Development and Implementation of Software Elements using State-of-the-Art Computational Methodology to Advance Modeling Heterogeneities and Mixing in Earth's Mantle

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CIG COMPUTATIONAL INFRASTRUCTURE for GEODYNAMICS

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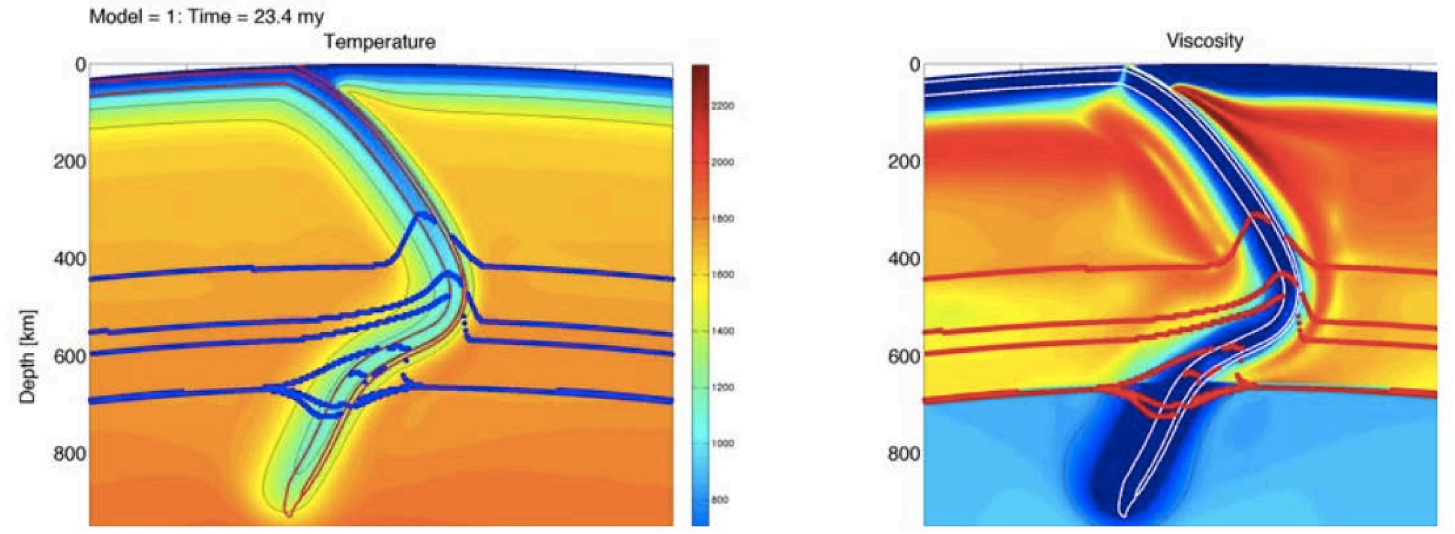
The Geophysical Motivation

At the elevated pressures and temperatures of Earth's deep interior, mantle rock responds to stress by slow, creeping, solid-state flow. The resulting convection in the Earth's mantle is the driving mechanism behind subducting slabs that occur at continental boundaries and are the cause of earthquakes and volcanism. There is a very large viscosity gradient near the slab tip and, since the viscosity depends exponentially on the temperature, numerical errors in the temperature such as *overshoot* severely degrade the accuracy temperature of the model. We have developed a high-order accurate, bound preserving method for solving the temperature equation in computational subduction models.



Crustal layers for slab dynamics, *Arredondo & Billen, J. Geodynamics (submitted)*

Model Problem

We approximate solutions of the incompressible Stokes equations with an advection-diffusion equation for the temperature and a Boussinesq approximation for the density.

$$-\nabla\cdot[2\eta\epsilon(\mathbf{u})]+\nabla p=\rho\mathbf{g}, \quad (1)$$

$$\nabla\cdot\mathbf{u}=0, \quad (2)$$

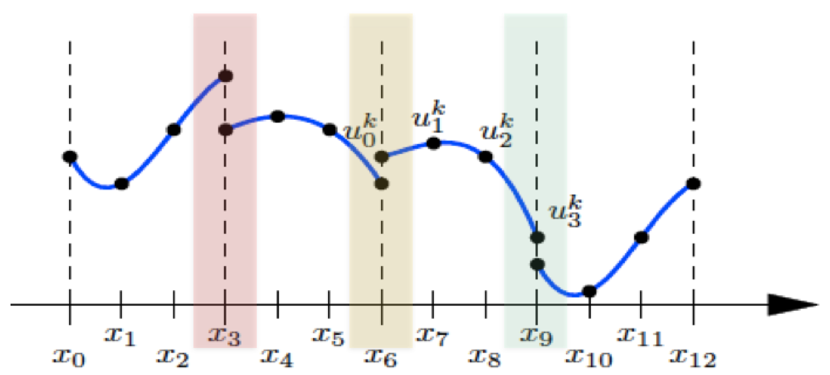
$$\frac{\partial T}{\partial t}+\mathbf{u}\cdot\nabla T=\nabla\cdot(\kappa\nabla T), \quad (3)$$

$$\rho=\rho_0(1-\alpha(T-T_0)). \quad (4)$$

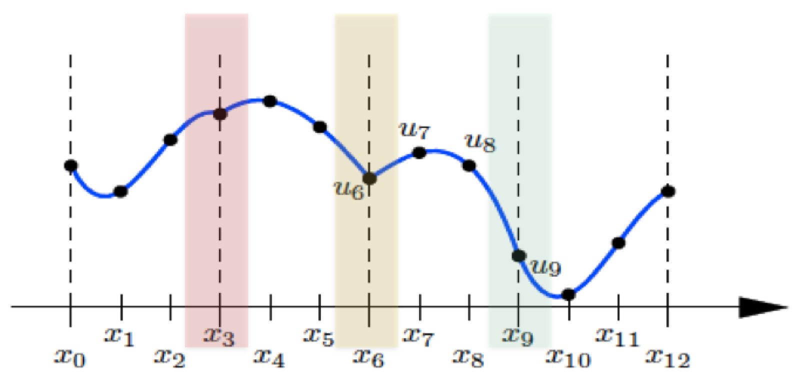
The temperature equation (3) is convection-dominated, if the Péclet number $P_e=\frac{L\|\mathbf{u}\|_\infty}{\kappa}>>1$. Note, if $\kappa=0$, there is no diffusion and $Pe=+\infty$.

The following two advection methods are implemented in ASPECT

Discontinuous Galerkin Method (DG)



Finite Element Method (FEM)



We have developed a *DG advection algorithm with a monotone flux and limiting*

The original advection-diffusion algorithm is an *FEM algorithm with* ***entropy viscosity*** *stabilization*

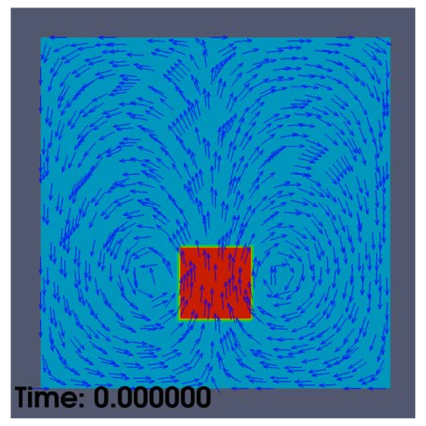
ASPECT (Advanced Solver for Problems in Earth's ConvecTion) is an open source C++ code to model problems in the Earth's mantle. https://aspect.dealii.org/

Numerical Results

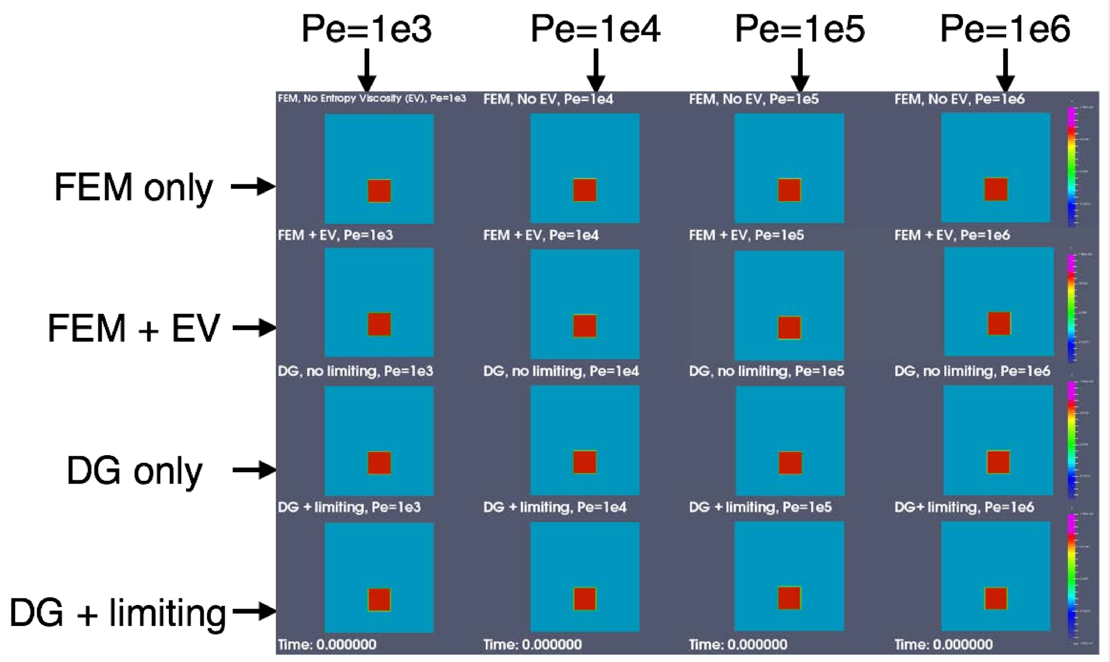
We solve the dimensionless problem

$$Pe=\frac{L\|u\|_\infty}{\kappa}=\frac{1}{\kappa},\ \text{if}\ L=\|u\|_\infty=1$$

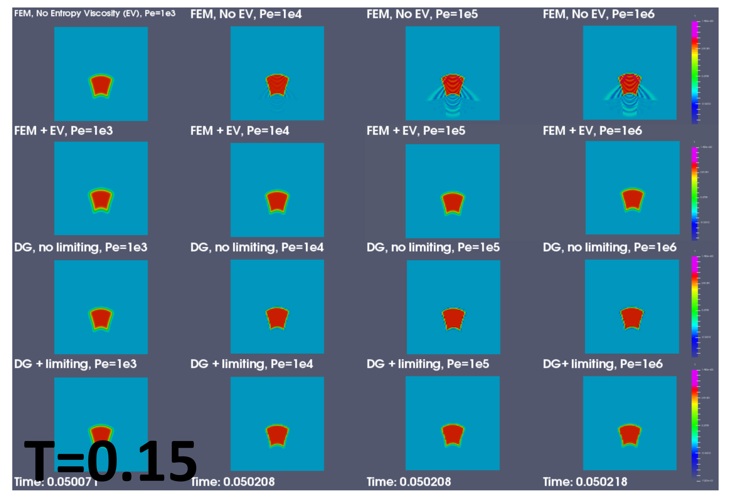
Initial Temperature *and velocity field in a unit box*



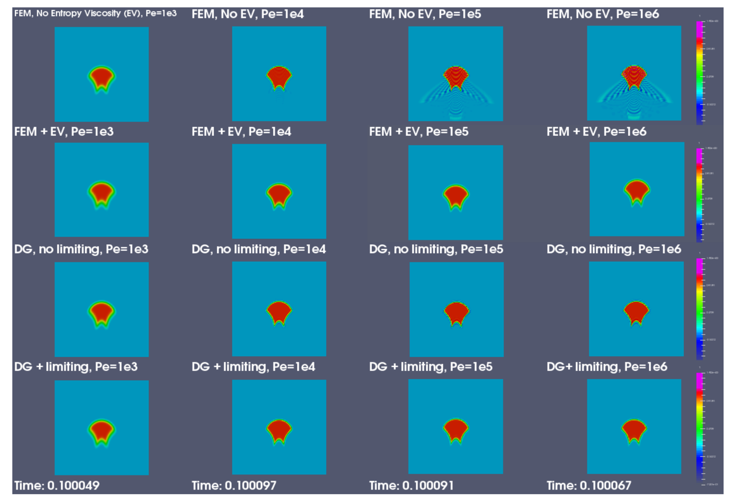
At time T=0



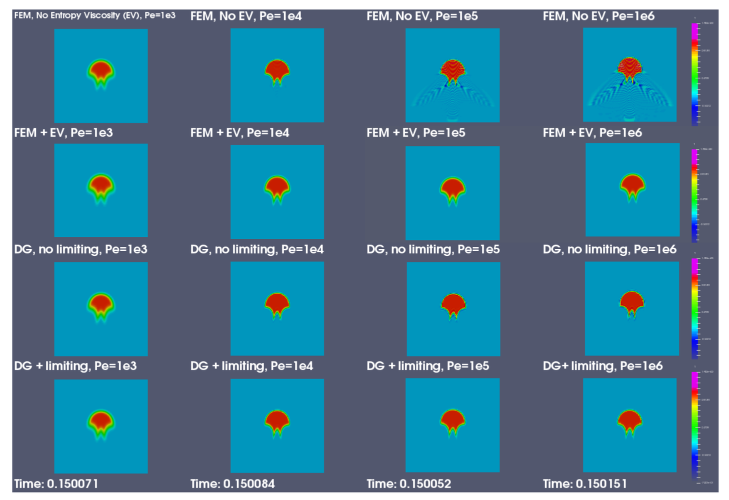
T=0.05

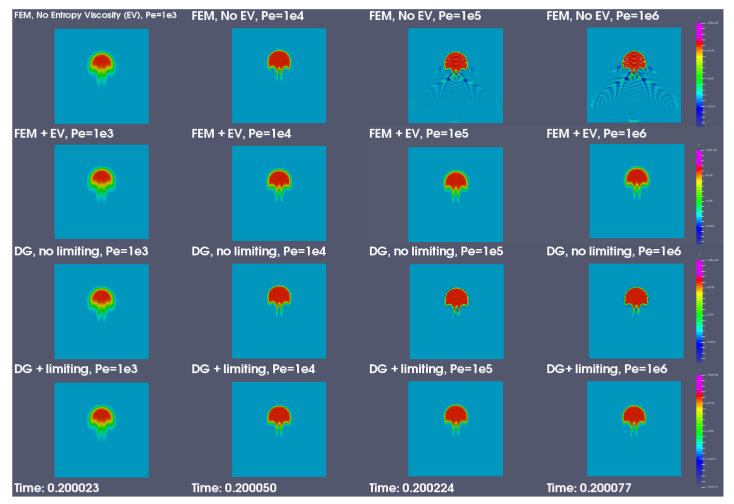


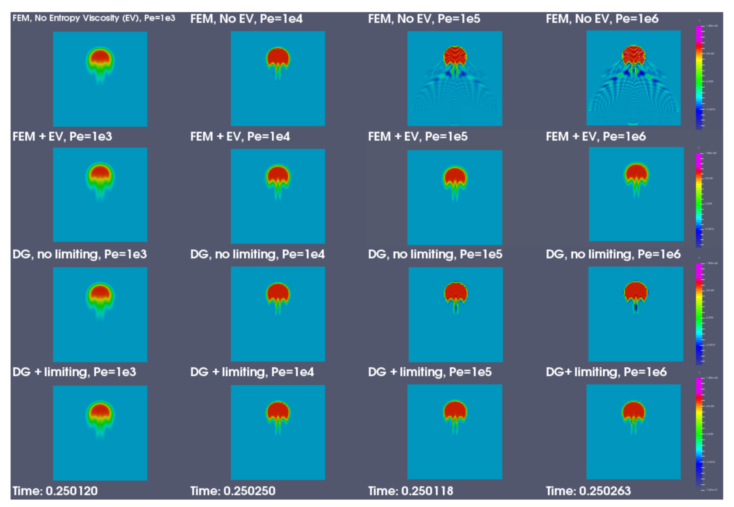
T=0.10

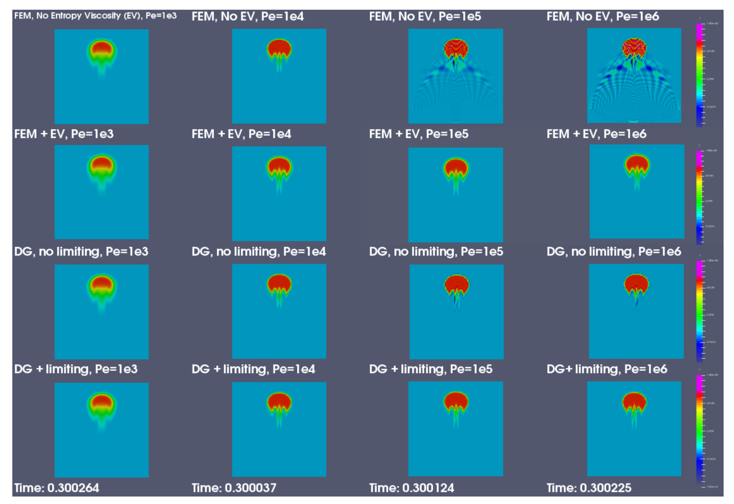


T=0.15

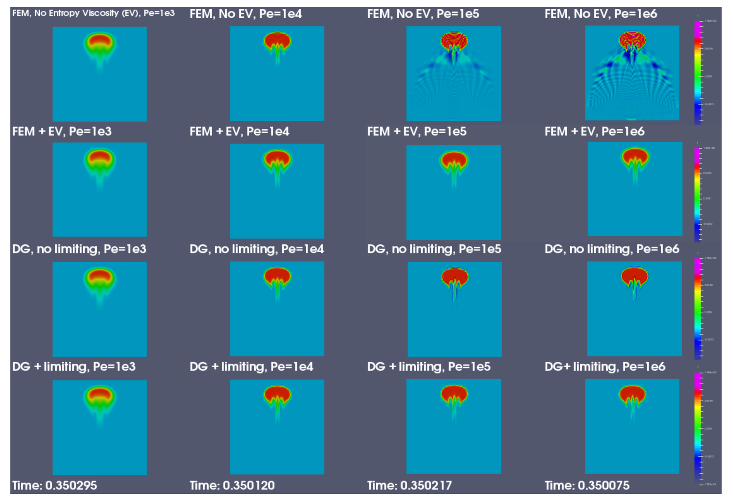




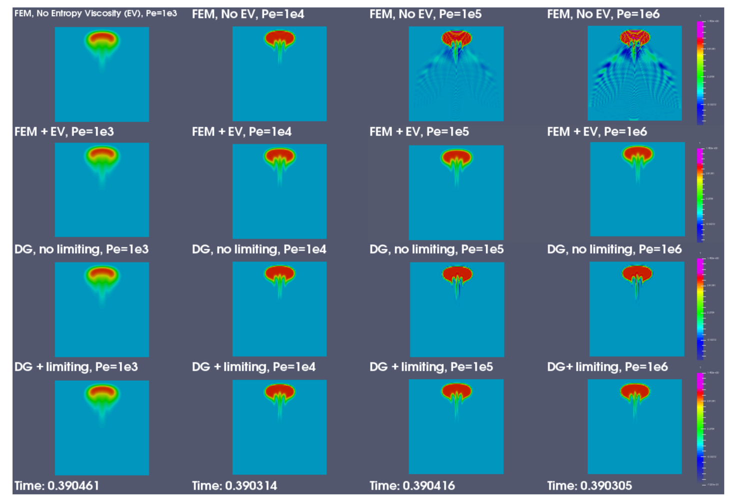




T=0.35

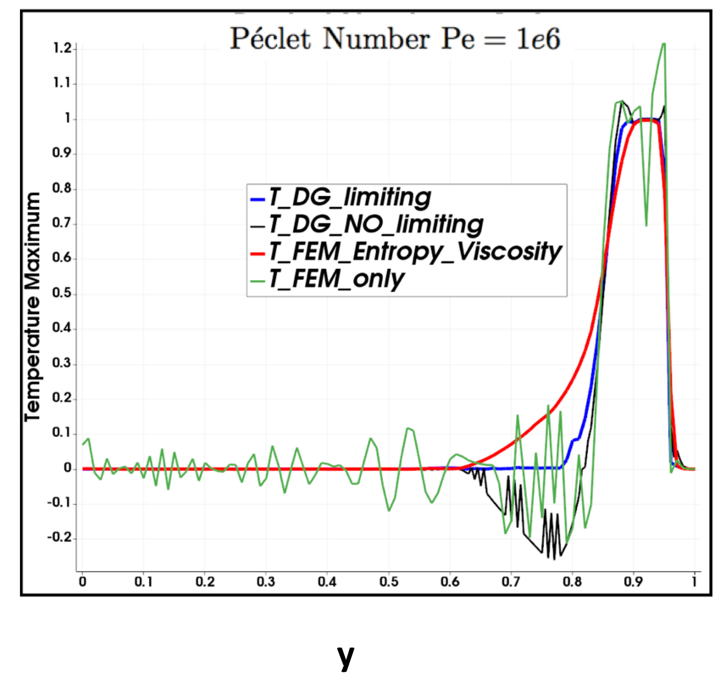


T=0.39

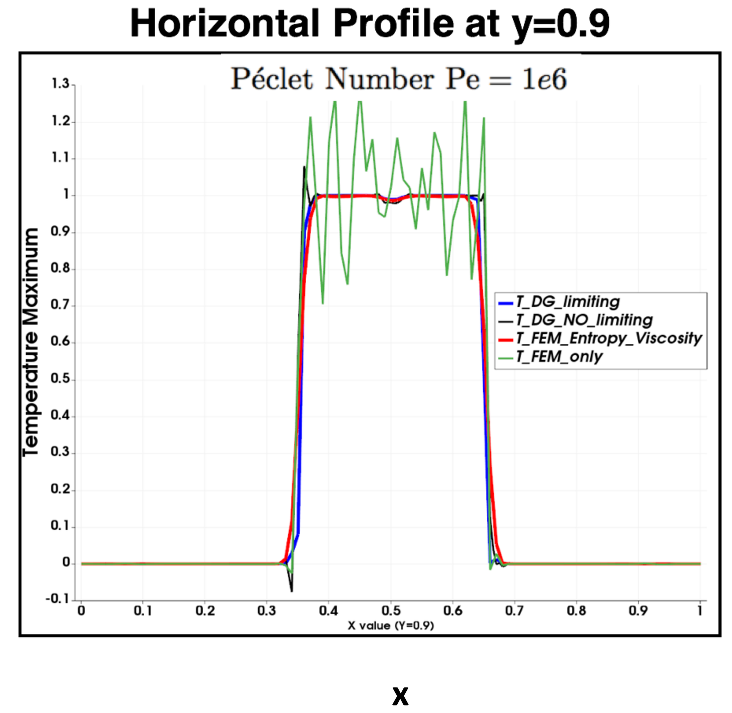


At time T=0.39

Vertical Profile at x=0.5



Horizontal Profile at y=0.9



Conclusions and Future work

- We studied isoviscous test cases involving a rising square with Péclet numbers from 1e3 to 1e6.
- Our numerical results have demonstrated that, compared to FEM with entropy viscosity, DG with limiting is more suitable for convection dominated flows; i.e., higher Péclet numbers.
- We are now using this algorithm in ASPECT to model subducting slabs, where there is a sharp viscosity contrast near the slab tip. The sharp viscosity contrast produces thin thermal boundary layers that "act" like isoviscous flows with large Péclet numbers.

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