Metacognition in the Mathematics Classroom

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Metacognition, a popular area in educational research, sparked an interesting panel discussion at the Joint Mathematics Meetings in Boston this January when it was applied to the teaching of math in the classroom. Annalisa Crannell from Franklin & Marshall College, a member of the Joint Committee on Women in the Mathematical Sciences (JCW) had suggested this topic and the JCW eagerly adopted it to organize a contribution to the meeting in Boston and to promote the JCW.

Founded in 1971, the JCW forms an umbrella organization for the various mathematical societies in our country: American Mathematical Association of Two Year Colleges (AMATYC), American Mathematical Society (AMS), American Statistical Association (ASA), Association for Women in Mathematics (AWM), Institute of Mathematical Statistics (IMS), Mathematical Association of America (MAA), National Association of Mathematics (NAM), National Council of Teachers of Mathematics (NCTM), and Society for Industrial and Applied Mathematics (SIAM).

The committee serves primarily as a forum for communication among member organizations about the ways in which each organization enhances opportunities for women in the mathematical and statistical sciences. The committee collects data, disseminates information, and facilitates discussion with a view towards developing best practices. (See https://jcwmath.wordpress.com/). Nancy Sattler (Terra Community College) and Jennifer Schultens (University of California, Davis) have been co-chairs of the JCW since 2020.

The term “metacognition” lacks the precise meaning we have come to expect in mathematics. Broadly speaking, the term refers to monitoring one’s thought processes. For example, in solving a mathematical problem, one might try different approaches, evaluate the likelihood of success of a certain strategy, spur oneself on. In a 2018 report by the National Academies *How People Learn II: Learners, Contexts, and Cultures* metacognition is described as “the ability to monitor and regulate one's own cognitive processes and to consciously regulate behavior, including affective behavior” (p. 70, [7]). There are variations in the usage of the term metacognition. Most importantly, in the context of education, the term is used to describe scaffolding activities intended to promote metacognition, *e.g.,* exit tickets, exam wrappers, keeping a learning journal. Teachers can design learning environments that include monitoring each student's performance or learning environments in which students monitor each other's performance. These types of environments are sometimes called metacognitive. It is worth noting that the thought processes of students in such environments are not, strictly speaking, metacognitive, but the hope is that they later become so.

While largely unfamiliar with research concerning metacognition, the members of the JCW eagerly adopted this theme for the panel discussion at the 2023 Joint Meetings of the Mathematical Societies because it evoked so many moments of awareness, and the promise of navigating around stumbling blocks both in mathematical research and in career trajectories. As working mathematicians we are all familiar with the frustrations of facing difficult problems and the exhilaration of solving them. As members of marginalized communities, we more often face these stumbling blocks in isolation. Metacognition can help pinpoint inequality. Even without being experts in psychology, or mathematics education, and perhaps even without thinking in terms of concepts as refined as metacognition, members of the JCW immediately perceived the relevance of the concept and the benefits of a panel discussion on the topic.

As readers of the Notices are aware, the discipline of mathematics holds a central role in education by virtue of its unique combination of practical applicability (to fields as disparate as medicine, construction, financial planning) and training in rational thinking, distinguishing between true and false, and searching for universal truth. This central role goes hand in hand with a broader pedagogical function in society, as a potentially stabilizing force in public discussion. Yet to appreciate and develop mathematical ways of thinking, we must encourage students to reflect on how they think as well as what it means to engage in mathematics. The panel on Metacognition in the Mathematics Classroom examined teaching practices that do so.

Participants in the panel discussion were Jo Boaler (Stanford University), Lakeshia Legette Jones (Clark Atlanta University), Yvonne Lai (University of Nebraska-Lincoln), and John Nardo (Oglethorpe University). Jennifer Schultens moderated. After the panel discussion on Metacognition in the Mathematics Classroom, panelists were asked to summarize their contributions. Some summaries included recollections from the questions and answers portion of the panel discussion. We now provide these summaries.

Metacognition
Jo Boaler (Stanford University)

In 1979, Stanford professor of psychology John Flavell created the theory of metacognition, and researchers have been investigating its impact ever since then. The word "meta" comes from the Greek prefix, meaning beyond, and metacognition regards the important processes that go beyond thinking, such as planning, tracking, and assessing. Flavell describes metacognition as including knowledge of ourselves, knowledge of the task at hand, and knowledge of strategies (Moritz & Lysaker 2018, see [6]), so it is no surprise that it boosts problem solving, and enhances mathematics achievement (Wilson & Conyers, 2016, see [8]).

In classrooms I find it easy to spot people who have learned metacognitive strategies and others who have not. I see some learners who are discouraged when they are given difficult challenges, assume that they cannot do well, and give up in the face of roadblocks. By contrast I see learners who are inquisitive and curious, they are eager to learn, and they appreciate diverse viewpoints. If they are stuck in a problem, they may circle back and think about what they know and need to know, or they may choose from other different strategies they have learned. Importantly, they enjoy the process of problem solving and learning. This complex combination of high-level problem solving, mindset, and planning that occurs when people are metacognitive, takes place in the anterior prefrontal cortex of our brains (Fleming, 2014, see [5]).

I see the potential of taking a metacognitive approach in three different areas of teaching and learning. First, the area most often associated with metacognition, is the self-awareness we have, of our own learning and interacting. At youcubed we have developed a mindset rubric to help people engage in this important self-reflection. [1] A second aspect of metacognition involves different ways of focusing on the task at hand, being willing and able to unpack it and think about what is involved. A metacognitive person will think in important ways—possibly going back to the question, considering what information is needed, thinking out loud, drawing the problem, or taking a smaller case. Someone who has developed and reflected on different strategies can choose among them, or try a few different approaches.

[1] https://www.youcubed.org/resources/mathematical-mindset-practices-rubric/

When we taught 82 middle school students in a youcubed summer camp we taught the students these strategies as they worked on open tasks. At the end of the four-week camp the students had increased their achievement on standardized tests by the equivalent of 2.8 years (Boaler et al, 2021, see [3]).

The third part of metacognition involves assessment, and being able to track one's own progress and reflect on what is needed to achieve goals. This is where teachers and parents play a critical role in setting out for students where they should be going, and ways to get there. The education leaders, Paul Black and Dylan Wiliam, who proposed the approach, which they called "assessment for learning" defined it in these ways – communicating to students where they are now, where they need to be, and ways to close the gap between the two (Black & Wiliam, 1986, see [1]). One of my favorite strategies for assessing in this way is to give students a rubric, that sets out their mathematical journey, and use the rubric to share feedback on where students are, and where they need to be, and ways they can get there. A K-8 school using this approach is shared on youcubed, with some example rubrics.

The most mathematically empowered people in the world take an approach to learning math that is different from those who are less successful (Boaler, 2024, see [2]). It is not typically the case that they achieve highly because they were born with special advantages, but because they have been given access to important approaches to learning. This selection of articles shares many of them.

Metacognition as a Soft Skill
Lakeshia Legette Jones (Clark Atlanta University)

Unknowingly, I have practiced metacognition in the classroom for quite some while. I have always been intentional about my teaching methods but didn't realize there is a name for it. Metacognition was formally introduced to me by my son's first-grade teacher after noticing similarities in our teaching styles. At that point, I began reading and researching the benefits, best practices, and strategies for implementation. I was pleased to find that a number of the strategies are already a part of my practice.

For example, each class I recite the narrative as a form of review, and to help students prepare for conceptual assessment questions. To help students understand the big picture, I discuss the purpose of the class and the overall expected outcomes. I also outline the details. I reiterate the types of problems we can expect, the general process toward solving such problems and the questions we should ask ourselves in the process. I explain how the current content connects to the previous and future content. I also ask students to explain their work, provide reasoning for their approaches and describe their thought processes. In a class such as Advanced Calculus or other proof-based courses, I ask students to communicate the best method of proof, which definitions are needed, or what previous results might be necessary.

Although it was great to learn that my teaching practices are generally in a good ballpark, what I also learned is that my metacognition strategies were largely being conducted orally. I was not allowing adequate opportunity for students to think about their processes or question their reasons. They were not journaling or creating a document that would grow and develop with them. Essentially, my classes were lacking in formal, structured time for reflection. Understanding that part of my responsibility is to help students discover how they learn best and gain an accurate account of their strengths and areas of improvement, I immediately committed to incorporating time for written reflections. Now, among other low-stakes summative assessments, I require students to keep a learning journal, participate in exit tickets and complete exam wrappers.

What I notice are what I believe some of the greatest benefits to strengthening students' metacognitive ability. That is, it also increases their agency and makes them proud to accept responsibility for their learning. Students become more aware of what learning strategies are most effective and when more or less time is needed for grasping a concept. Students are honest in their assessments and will admit to not studying enough when they knew more time was required. They can more intelligently articulate what they don't understand and express when there are gaps in their learning. They respect the notion that sometimes struggle (without notes or other resources) is necessary. There is an overall improvement in work ethic.

Although introducing metacognitive strategies to our students is one of the greatest lifelong gifts we can share, I think it is imperative that instructors understand that developing strong metacognitive abilities requires a level of vulnerability from students. Therefore, care must be taken to create a safe space and welcoming environment inside the classroom. Students will be more likely to operate in honesty and with an eye towards self-improvement when they know there is sincerity and true care and concern for their success.

Audience Question: Are the strategies different for majors versus non-majors?

Answer: In my opinion, metacognition may be viewed as the umbrella term that encompasses many of the other soft skills we desire for our students, including communication, organization, leadership, work ethic, integrity, time & stress management, and collaboration. Strengthening metacognition will consequently also positively affect each of our soft skills in some way. With this understanding, it is clear that course content is less of a factor in metacognition. We are contending with the self and not necessarily the textbook, or course content. For this reason, in a class filled with non-majors, my metacognition strategies are largely unaffected.

Audience Question: How do you know whether your strategies are effective?

Answer: Ask the students! This is a great way to allow individual, as well as the collective student voice to be heard, included, and considered. It is an explicit show of inclusivity and creates buy-in from the students. They appreciate any opportunity to contribute feedback that will ultimately lead to a better learning experience. Students find comfort in knowing their instructor is willing to accommodate their most reasonable requests. The solicitation of feedback can be achieved in several different ways. For example, securing instantaneous feedback through Mentimeter, Poll Everywhere, iClicker or similar platform. There is also the option of an exit question or journal prompt.

After each major assessment I ask my students, not if it was easy, but if it was fair. They recognize and appreciate the difference in those two questions. The former speaks to their responsibility and the latter speaks to my responsibility. Students are refreshingly reflective in their responses to my question and 100% of the time state that the assessments are fair and within bounds. However, they also trust that if the answer is "no," then I am willing to make adjustments to get it right. At multiple points of the semester, I also check in with students with a "temperature check" survey. It is interesting to notice the changes in responses as the semester progresses. Students become aware of strategies that no longer serve them well and realize others that are more helpful. They will earnestly respond to the effectiveness of a given strategy. I find solace in this because it lets me know they understand we are all playing for the same team.

Metacognition for learning–and teaching

Yvonne Lai (University of Nebraska Lincoln)

As mathematics faculty, we may hope for our students to know beauty through mathematics, and joy through mathematical community. In proof-based courses, we may hope for proofs to be a vehicle for mathematical challenge and fulfillment. We may also hope for students to begin to understand and embrace the reasons for why proof is essential to knowledge building in mathematics. Yet proof-based courses are a barrier to too many students, including prospective middle and high school teachers. These courses may also be disproportionately a barrier to women and students of color.

I am not a researcher in metacognition. However, I find the concept useful for teaching. I will argue here that developing students' metacognition in mathematics learning requires developing our own metacognition in mathematics teaching.

Broadly speaking, to my understanding, metacognition can be thought of as knowledge and regulation. That is, mathematical metacognition involves knowledge about mathematical processes and one's own thinking processes as they relate to mathematics. Moreover, mathematical metacognition includes how one might regulate these processes. I think about metacognition when I try to find ways to understand why mathematical proof and reasoning is so hard and frightening for students, and to find inroads to helping students to embrace proof.

Here is one example of a routine I use to open the proof and reasoning doors to future high school teachers and math major. It is not my own; I learned it from Sameer Shah. I will then say my view of what metacognition has to do with undergraduate students' learning and also instructors' teaching.

Attacks and Counterattacks

Sameer Shah designed a brilliant extension of the idea of asking students to come up with the definitions of key terms.

Prior to reading Shah's work[2], the way that I understood this idea was that students generated drafts of definitions and, akin to the fictional students of Lakatos[3] though perhaps with more informal language, the student found examples to motivate reworking drafts. After enough drafts, a satisfactory definition would emerge. I've never been happy with this model. In practice, I've found that it takes too long for the result. Moreover, it seemed pedagogically inconsistent to ask students to go through this process, but then use a textbook or other source that had its own definition. Shah's extension retains the advantages of students making definitions while also

[2]See his blog for a full description of the routine: https://samjshah.com/2014/10/19/attacks-and-counterattacks-in-geometry

[3]in his book *Proofs and Refutations*

engaging them with the language of their own mathematical textbook.

For Shah, generating drafts of definitions (the “attacks”) and coming up with examples that satisfy the drafts but are not actually the desired object to be defined (the “counterattacks”) is only Part One. He proposes a Part Two and Part Three. In Part Two, you show students the definition from the textbook you are using (or another one), and ask them to reflect on the language chosen. In Part Three, you take these textbook definitions, cross out a condition of the definition, and ask them to produce a counterattack for the amended definition. (For instance, if defining triangle, you might cross out the condition that the vertices must be non-collinear.)

I have now used Shah’s materials (which ask students to define circle, triangle, and polygon) and adapted his routine to multiple other terms (including vector, inverse vector, angle measure, isometry, among others). My personal reflection is that through this routine, students reconfigure their relationship with mathematical language from receiving to doing. Part Two, in combination with Part One, helps them understand why mathematics is written the way it is. I find that their ability to read and understand technical mathematical language, even outside of this routine, is changed. They are more patient with the stilted language that mathematics sometimes requires. They also see counterattacking as a way of unpacking the meaning of a definition, even when they know they are reading a textbook definition. It used to be that only the students with the most proof-based courses in their background would find mathematical errors; now I find that more students spot and are willing to bring up potential mathematical errors.

There may not be time in every mathematics course to do this routine for every definition. That said, I think it is worth doing at least once with every proof-based course. It is a way for students to get to know each other mathematically, and for you to see all students have the potential to see everyone – including themselves – contribute meaningfully to the classroom discourse. This routine is also a way for students to build for themselves an understanding of why mathematics might sound different from natural language. It is an invitation to talk about and ultimately participate in mathematical language.

Of Horses and Zebras

I read recently a New Yorker article[4] referencing the medical school adage, “When you hear hooves, think horses, not zebras.” It means that

[4] https://www.newyorker.com/magazine/2023/01/30/nobody-has-my-condition-but-me

when you are a doctor, and you observe particular symptoms in a patient, you should first think of the most common explanation for these symptoms rather than the most exotic one. I think that there is a lot of work that we as mathematics faculty can do to improve how we see "horses" and "zebras" in student thinking, especially if we use activities such as Shah's Attacks or Counterattacks, or any other routine. I'll use myself as an example.

When I first began teaching, the "horses" in my mind were diagnoses such as, "made a careless mistake", "doesn't know what they are doing", or "isn't reading the question". While these diagnoses may be true in some sense, they are also unhelpful for taking action, both because I'm describing what students are not doing rather than what they are doing, and also because this is like describing "horses" as "some animals of some sort that are not doing what I think they should be doing". Now, when I teach introduction to proof or abstract algebra, some typical "horses" might be "needs support structuring a proof", "has worked out examples to show the theorem and may not know how to generalize", or "may think that 'unit' only refers to '1' rather than any element with a multiplicative inverse". In general, I have tried to shift my own way of observing towards describing both what I see students doing as well as what they might move toward doing, rather than only the latter. In this way, I can begin building on what they are doing rather than only imposing my own ideas of where they "should" be.

When it comes to "zebras", I have in my early years thought that a student's question alluded to open or difficult problems, like thinking that a student was asking about Fermat's Last Theorem when they were only asking about a detail of the Pythagorean Theorem. Although it does happen that students ask about mathematics years beyond what we are formally studying in the course, we should first check whether it is horse. And only after we are sure that it is not a horse, should we hypothesize that it is a zebra.

When we teach proof-based courses, we have the opportunity to shape – for better or worse – the mathematical experiences of a future generation. I also try to remember that students include future parents and teachers. In finding ways to tune my own metacognition for teaching, I try to think about how my own ways of interpreting student talk and thinking can help me be more responsive to what students need for a more joyful, beautiful, and community-oriented experience.

How to Begin using Metacognition to Help Your Students

Thrive
John Nardo (Oglethorpe University)

As a professor at a small liberal arts college, I proudly view myself as a teacher-scholar, and I approach this work on metacognition in mathematics firmly through an applied lens. My focus on metacognition is central to my career-long goal: to help students learn and grow. By thinking and writing about their learning, my students have gained both confidence and skills; this metacognitive work has also helped shift their view of mathematics from a collection of algorithms and processes into a richer, integrated view of our field.

I am privileged to have a close cohort of colleagues who are experimental in the classroom, and we share our successes and challenges frankly with each other. We visit each other's classrooms often. I routinely adopt best practices of others and implement them in my own classes; many of my ideas have been improved after talking them over with my peers or hearing how they implemented them differently in their classrooms. I have benefitted as both generator and receiver of ideas.

Even after decades in the profession, I struggle with perfectionism in my work. Often, I have wanted to make sure that an activity or assignment is "just right" before deploying it to students. I have lately realized the trap in this type of thinking: perfectionism can make me timid and hinder my actions to innovate in the classroom.

I have learned that students can be potent allies in course reform. You do not have to wait until official course evaluations at the end of the semester: you can use your own informal "check in" surveys. Students are not shy in telling you how class is functioning. Fully engaging with student feedback, by mentioning (anonymously) things they have shared or modifying a class strategy based on their insights, can strengthen their sense of belonging.

To get started, it helps to think small. The commitment to sustain multiple experimental approaches over a semester can be daunting to the point of inaction. Pick a single activity or assignment for experimentation and see how it goes. Here are some experiments that have enriched my students' learning.

General Education: In Oglethorpe's "Mathematics and Human Nature" course, writing and reflection are central. We start the course with a tried-and-true activity: writing mathematical autobiographies. This fortifies students' identities as mathematicians and exorcises demons from past interactions with our discipline. This often cathartic experience is revisited multiple times through discussions and revisions; thus, it helps them take agency in learning mathematics. When students share their biographies in small groups, they see common experiences and build community. They realize how quickly they attribute their successes or failures to other people instead of taking ownership of their education. Rather than a "one and done", professor-validated assignment, this essay becomes a living, breathing document, and students become agents for change in our beloved discipline.

Transition to the Major Course: To encourage metacognition for majors, I use a summative portfolio assignment instead of a final. The portfolio showcases the proof techniques learned in class on a subset of the battery of problems given at the beginning of the semester. At regular checkpoints, students practice communicating formal mathematics by submitting solutions for feedback. Discussing how to tackle problems improves both grades and proofs; it helps to instill pride in their increasing sophistication. These reflections and discussions are routinely praised in course evaluations. Moreover, the portfolio is a resource for their future success. When struggling in an upper-level course on a proof technique, why go to a textbook or the internet for an example? They can refer to their own work in the portfolio to refresh their familiarity with the proof technique and to jump-start their thinking.

Introductory/Intermediate-Level Courses: I adopted a colleagues' suggestion to allow variable point allocations on take-home tests: students pick point values from target ranges. Students pick a grading scheme that aligns with their conceptual status, thereby celebrating their successes and minimizing the fallout from their challenges; it has also allowed them to have agency in even their high-stakes work. To reduce anxiety and encourage reflection over the entire course, I have also changed the format of final exams. Each final is explicitly structured with sections covering the material from each unit test along with a section for the material since the last test. The final is scored on its own merit as a whole, but students are allowed to pick a section to replace the corresponding test, if it is better. They know about this opportunity ahead of time. In making that selection, they must balance the desire to target the section corresponding to their lowest test grade with an assessment of which section on the final will have the highest success and percentage score when tallied by itself.

These are just a few experiments that I have tried in my teaching, and my students have encouraged me to keep experimenting. Students generally respond with grace when experiments fall short, knowing that we will improve things together (or in the next iteration of the class).

Question: How can you be supported to do this metacognitive work if you have a very traditional department or lack a group of supportive peers?

Answer: My close professional interactions keep me energized, especially in the wake of challenges posed to education/educators during the COVID-19 pandemic. However, not everyone has a supportive department; traditionally oriented departments make it difficult for innovative work by contingent faculty or beginning tenure-track colleagues. If you are not fortunate to have a close department or one open to experimentation, then you can grow an external network online through the video technologies normalized during the pandemic. Surround yourself with people who will both challenge and nurture you.

The discussion proved both lively and informative. We invite you to look out for our next panel discussion, to be held on Thursday, January 4, 2024, 3-4:30pm, at the Joint Mathematics Meetings in San Francisco. If you have a topic you would like to suggest, please write to us at jcs@math.ucdavis.edu or jcw-comm@ams.org.

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