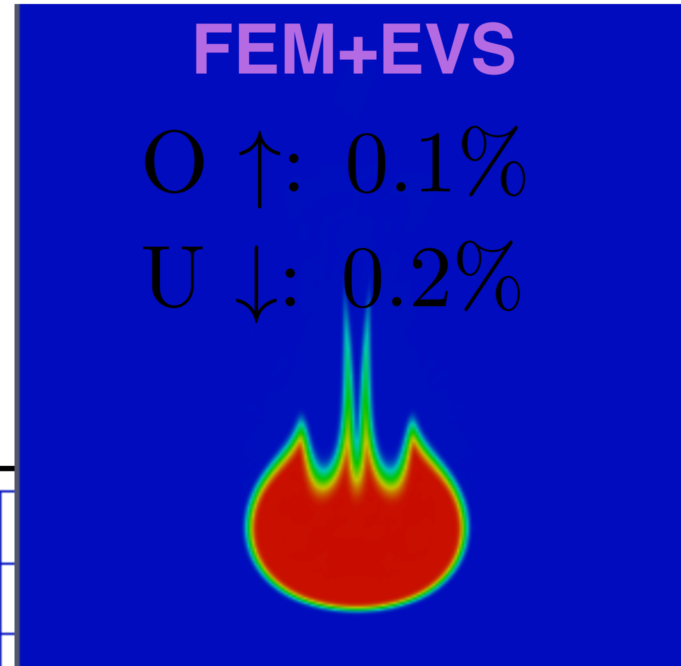
Less
number of cells
degrees of freedom



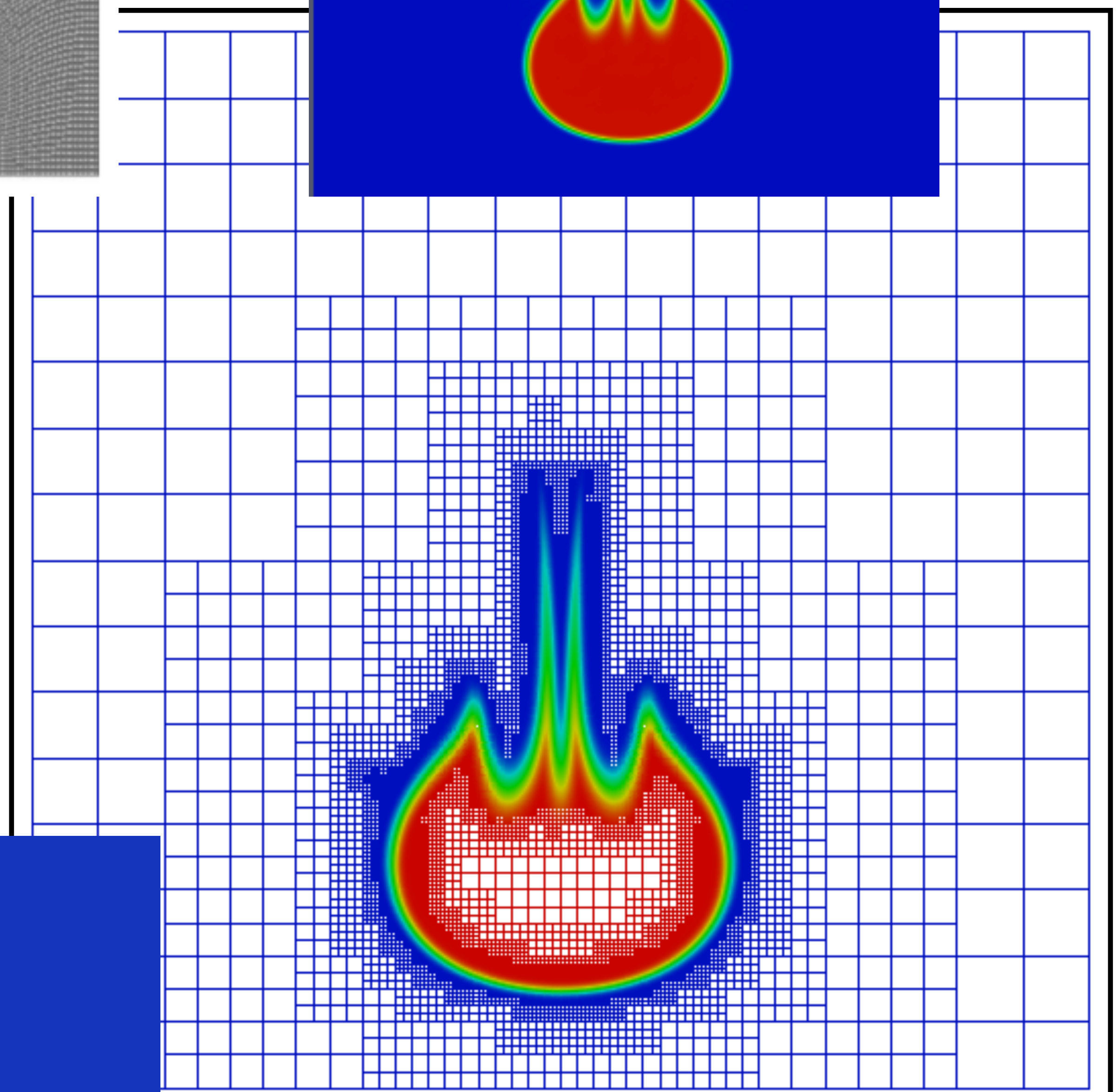
Stabilization?
Need further verification…

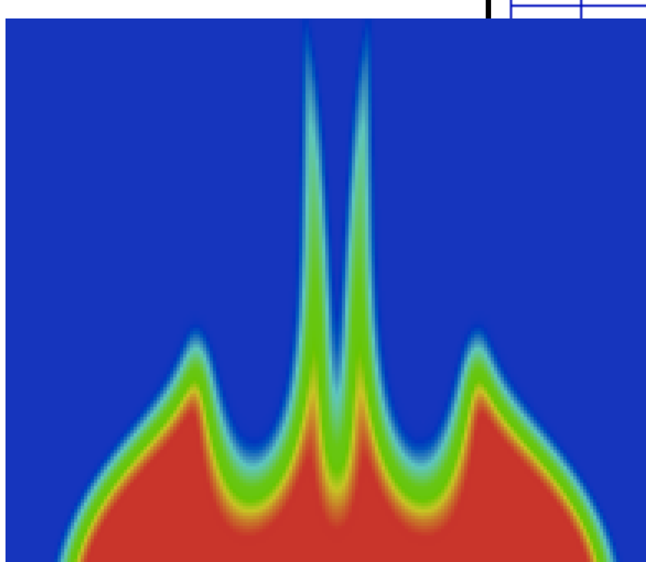






FEM with Entropy Viscosity Stabilization (EVS) in deal.ii



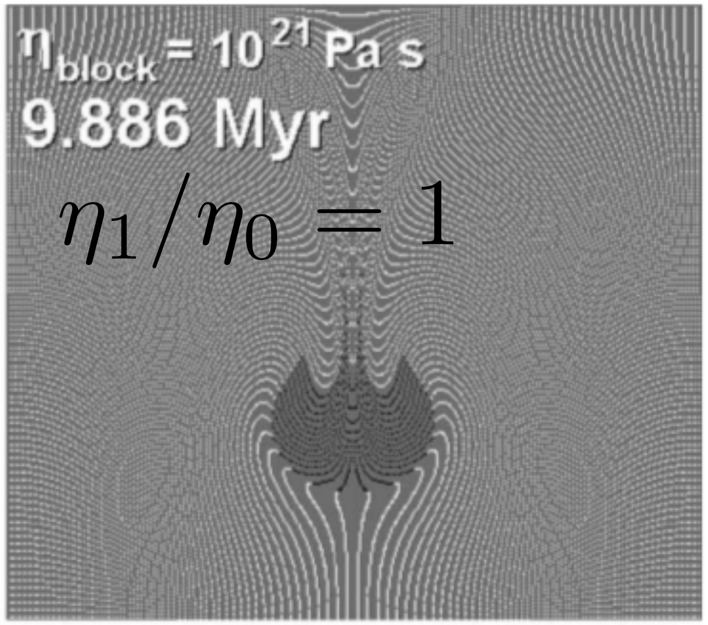


Less
overshoot 0.1%
undershoot 0.2%

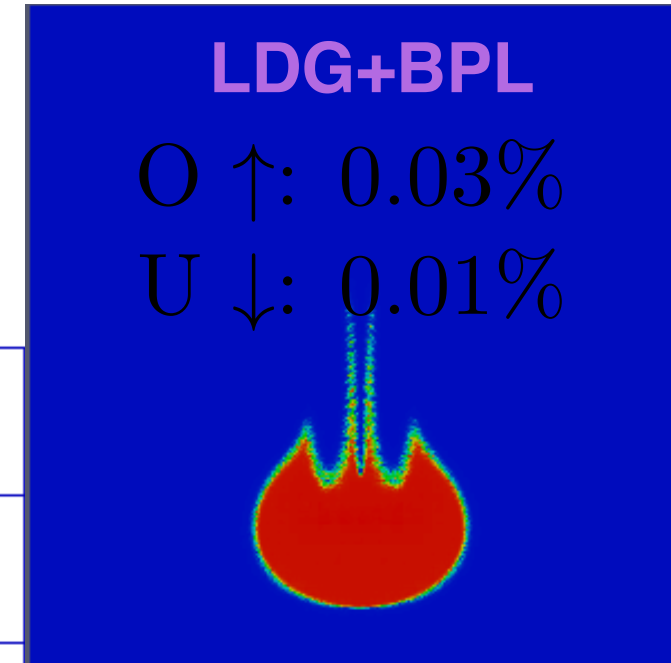
#cells: 30k
#DoFs(C): 125k
#DoFs(u+p): 283k

More
number of cells
degrees of freedom

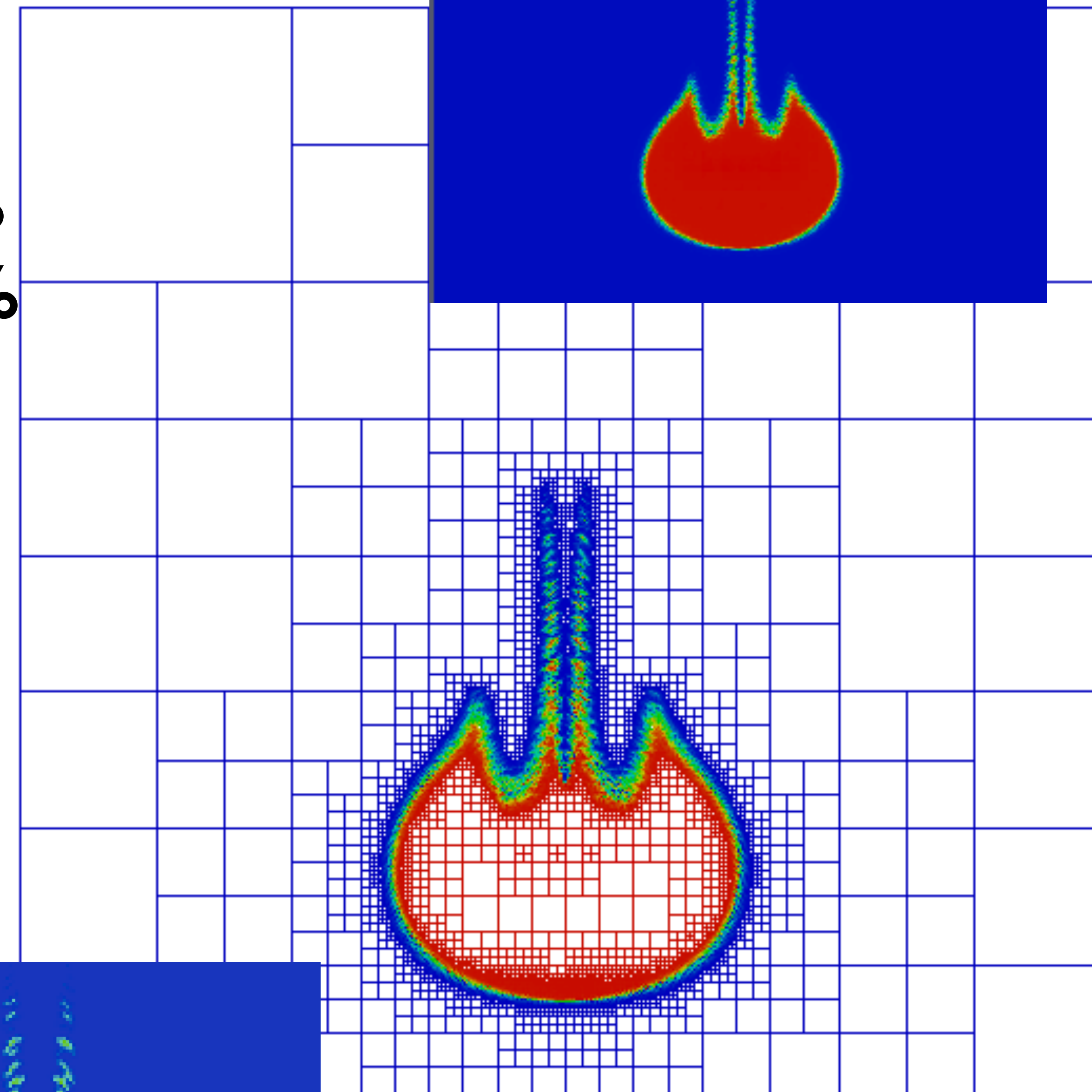
Control overshot/undershoot by EVS
but lose sharp boundary



LDG with bound preserving limiter in deal.ii



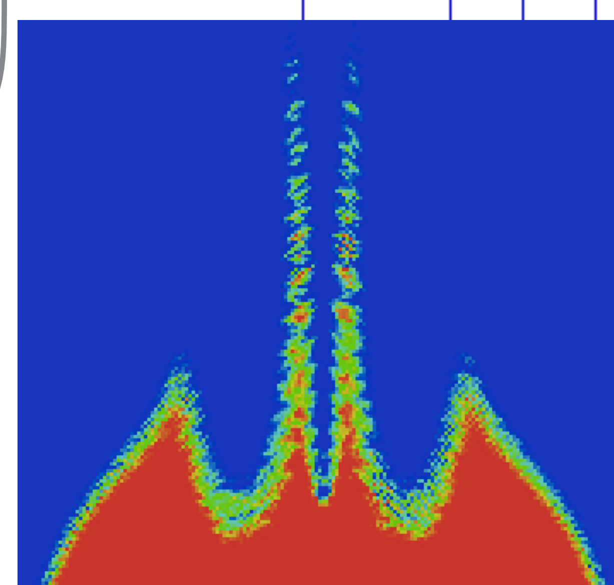
Less
overshoot 0.1%
undershoot 0.2%

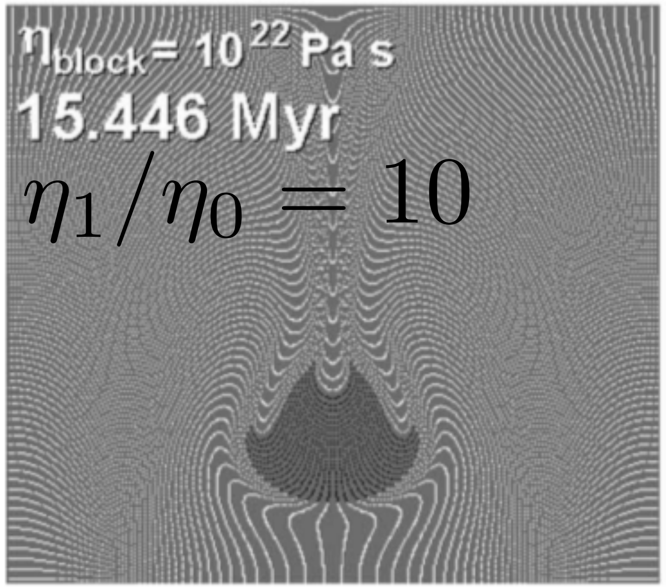


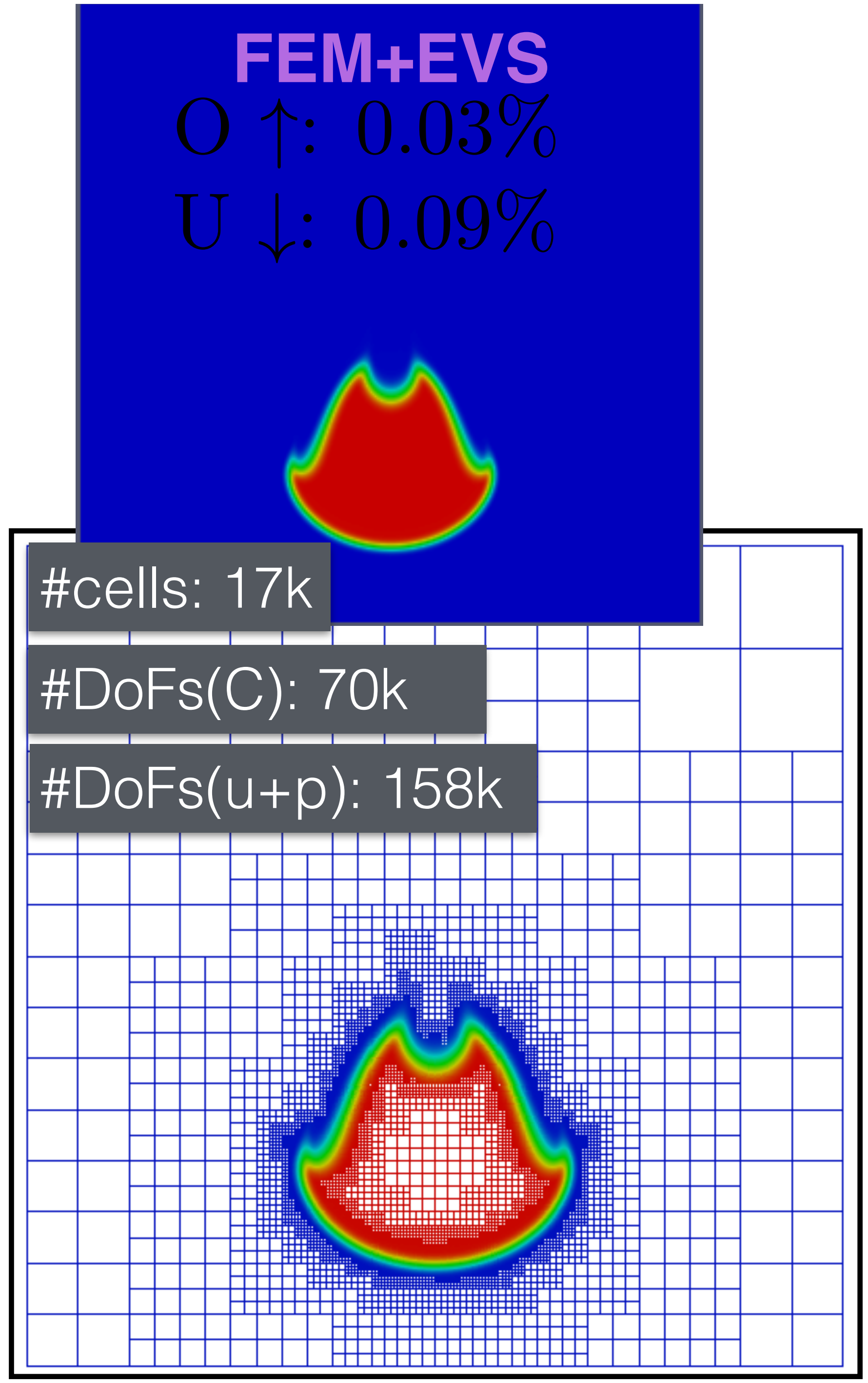
#cells: 19k
#DoFs(C): 167k
#DoFs(u+p): 178k

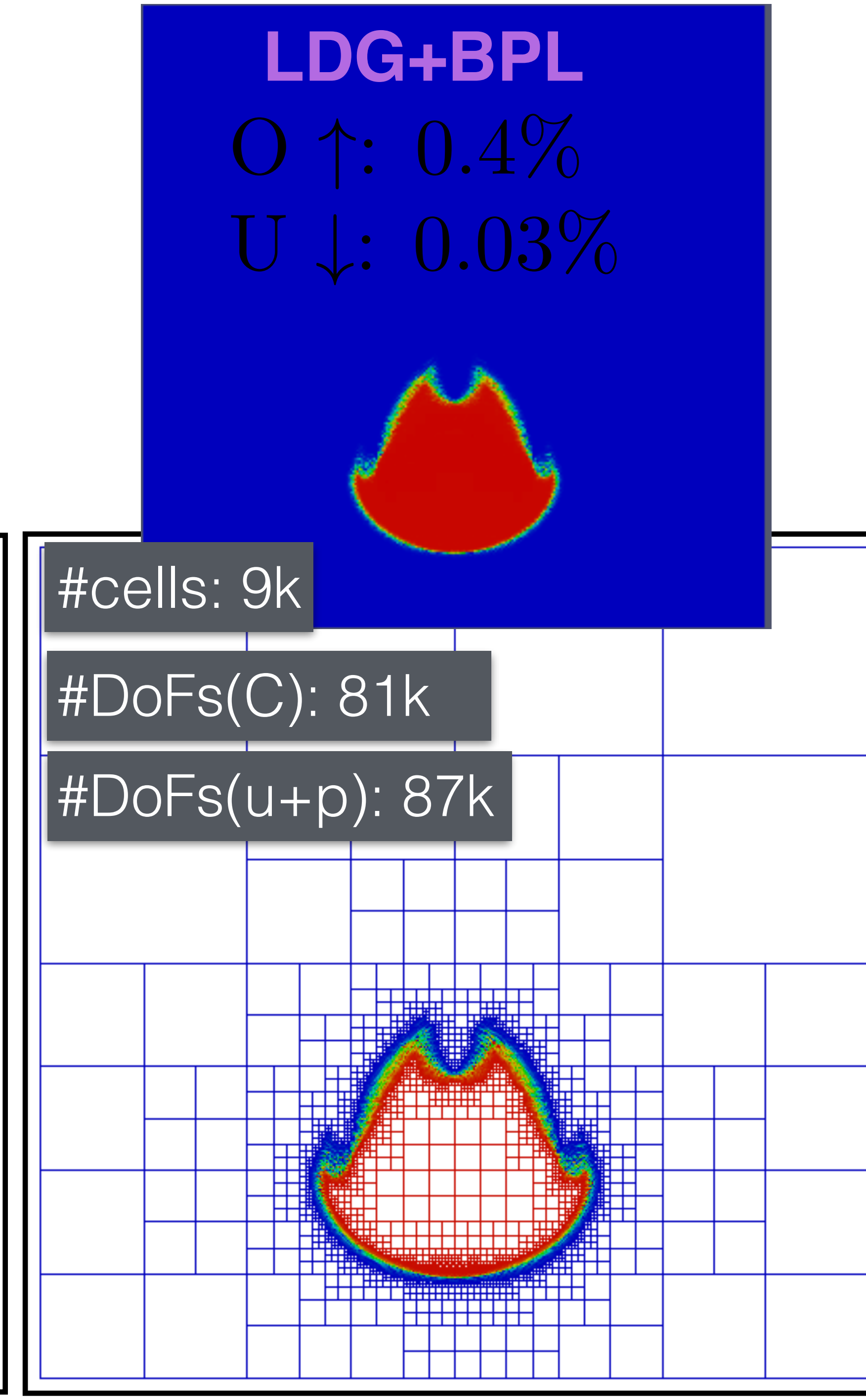
Less
number of cells
degrees of freedom

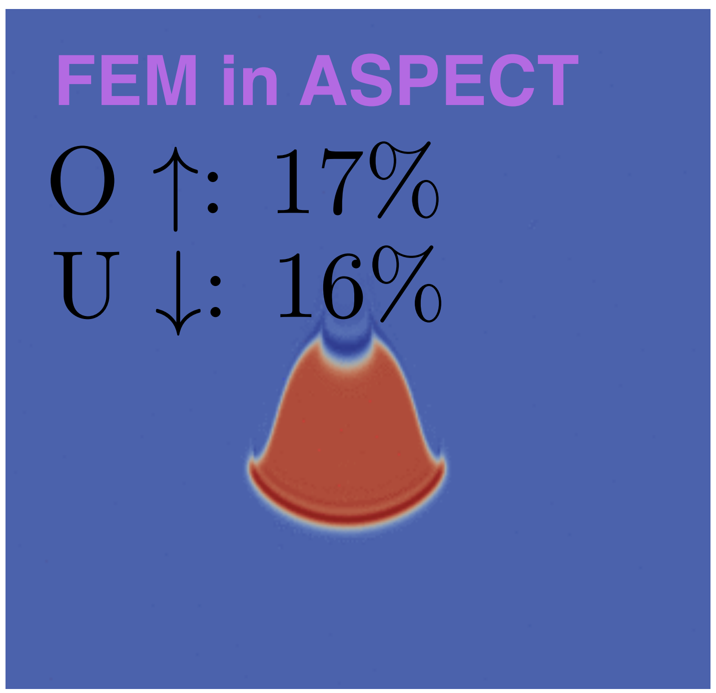
Control overshot/undershoot by BPL
better sharp boundary

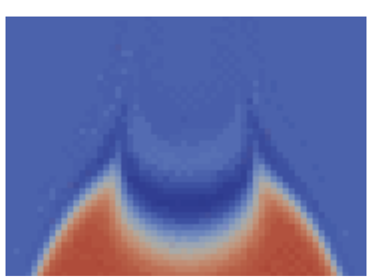


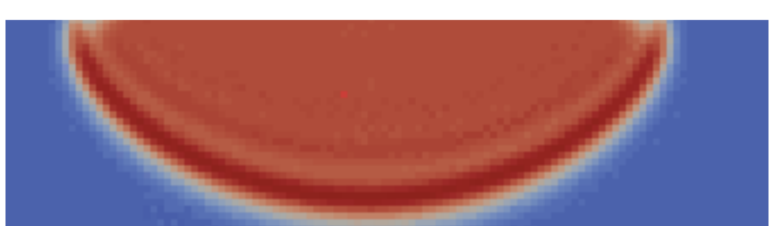

η_{block}= 10^{22} Pa s
15.446 Myr
$\eta_1 / \eta_0 = 10$

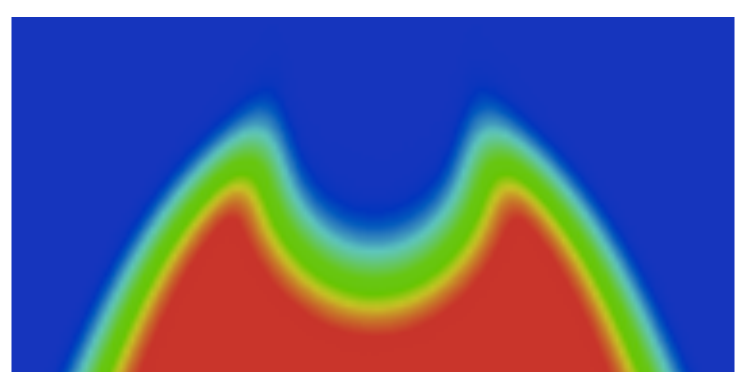

FEM+EVS
O ↑: 0.03%
U ↓: 0.09%
#cells: 17k
#DoFs(C): 70k
#DoFs(u+p): 158k

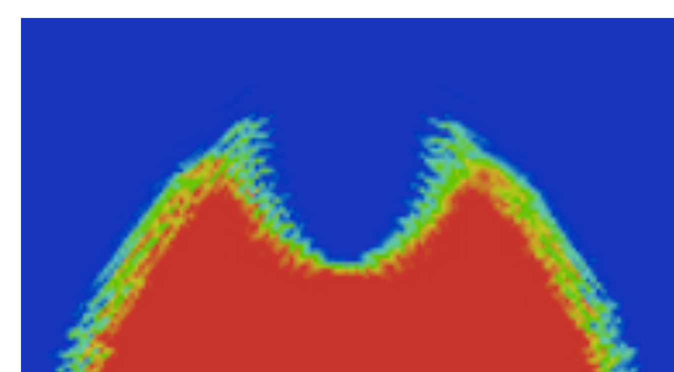

LDG+BPL
O ↑: 0.4%
U ↓: 0.03%
#cells: 9k
#DoFs(C): 81k
#DoFs(u+p): 87k

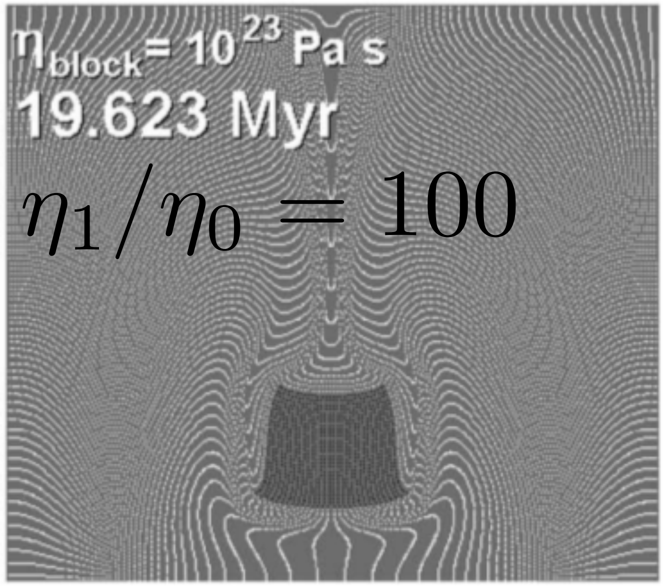

FEM in ASPECT
O ↑: 17%
U ↓: 16%



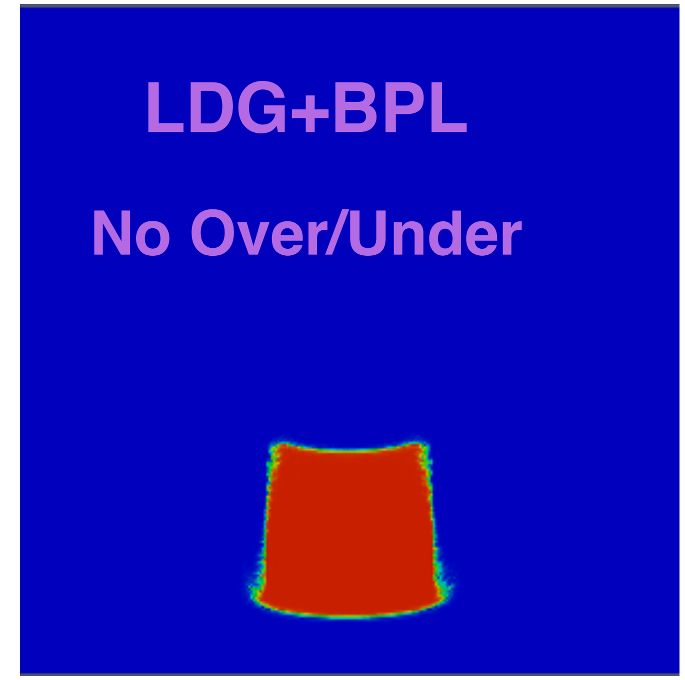


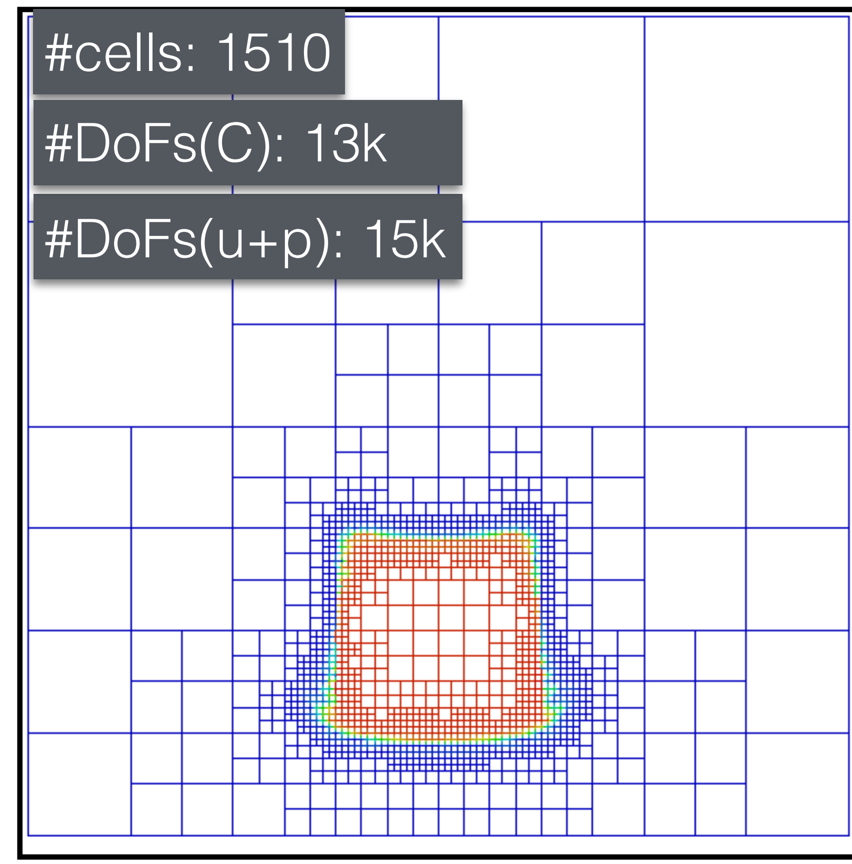






FEM+EVS
failed to converge





FEM in ASPECT
overshoot >20%

Simple & cheaper preconditioner

better & expensive preconditioner

LDG+BPL
convergent limited by mesh size

Future work: preconditioner studies with stabilization

Conclusion and future work

Developed a stable, accurate, and efficient method for compositional field advection equation: LDG+bound preserving limiter.

Numerical results demonstrated that the proposed numerical method reduces the #cells and #DoFs significantly when AMR is used.

To study more advanced preconditioner for Stokes Solver with LDG method.

Stabilization + discrete bound preserving limiter is necessary: apply entropy viscosity + limiter (J-L Guermond 2014, 2015) to the FEM approach. Make the comparison again.

Method to be added in ASPECT to be accessible for geodynamics community.