

ESP  
Kouba  
Worksheet 8

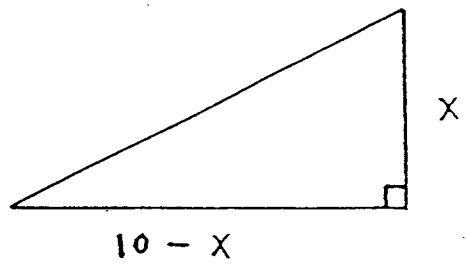
1. Sketch a graph of each of the following functions. Use  $\frac{f(2+h)-f(2)}{h}$  to find the slope of the line tangent to each graph of  $y = f(x)$  at  $x = 2$ .

- a.  $f(x) = 1/(x+1)$
- b.  $f(x) = \sqrt{x}$

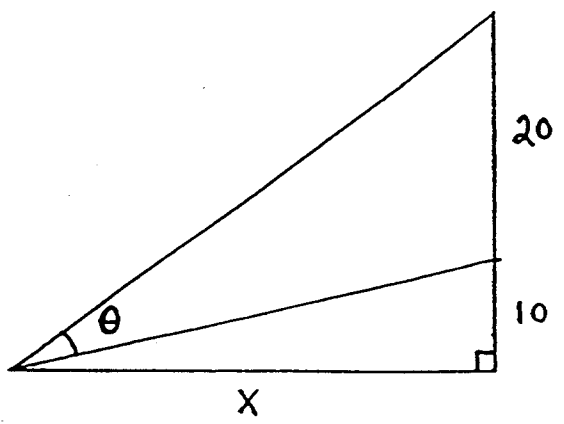
2. Derive an equation for the line

- a. passing through the points  $(-1,0)$  and  $(2/3, -1/4)$ .
- b. passing through the point  $(-1, 0)$  and perpendicular to the line  $x + 2y - 3 = 0$ .

3. Derive formulas for the area  $A$  and perimeter  $P$  of the given right triangle as a function of  $x$ .



4. Derive a formula for the measure of angle  $\theta$  as a function of  $x$ .



5. Find all points  $(x, y)$  which are equidistant from the three given points  $(0, 0)$ ,  $(4, 0)$ , and  $(3, 2)$ .

6. Sketch a graph of a function with all of the following properties.

- a. y-intercept at 3
- b. x-intercepts at 2 and -7
- c.  $\lim_{x \rightarrow 10^-} f(x) = -\infty$  and  $\lim_{x \rightarrow 10^+} f(x) = -\infty$
- d. f is not continuous at  $x = 4$  but  $\lim_{x \rightarrow 4} f(x) = 5$
- e. f is continuous but not differentiable at  $x = -3$
- f.  $\lim_{x \rightarrow +\infty} f(x) = 1$

7. Let  $f(x) = \begin{cases} 1 & \text{for } x \leq 0 \\ 3x & \text{for } x > 0 \end{cases}$  and let  $g(x) = (f \circ f)(x) = f(f(x))$ .

- a. Sketch the graph of f.
- b. Sketch the graph of g.

8. Use  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to compute the derivative of each of the following functions.

- a.  $f(x) = 7$
- b.  $f(x) = 3x + 2/3$
- c.  $f(x) = x + (x+3)^2$
- d.  $f(x) = x^2 + \frac{5}{x}$
- e.  $f(x) = x^2 \sin x$
- f.  $f(x) = \sqrt{1 + \sqrt{x}}$

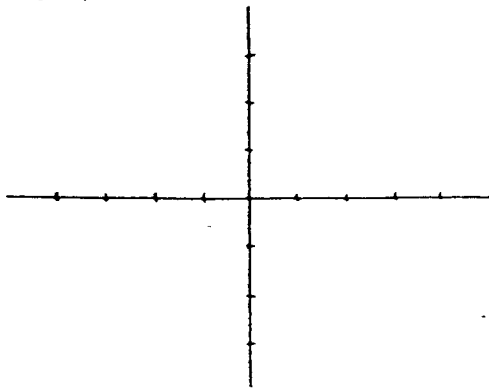
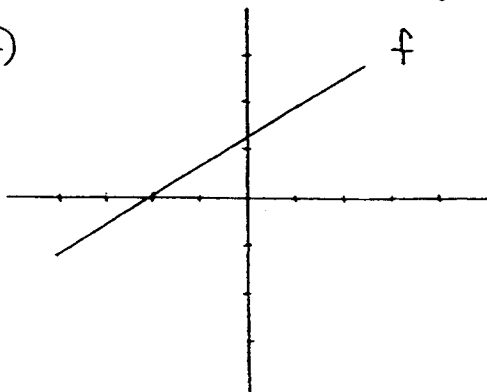
9. Let  $f(x) = \begin{cases} x^2 & \text{for } x > 0 \\ x & \text{for } x \leq 0 \end{cases}$  and  $g(x) = [f(x)]^2$ .

- a. Sketch the graph of f.
- b. Is f continuous at  $x = 0$ ?
- c. Is f differentiable at  $x = 0$ ?
- d. Sketch the graph of g.
- e. Is g continuous at  $x = 0$ ?
- f. Is g differentiable at  $x = 0$ ?

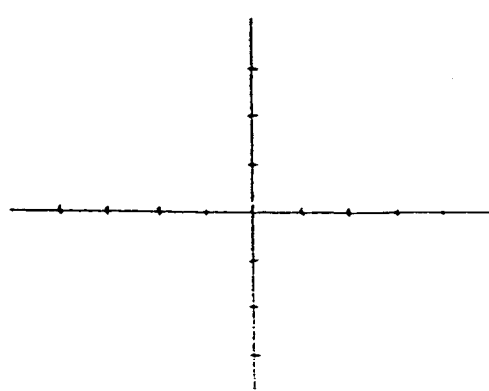
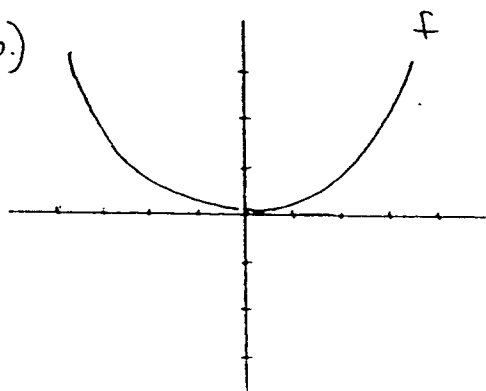
10. A projectile is being fired from the point  $(4, 0)$  towards a barrier sitting on the curve  $y = \sqrt{x}$ . What point on the curve should be targeted in order that the projectile strike the barrier orthogonally?

11. Sketch a rough graph of  $f'$  by using the graph of  $f$ .

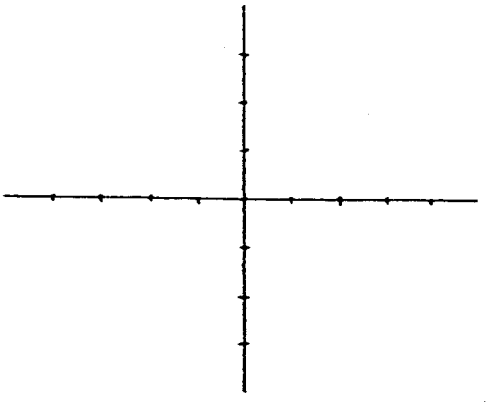
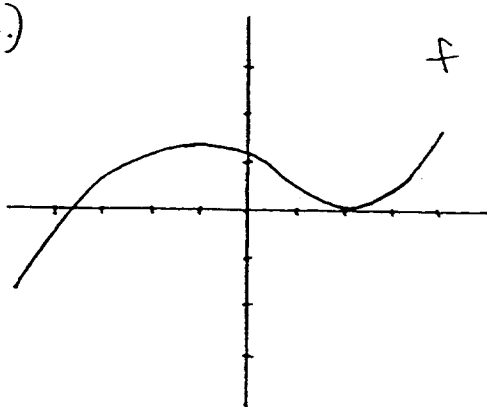
a.)



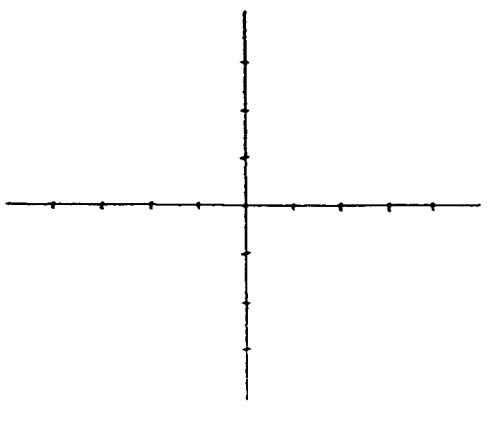
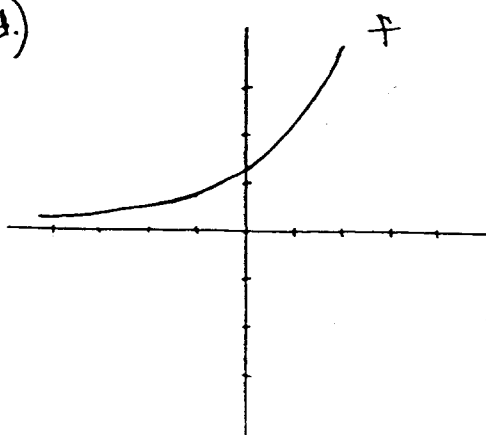
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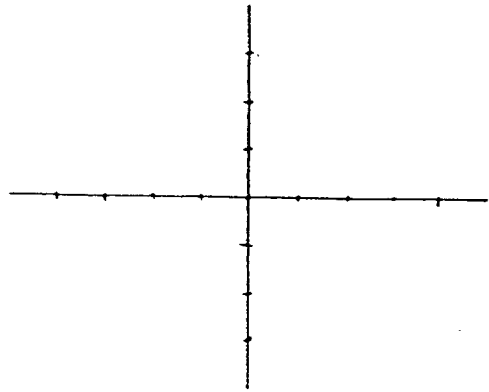
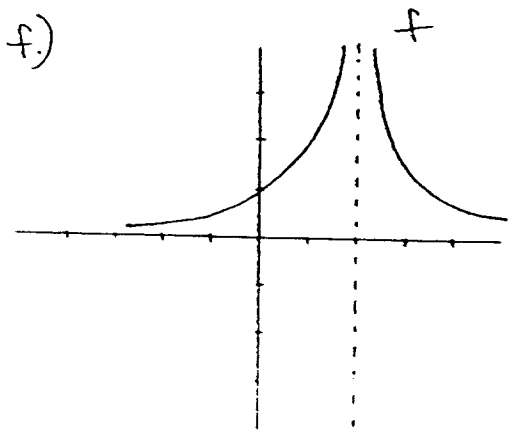
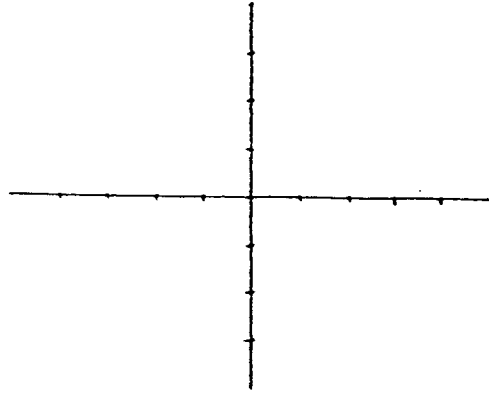
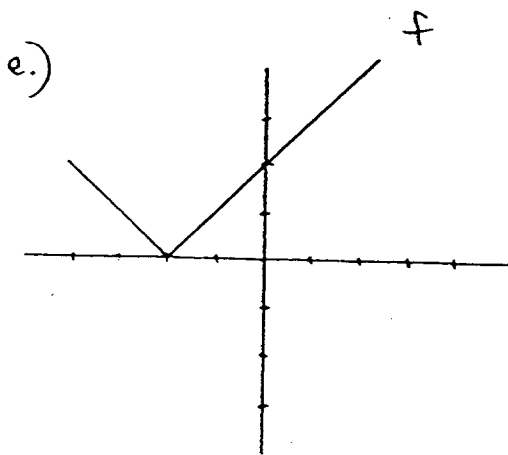


c.)



d.)





12. True or False :  $x^2 - xy + y^2 \geq xy$  for all values of  $x$  and  $y$ .

13. Three spheres of radius 2 feet sit in contact with each other on a flat surface. What is the radius of the largest hemisphere which can fit (flat side down) beneath these three spheres without displacing them ?