New Numerical Approaches to Thermal Convection in a Compositionally Stratified Fluid

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Background

A major unresolved question in geodynamics is whether mantle convection consists of a single layer or two separate layers. Seismic studies have shown deep penetration of subducted lithosphere which favors whole mantle convection. However isotopic studies provide strong evidence for a well mixed upper mantle but a lower mantle reservoir containing primordial material and unmixed subducted material [5].

Recent studies utilizing seismic imaging [1,3,8] have revealed large and small scale heterogeneities in the lower mantle: specifically structures known as large low shear velocity provinces (LLSVP) below Africa and the South Pacific. Most interpretations propose that the heterogeneities are compositional in nature, differing from the overlying mantle, an interpretation that would be consistent with chemical geodynamic models. The LLSVP's are thought to be very old, meaning they have persisted throughout much of Earth's history. Numerical modeling of persistent compositional interfaces present challenges to even state-of-the-art numerical methodology. It is extremely difficult to maintain sharp composition boundaries which migrate and distort with time dependent fingering without compositional diffusion and/or artificial diffusion. The compositional boundary must persist indefinitely.

A number of studies of the role of compositional discontinuities on mantle convection have been carried out. Numerical studies include Montague and Kellogg (2000), McNamara and Zhong (2004), Tan and Guruis (2005) and Galsa et al (2015) and Davaille (1999) carried out an extensive laboratory study. As mentioned above a major difficulty in the numerical studies is analyzing the motion of a sharp discontinuity between immiscible fluids with different densities. Numerical methods introduce numerical artificial diffusion that blurs the sharp boundaries. In this paper, we discuss alternative methods of addressing this problem.

Our Problem

In order to study alternative numerical methods we will consider a problem that emphasizes the role of a density difference on thermal convection. We consider a two-dimensional flow in a horizontal fluid layer with a thickness d. The region $0 \leq y < \frac{d}{2}$ has a density $\rho_0 - {}^{\Delta\rho}/_2$ and the region $-\frac{d}{2} \leq y < 0$ has a density $\rho_0 + {}^{\Delta\rho}/_2$ $(\Delta\rho \ll \rho_0)$. The upper boundary has a temperature T_0 and the lower boundary has a temperature T_1. The width of the layer is taken to be $3d$.

This is the superposition of a Rayleigh-Taylor problem and a Rayleigh-Bernard problem. In the isothermal limit $(T_0 = T_1)$ it is the classic Rayleigh-Taylor problem (pp. 285-289 of [10]). If $\Delta\rho$ is positive, a light fluid is above the heavy fluid and in a downward gravity field the fluid layer is stable. If $\Delta\rho$ is negative, a heavy fluid lies over a light fluid and the layer is unstable. Flows will transfer the heavy fluid to the lower half and the light fluid to the upper half. The density layer will overturn.

We assume infinite Prandtl number (very viscous) and the Boussinesq approximation (small density changes). Thermal convection in the layer is controlled by the Rayleigh number (pp. 309-313 of [10])

$$Ra = \frac{\rho_0 g \alpha (T_1 - T_0) d^3}{\mu\kappa} \qquad (1)$$

Where g is the acceleration of gravity, α is the coefficient of thermal expansion, μ is the viscosity, and κ is the thermal diffusivity. The maximum density difference due to thermal expansion is $\rho_0\alpha(T_1 - T_0)$, the density difference due to composition is $\Delta\rho$. The ratio of these density differences is the buoyance ratio

$$B = \frac{\Delta\rho}{\rho_0 \alpha (T_1 - T_0)} \qquad (2)$$

If B is small, the density difference boundary will not block the flow driven by thermal convection. Kinematic mixing will occur and the composition will homogenize so that the density is constant. Whole layer convection will occur.

If B is large, the density difference boundary will block the flow driven by thermal convection. The compositional boundary will be displaced vertically but will remain intact. Layered convection will occur with the compositional boundary, the boundary between the convecting layers. The Rayleigh number defined in Eq. (1) is based on the thickness d. This is the case for which we will give numerical solutions. We also assume free slip velocity boundary conditions on horizontal boundaries and reflective boundary conditions on the side walls.

Numerical Results

We carry out numerical solutions for a representative case in which two-layer thermal convection is stable. In these computations we compare three numerical approaches to modeling the movement of two distinct, thermally driven, compositional fields; namely, a high-order Finite Element Method (FEM) that employs artificial viscosity to preserve the maximum and minimum values of the compositional field, a Discontinuous Galerkin (DG) method with a Bound Preserving (BP) limiter, and a Volume-of-Fluid (VOF) interface tracking algorithm. Our results are given in Figure 1. We give color coded plots of the temperature field, density distribution, and composition distribution. There are three convection cells in our results. Ascending flows in the lower layer impinge on the descending flows in the upper layer. A well defined thermal boundary layer transports heat across the compositional boundary at $y = 0$. Since the fluids in the two layers are immiscible, they cannot mix. A valid solution should be one composition in the upper layer and the other composition in the lower layer. Our computations demonstrate that the FEM approach has far too much numerical diffusion to yield meaningful results, the DGBP method yields much better results but with small amounts of each compositional field being (numerically) entrained within the other compositional field, while the VOF method maintains a sharp interface between the two compositions throughout the computation. In the figure, we show a comparison of between the three methods for a computation made with $B = 1.111$ and $Ra = 10,000$ after the flow has reached 'steady state'. (R) the images computed with the standard FEM method (with artificial viscosity), (C) the images computed with the DGBP method (with no artificial viscosity or diffusion due to discretization errors) and (L) the images computed with the VOF algorithm.

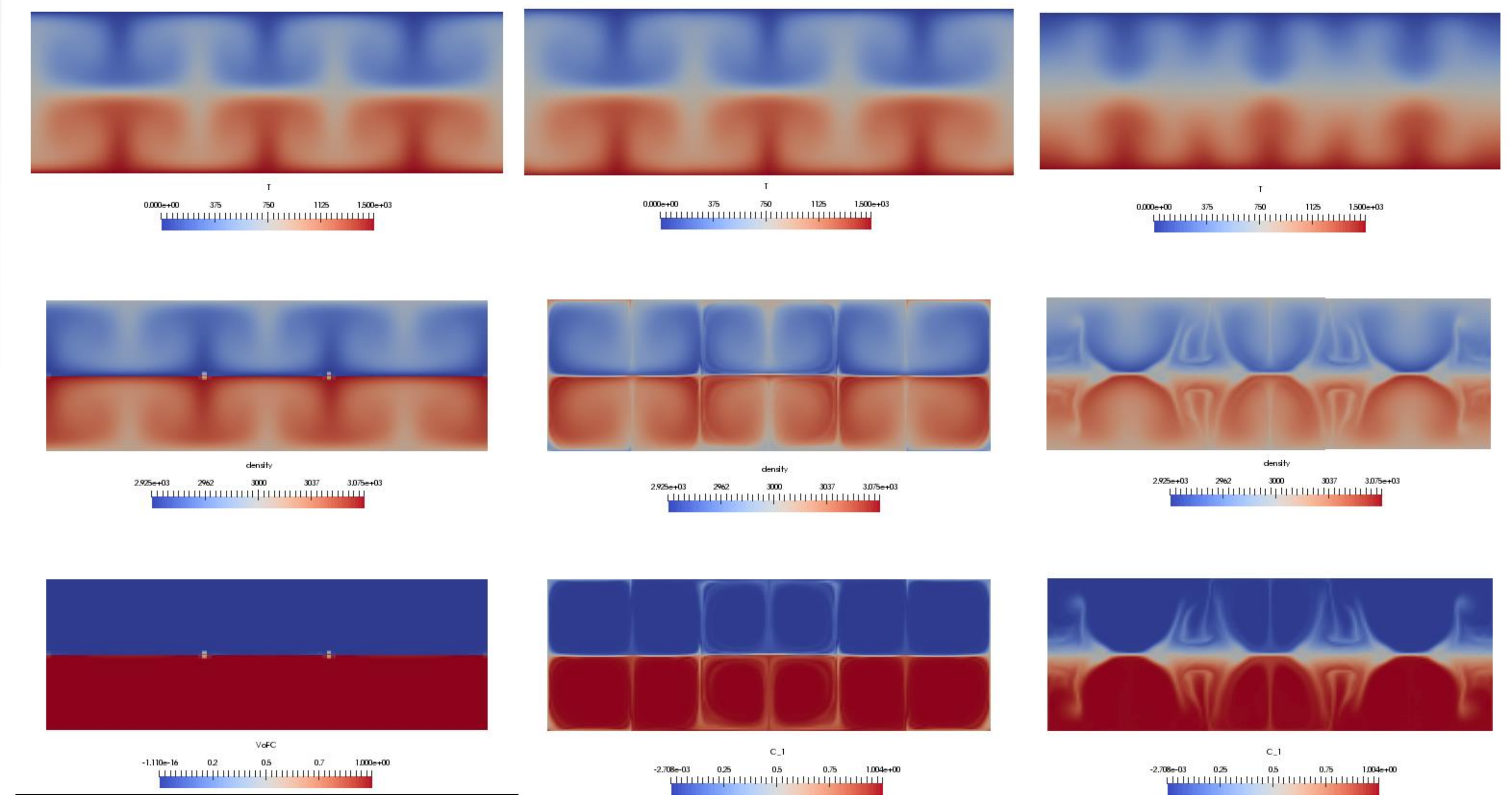

Figure 1. Results for two layer thermal-chemical convection with $Ra = 10000$ and $B = 1.111$

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