Development and Implementation of Software Elements using State-of-the-Art Computational Methodology to Advance Modeling Heterogeneities and Mixing in Earth's Mantle

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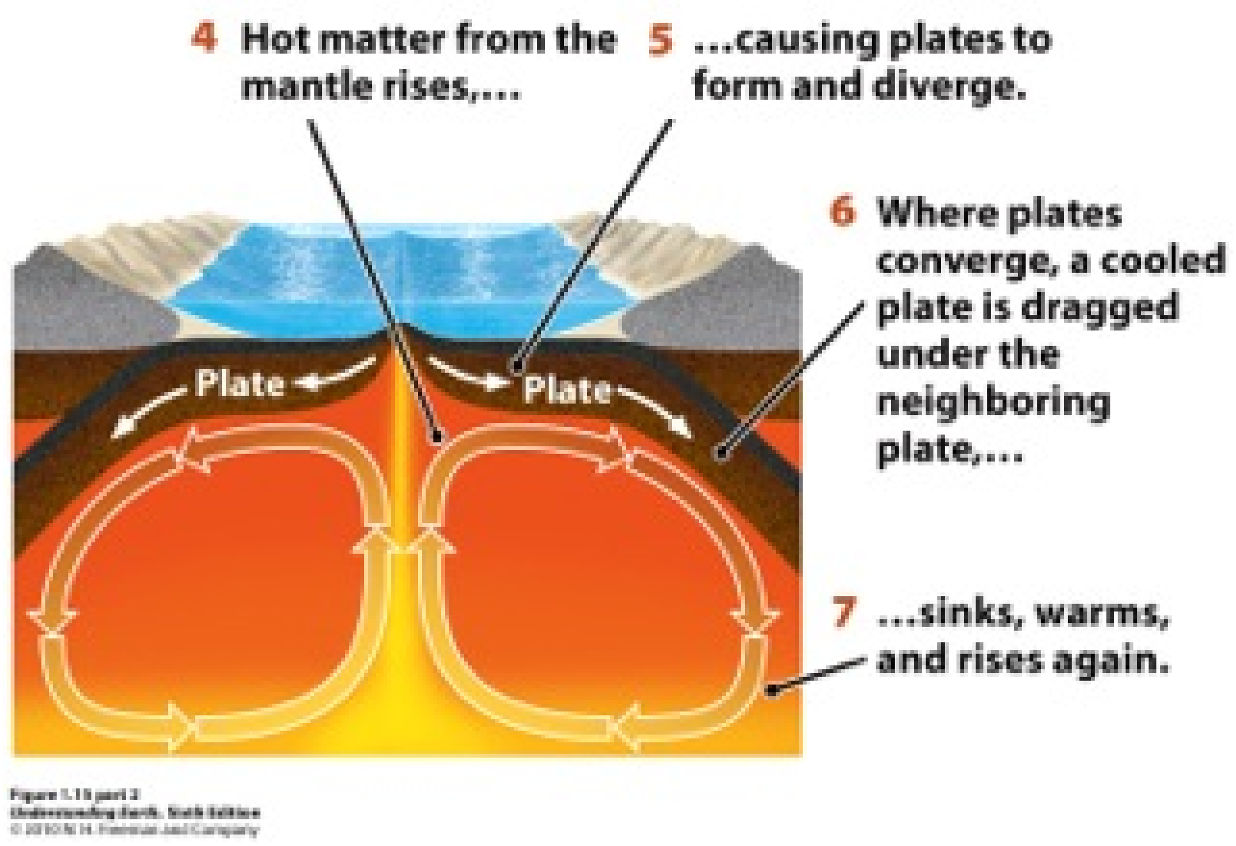
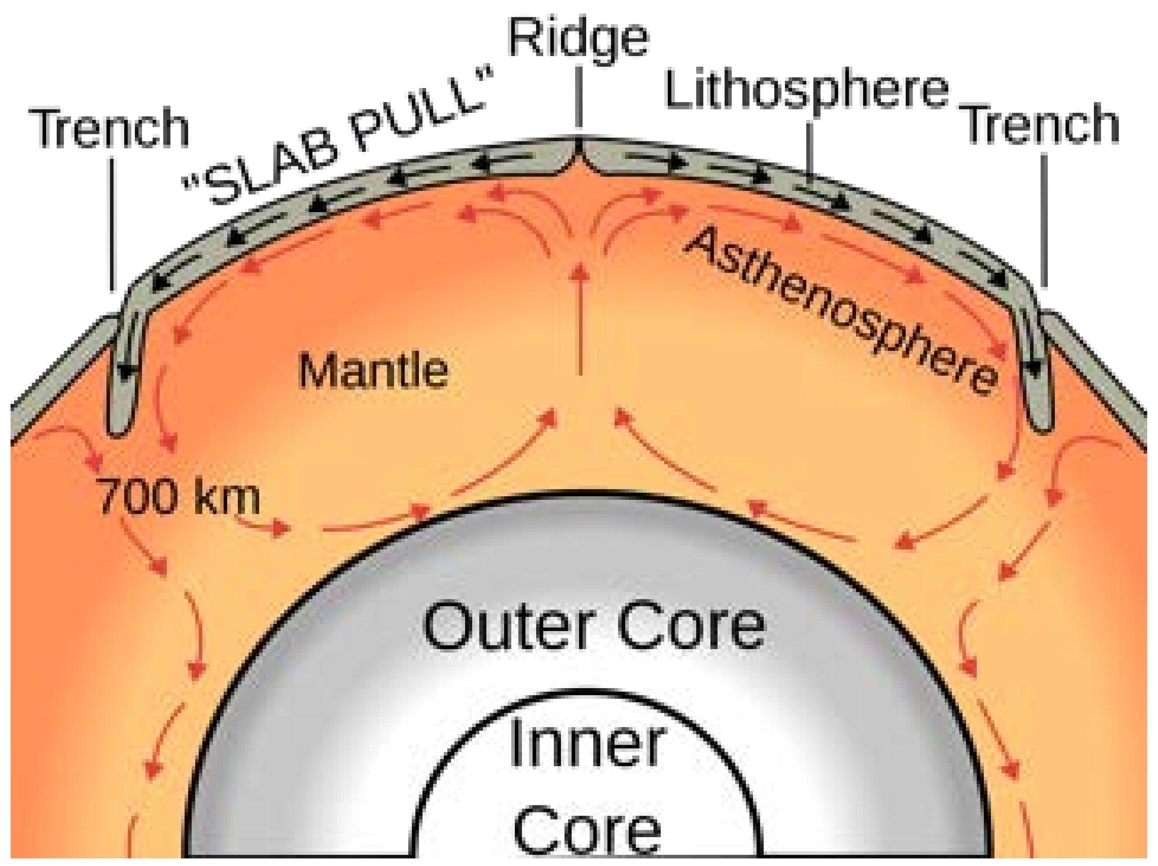
The Geophysical Motivation

At the elevated pressures and temperatures of Earth's deep interior, mantle rock responds to stress by slow, creeping solid-state flow. The resulting convection in the Earth's mantle is the driving mechanism of plate tectonics, volcanism, earthquakes, mountain building, and other geologic activity.

Mantle Convection and Plate Tectonics



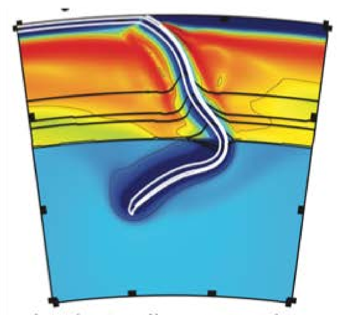
Subduction



Subduction occurs when a cold tectonic plate plunges into the hot interior of the mantle.

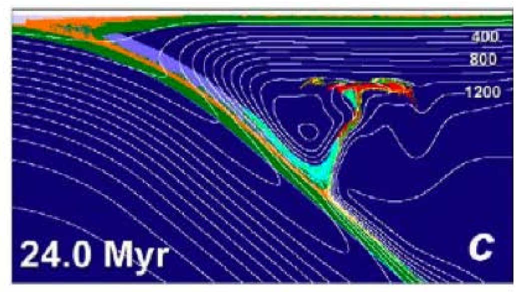
Geodynamics Simulations Need to Track Compositions

Crustal layers for slab dynamics



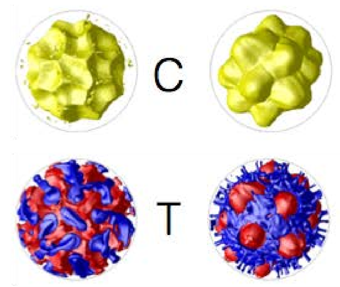
Arredondo & Billen, J. Geodyn. submitted

Sediments, water, melts for mantle wedge dynamics



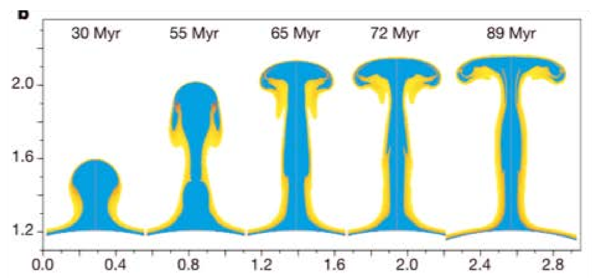
Gerya & Yuen, EPSL, 2003

Chemical piles for deep mantle dynamics



McNamara & Zhong, JGR, 2004

Chemical entrainment for plume dynamics



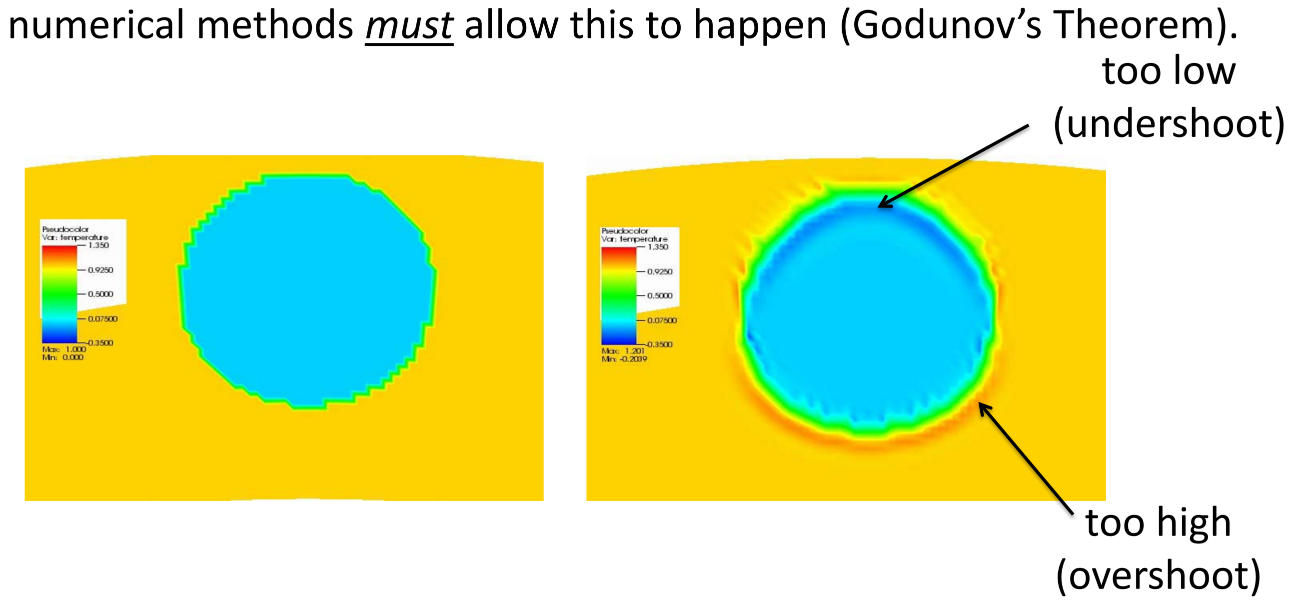
Lin & van Keken, Nature, 2005

Numerical Challenges

- Multi-scale problem
- Nonlinear system
- Strong Discontinuities in the composition field, viscosity and density
- Eliminating the overshoot / undershoot requires a bound preserving (aka monotone) flux limiter

A Simple Example of Overshoot / Undershoot

A circle of fluid initially at temperature T = 0.0 convects in a hot fluid at temperature T = 1.0. Physically, at no time can the temperature rise above T = 1.0 or go below T = 0.0. But linear, high-order accurate numerical methods *must* allow this to happen (Godunov's Theorem).



Two Computational Approaches

- Tracer Particles
  - **Pro:** Separate from the underlying FEM solve
  - **Con:** Can be expensive if there are a lot of tracers
- Piecewise Continuous Compositional Field
  - **Pro:** Less expensive
  - **Con:** Eliminating the overshoot / undershoot requires a bound preserving (i.e., monotone) flux limiter (**BPL**)

Overview of this Study

Goal:

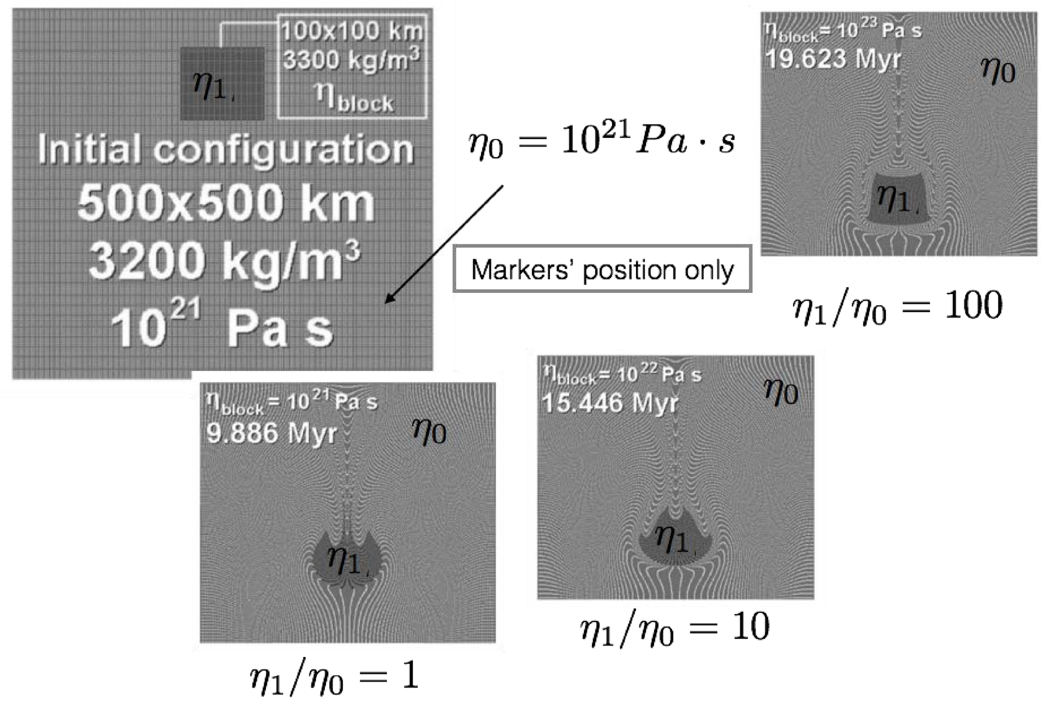
Develop a stable, accurate, and efficient mixed Continuous Galerkin – Discontinuous Galerkin (**CG-DG**) Finite Element Method (**FEM**) for the advection equation of a composition in mantle dynamics problems

Approach:

- Apply the Discontinuous Galerkin (**DG**) method to solve the advection equation for the composition with a bound preserving limiter (Zhang & Shu 2010, 2013).
- Apply the CG FEM to the Stokes equation for the velocity field.
- Use Adaptive Mesh Refinement (**AMR**)

Test example: sinking hard box problem

T. V. Gerya, D. A. Yuen 2003



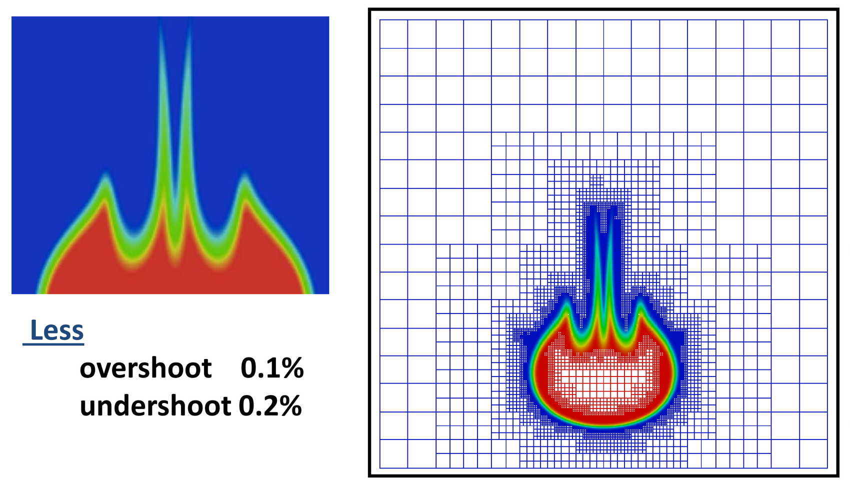
$\eta_0 = 10^{21} Pa \cdot s$

$\eta_1/\eta_0 = 100$ $\quad$ $\eta_1/\eta_0 = 1$ $\quad$ $\eta_1/\eta_0 = 10$

Gerya & Yuen's falling square benchmark comparing the FEM with Entropy Viscosity Stabilization (EVS) (J-L Guermond 2011) and the DG with Bound Preserving Limiter (BPL). These tests were run in deal.ii.

First Test Case: 1 to 1 viscosity ratio $\eta_1/\eta_0 = 1$

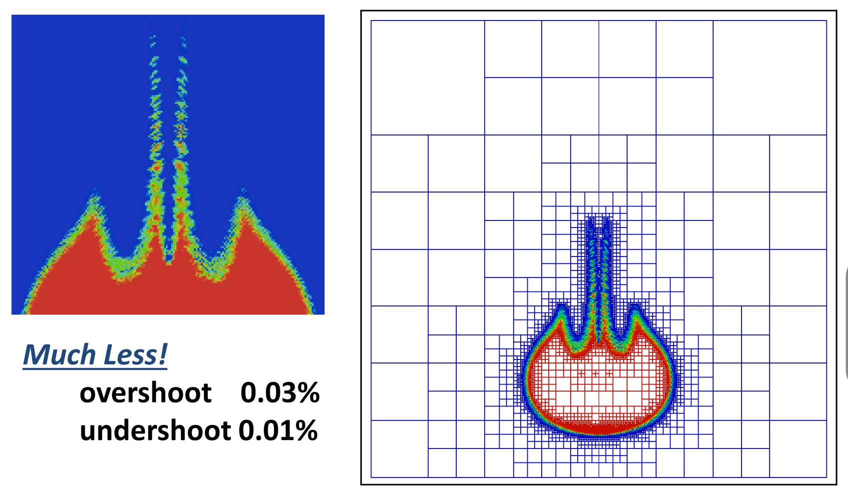
FEM with EVS in deal.II



Increases the number of
- Cells
- Degrees of Freedom (DoF)

EVS Controls the overshoot & undershoot but it *smears* the sharp boundary

DG with BPL in deal.II

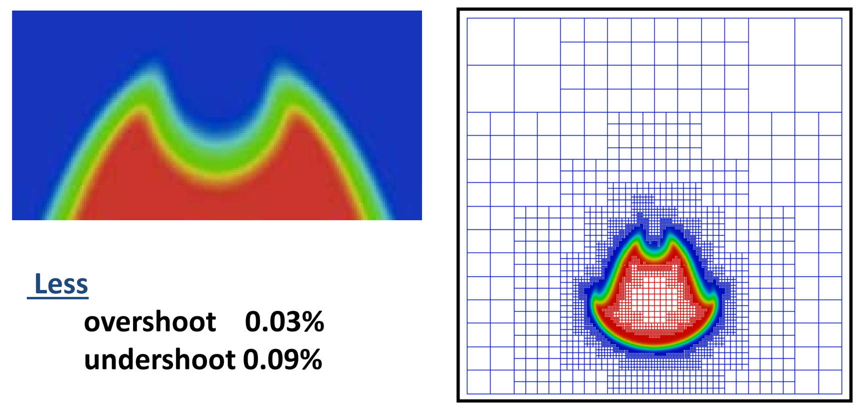


Fewer
- number of cells
- Degrees of Freedom

Much sharper boundary

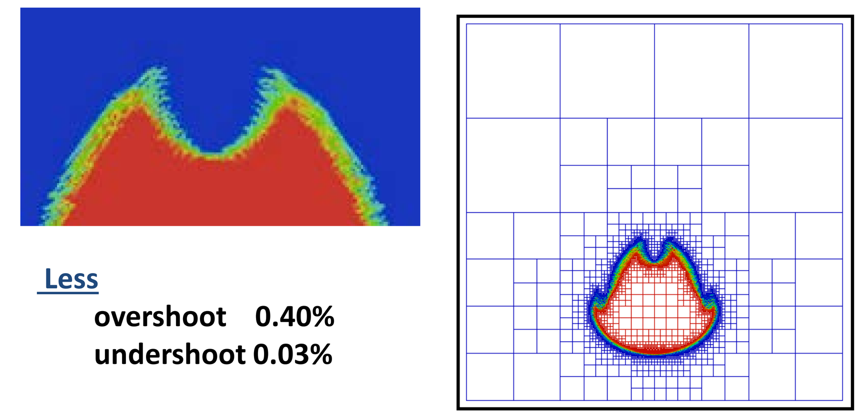
Test Case with a 10 to 1 viscosity ratio $\eta_1/\eta_0 = 10$

FEM with EVS in deal.ii



#cells: 17k
#DoFs(C): 70k
#DoFs(u+p): 158k

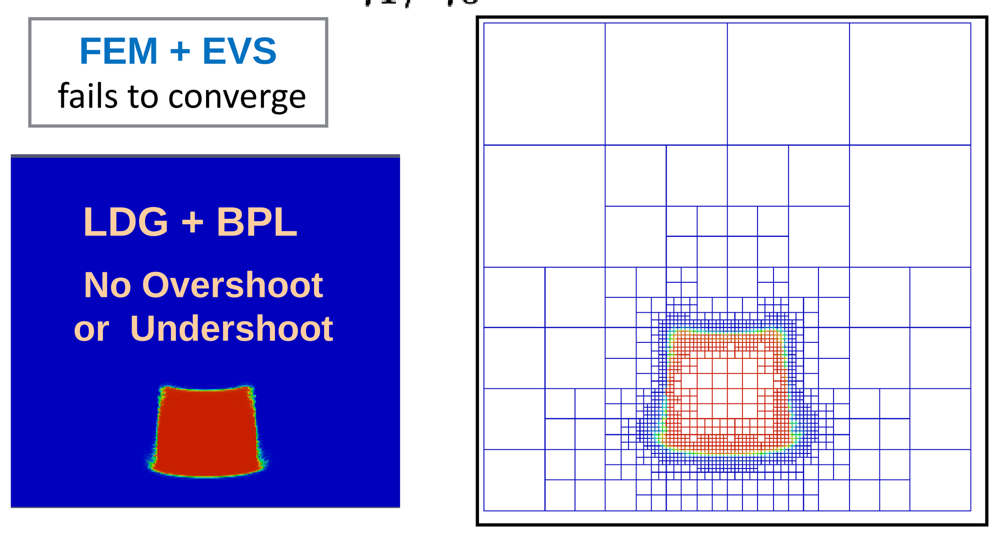
DG with BPL in deal.ii



#cells: 9k
#DoFs(C): 81k
#DoFs(u+p): 87k

Test Case with a 100 to 1 viscosity ratio $\eta_1/\eta_0 = 100$

FEM + EVS fails to converge



DG + BPL

- Convergence is limited by the mesh size.
- A better more expensive preconditioner is required.

Conclusions and Future Work

- We have developed a stable, accurate, and efficient method for the advection of a compositional field in ASPECT and other Finite Element methods using the Local Discontinuous Galerkin method with a Bound Preserving Limiter.
- Our numerical results have demonstrated that the proposed numerical method significantly reduces the number of cells and DoFs when we use AMR.
- We plan to study more advanced preconditioners for the Stokes solver with the LDG method
- This SSE will be added to the open source Mantle Convection Code ASPECT, which is accessible to the entire geodynamics community.

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