ESP

Kouba

Worksheet 5 Solutions

1.) a.) $\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$ and

$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1$ so that

$\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

b.) $$x^{16}-1 = (x^8+1)(x^8-1) = (x^8+1)(x^4+1)(x^4-1)$$
$$= (x^8+1)(x^4+1)(x^2+1)(x^2-1)$$
$$= (x^8+1)(x^4+1)(x^2+1)(x+1)(x-1) \quad \text{so}$$

$$\lim_{x \to 1} \frac{x^{16}-1}{x-1} = \lim_{x \to 1} \frac{(x^8+1)(x^4+1)(x^2+1)(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$
$$= (2)(2)(2)(2) = 16$$

c.) $0 = \frac{0}{x^2} \le \frac{1+\sin x}{x^2} \le \frac{2}{x^2}$ and $\lim_{x \to +\infty} \frac{2}{x^2} = 0$

so that $\lim_{x \to +\infty} \frac{1+\sin x}{x^2} = 0$.

d.) $$\lim_{x \to 0} \frac{x}{1-\sqrt{x^2+1}} \cdot \frac{(1+\sqrt{x^2+1})}{(1+\sqrt{x^2+1})}$$
$$= \lim_{x \to 0} \frac{x(1+\sqrt{x^2+1})}{1-(x^2+1)} = \lim_{x \to 0} \frac{x(1+\sqrt{x^2+1})}{-x^2}$$
$$= \lim_{x \to 0} \frac{1+\sqrt{x^2+1}}{-x} = \text{"}\frac{2}{\pm 0}\text{"} \quad \text{(does not exist)}$$

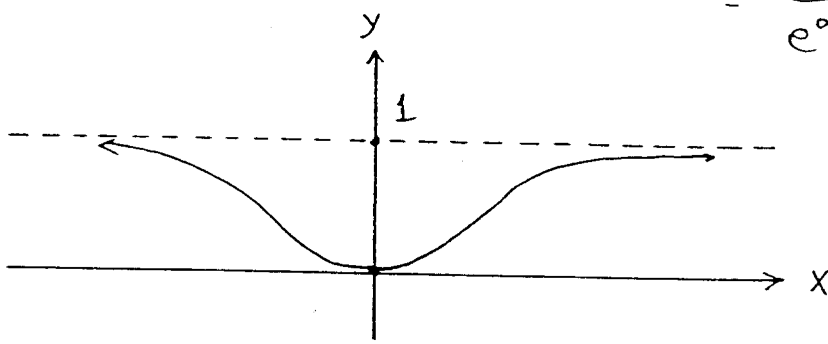
2.) a.) i.) $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} e^{\frac{-1}{x^2}} = e^0 = 1$

ii.) $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} e^{\frac{-1}{x^2}} = e^0 = 1$

iii.) $\lim_{x\to 0} f(x) = \lim_{x\to 0} e^{\frac{-1}{x^2}} = e^{\frac{-1}{+0}} = e^{-\infty}$

$= \dfrac{1}{e^{\infty}} = \dfrac{1}{\infty} = 0$

b.)



3.) a.) $y = x^3$ and $y = \sin x$ are continuous for all x-values, so $f(x) = x^3 + \sin x$ is continuous for all x-values (sum of continuous functions)

b.) $y = 2x$ and $y = \cos x$ are continuous for all x-values, so $f(x) = 2x \cdot \cos x$ is continuous for all x-values (product of continuous functions)

c.) $y = 3x^5$ and $y = x^2 - 4x$ (polynomials) are continuous for all x-values, so

$f(x) = \dfrac{3x^5}{x^2-4x} = \dfrac{3x^5}{x(x-4)}$ is continuous (quotient of continuous functions)

for all x-values except $x=0$, $x=4$.

d.) $y = \sin x$ and $y = \cos x$ are continuous for all x-values, so $y = \tan x = \dfrac{\sin x}{\cos x}$ (quotient of continuous functions) is continuous for all x-values except where $\cos x = 0$: $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

e.) $Y = 3$ and $Y = 1 + \cos X$ are continuous for all x-values, so $y = \frac{3}{1+\cos x}$ is continuous for all x-values except where $\cos x = -1$:
$x = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$ (quotient of continuous functions)

f.) $Y = 1$ and $Y = \cos 2X$ are continuous for all x-values, so $g(x) = \sec 2X = \frac{1}{\cos 2X}$ is continuous for all x-values except where $\cos 2X = 0 \rightarrow 2X = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \rightarrow$

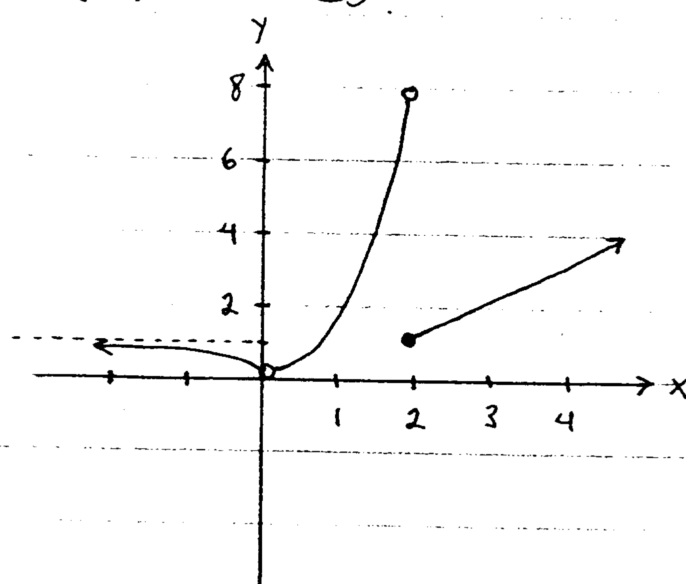
$x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \ldots$ (quotient of continuous functions)

g.) $f(x) = x^5$ and $g(x) = 5 + \cos x$ are continuous for all x-values, so $(f \circ g)(x) = f(g(x)) = (5 + \cos x)^5$ is continuous for all x-values.

h.) $f(x) = x^{20} + 2$ and $g(x) = 4x^3 - 1$ and $b(x) = x^{10}$ are continuous for all x-values, so
$(b \circ (f \circ g))(x) = b(f(g(x))) = b((4x^3-1)^{20}+2) = [(4x^3-1)^{20}+2]^{10}$
is continuous for all x-values.

(composition of continuous functions)

i.) $f(x) = \begin{cases} x - 1, & x \geq 2 \\ 2x^2, & 0 < x < 2 \\ \frac{x}{x-1}, & x < 0 \end{cases}$

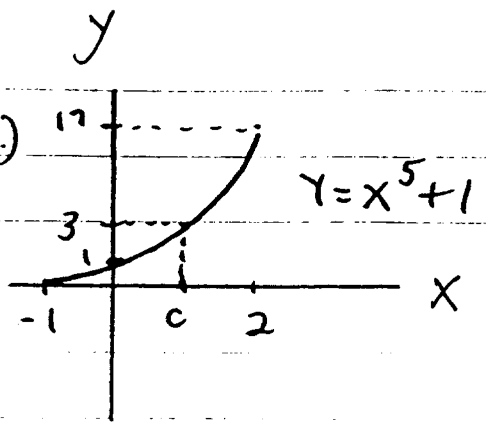


$\lim_{x \to 2^+} f(x) = 1$, $\lim_{x \to 2^-} f(x) = 8$,

$\lim_{x \to 0^+} f(x) = 0$, $\lim_{x \to 0^-} f(x) = 0$,

f is not continuous at $x = 0$ ($f(0)$ is not defined) and at $x = 2$ ($\lim_{x \to 2} f(x)$ does not exist).

4.) a)



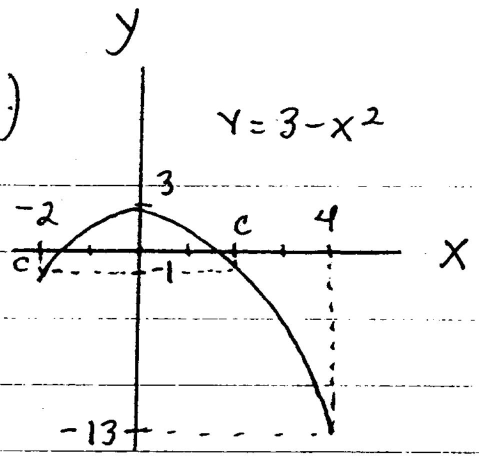
$x^5 + 1 = 3 \Rightarrow$

$x^5 = 2 \Rightarrow$

$x = 2^{1/5}$

$c = 2^{1/5}$

b.)



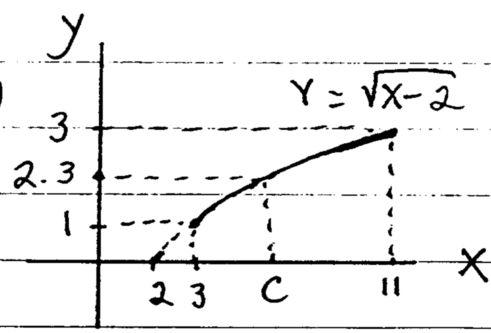
$3 - x^2 = -1 \Rightarrow$

$x^2 = 4 \Rightarrow$

$x = -2$ or 2

$c = -2$ or 2

c.)



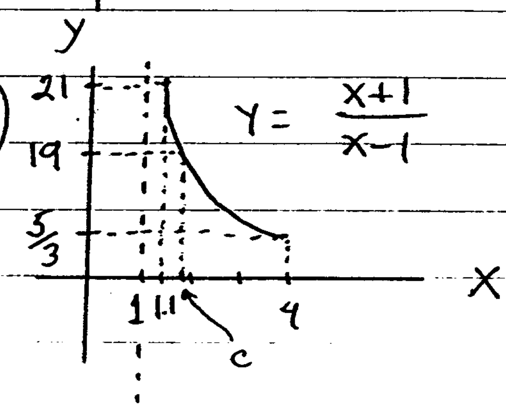
$\sqrt{x-2} = 2.3 \Rightarrow$

$x - 2 = 5.29 \Rightarrow$

$x = 7.29$

$c = 7.29$

d.)



$\frac{x+1}{x-1} = 19 \Rightarrow$

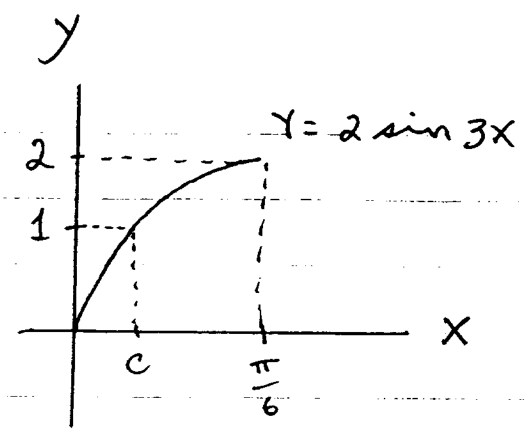
$x + 1 = 19x - 19 \Rightarrow$

$20 = 18x \Rightarrow$

$x = \frac{10}{9}$

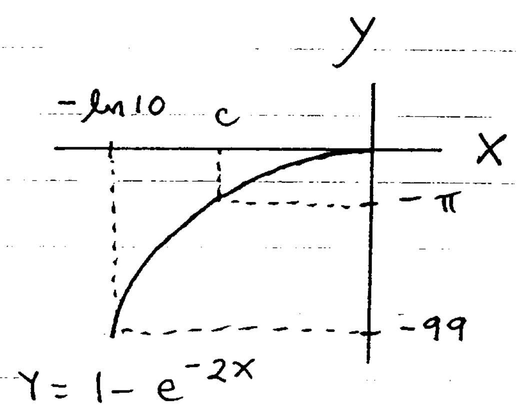
$c = \frac{10}{9}$

e.)



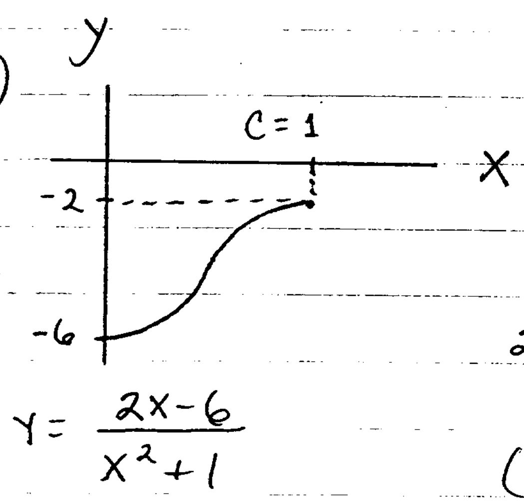
$$2\sin 3X = 1 \Rightarrow$$
$$\sin 3X = \frac{1}{2} \Rightarrow$$
$$3X = \frac{\pi}{6} \Rightarrow$$
$$X = \frac{\pi}{18}$$
$$c = \frac{\pi}{18}$$

f.)



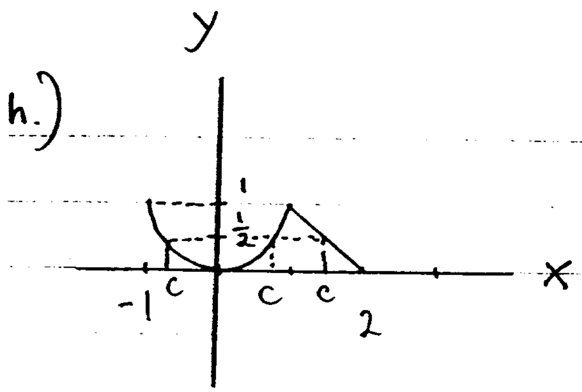
$$1 - e^{-2X} = -\pi \Rightarrow$$
$$e^{-2X} = \pi + 1 \Rightarrow$$
$$\ln e^{-2X} = \ln(\pi+1) \Rightarrow$$
$$-2X = \ln(\pi+1) \Rightarrow$$
$$X = \frac{-1}{2}\ln(\pi+1)$$
$$c = \frac{-1}{2}\ln(\pi+1)$$

g.)



$$\frac{2X-6}{X^2+1} = -2 \Rightarrow$$
$$2X - 6 = -2X^2 - 2 \Rightarrow$$
$$2X^2 + 2X - 4 = 0 \Rightarrow$$
$$X^2 + X - 2 = 0 \Rightarrow$$
$$(X-1)(X+2) = 0 \Rightarrow$$
$$\downarrow \qquad \downarrow$$
$$X = 1 \qquad X = -2$$
$$C = 1$$

h.)



$$f(x) = \begin{cases} x^2, & x \le 1 \\ 2-x, & x > 1 \end{cases}$$

$x^2 = 1/2 \Rightarrow$

$x = \pm 1/\sqrt{2}$ $\quad c = \pm \frac{1}{\sqrt{2}}$

$2 - x = \frac{1}{2} \Rightarrow$

$x = 3/2$ $\quad c = 3/2$