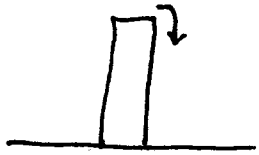
ESP Kouba

Worksheet 10½

Solutions

1.) Let $s(t)$ be height above ground at time t. Then $s(0) = 64$ ft. and $s'(0) = 0$ ft./sec.:



$s''(t) = -32$ ft./sec.^{2} $\rightarrow$

$s'(t) = -32t + c$ (and $s'(0) = 0$) $\rightarrow$

$\boxed{s'(t) = -32t}$ $\rightarrow$

$s(t) = -16t^2 + c$ (and $s(0) = 64$) $\rightarrow$

$\boxed{s(t) = -16t^2 + 64}$

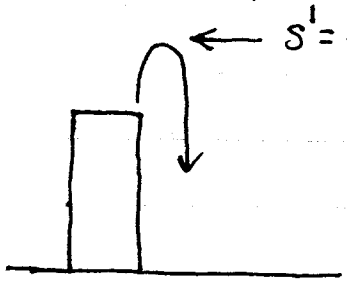
a.) <u>strike ground</u>: $s'(t) = 0$ $\rightarrow$

$-16t^2 + 64 = 0 \rightarrow t^2 = 4 \rightarrow$ (t = 2 sec.)

b.) $s'(2) = -64$ ft./sec.

$= -64 \frac{\text{ft.}}{\text{sec.}} \cdot \frac{1 \text{ mi.}}{5280 \text{ ft.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx -43.6$ mph

2.) Let $s(t)$ be height above ground at time t. Then $s(0) = 64$ ft. and $s'(0) = 48$ ft./sec.:



$s''(t) = -32$ ft./sec.^{2} $\rightarrow$

$s'(t) = -32t + c$ (and $s'(0) = 48$) $\rightarrow$

$\boxed{s'(t) = -32t + 48}$ $\rightarrow$

$s(t) = -16t^2 + 48t + c$ (and $s(0) = 64$) $\rightarrow$

$\boxed{s(t) = -16t^2 + 48t + 64}$;

a.) <u>highest point</u>: $s'(t) = 0$ $\rightarrow$

$-32t + 48 = 0 \rightarrow$ (t = 3/2 sec.)

b.) $s(3/2) = -36 + 72 + 64 =$ (100 ft.)

c.) strike ground: $s(t) = 0 \rightarrow$

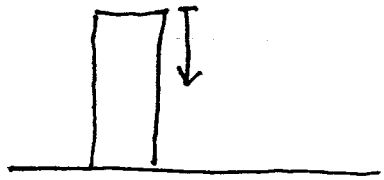
$-16t^2 + 48t + 64 = 0 \rightarrow$

$-16(t^2 - 3t - 4) = 0 \rightarrow$

$-16(t-4)(t+1) = 0 \rightarrow$ $\boxed{t = 4 \text{ sec.}}$

d.) $s'(4) = -128 + 48 = -80$ ft./sec.

3.) Let $s(t)$ be height above ground at time t. Then $s(0) = 64$ ft. and $s'(0) = -48$ ft./sec.:



$s''(t) = -32 \rightarrow$

$s'(t) = -32t + c \quad (s'(0) = -48)$

$\rightarrow$ $\boxed{s'(t) = -32t - 48} \rightarrow$

$s(t) = -16t^2 - 48t + c \quad (s(0) = 64) \rightarrow$

$\boxed{s(t) = -16t^2 - 48t + 64}$

a.) strike ground: $s(t) = 0 \rightarrow$

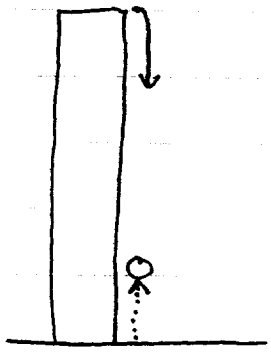
$-16t^2 - 48t + 64 = 0 \rightarrow$

$-16(t^2 + 3t - 4) = 0 \rightarrow$

$-16(t-1)(t+4) = 0 \rightarrow$ $\boxed{t = 1 \text{ sec.}}$

b.) $s'(1) = -80$ ft./sec.

4.) Let $p(t)$ be height of pebble above ground at time t. Let $b(t)$ be height of balloon above ground at time t. Then $p(0) = 1000$ ft. and $p'(0) = 0$ ft./sec.:



balloon: $\boxed{b(t) = 10t}$

pebble: $p''(t) = -32 \rightarrow$

$p'(t) = -32t + c \quad (p'(0) = 0) \rightarrow$

$\boxed{p'(t) = -32t} \rightarrow$

$p(t) = -16t^2 + c \quad (p(0) = 1000) \rightarrow$

$\boxed{p(t) = -16t^2 + 1000}$;

a.) They meet when $p(t) = b(t) \rightarrow$

$-16t^2 + 1000 = 10t \rightarrow$

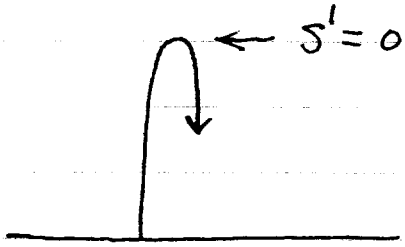
$0 = 16t^2 + 10t - 1000 \rightarrow$

$0 = 2(8t^2 + 5t - 500) \rightarrow$

$t = \dfrac{-5 + \sqrt{25 - 4(8)(-500)}}{16} \approx$ 7.6 sec.

b.) $p(7.6) \approx 243.2$ ft./sec.

5.) Let $s(t)$ be the height above ground at time t. Then $s'(5) = 0$ and $s(0) = 0$:



$s''(t) = -32 \rightarrow$

$s'(t) = -32t + c \quad (\text{and } s'(0) = I\ ?) \rightarrow$

$\boxed{s'(t) = -32t + I} \rightarrow$

$s(t) = -16t^2 + It + c \quad (\text{and } s(0) = 0) \rightarrow$

$\boxed{s(t) = -16t^2 + It}$;

But $s(5) = 0 \rightarrow -400 + 5I = 0 \rightarrow I = +80$

$\rightarrow \boxed{s'(t) = -32t + 80}$ and $\boxed{s(t) = -16t^2 + 80t}$

a.) highest point: $s'(t) = 0 \rightarrow$

$-32t + 80 = 0 \rightarrow$ $t = 2.5$ sec. $\rightarrow$

$s(2.5) = -16(2.5)^2 + 80(2.5) =$ 100 ft.

b.) $s'(0) = 80$ ft./sec.

6.) Let $s(t)$ be the distance (ft.) the vehicle travels after the brakes are applied. Then $s(0) = 0$ ft. and $s'(0) = 80$ ft./sec. Since the deceleration is -20 ft./sec. per sec. it takes $80/20 = 4$ sec. to go from 80 ft./sec. to 0 ft./sec. we have $s'(4) = 0$ ft./sec. :

$s''(t) = -20$ ft./sec.^{2} $\rightarrow$

$s'(t) = -20t + c$ $(s'(0) = 80) \rightarrow c = 80 \rightarrow$

$\boxed{s'(t) = -20t + 80} \rightarrow$

$s(t) = -10t^2 + 80t + c$ $(s(0) = 0) \rightarrow c = 0 \rightarrow$

$\boxed{s(t) = -10t^2 + 80t}$;

after 4 seconds the vehicle has traveled

$s(4) = -160 + 320 =$ 160 ft.