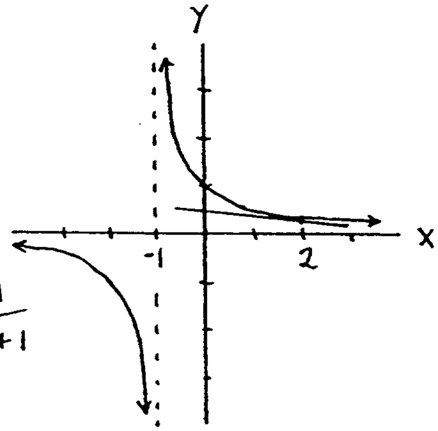
ESP
Kouba
Worksheet 8 Solutions

1.) a.)



$Y = \frac{1}{x+1}$

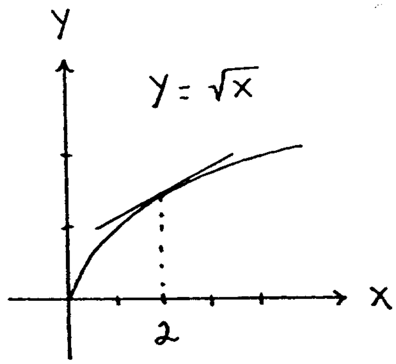
$$\frac{f(2+h)-f(2)}{h} = \frac{\frac{1}{2+h+1} - \frac{1}{3}}{\frac{h}{1}}$$

$$= \frac{3-(3+h)}{(3+h)\,3} \cdot \frac{1}{h} = \frac{-\cancel{h}}{(3+h)\,3\,\cancel{h}}$$

(Let h be very small.)

so slope $= \frac{-1}{(3+0)\,3} = \frac{-1}{9}$.

b.)



$$\frac{f(2+h)-f(2)}{h} = \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

$$= \frac{(\sqrt{2+h}-\sqrt{2})}{h} \frac{(\sqrt{2+h}+\sqrt{2})}{(\sqrt{2+h}+\sqrt{2})}$$

$$= \frac{(2+h)-2}{h(\sqrt{2+h}+\sqrt{2})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{2+h}+\sqrt{2})}$$ (Let h be very small.)

so slope $= \frac{1}{\sqrt{2+0}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$

2.) a.) $m = \frac{0-(\frac{-1}{4})}{-1-\frac{2}{3}} = \frac{\frac{1}{4}}{\frac{-5}{3}} = \frac{1}{4} \cdot \frac{-3}{5} = \frac{-3}{20} \rightarrow$

$Y = mx + b \rightarrow 0 = \frac{-3}{20}(-1) + b \rightarrow b = \frac{-3}{20}$ so

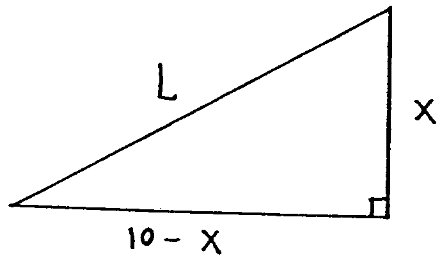
$Y = \frac{-3}{20}x + \frac{-3}{20}$.

6.) $X + 2Y - 3 = 0 \rightarrow Y = \frac{-1}{2}X + \frac{3}{2}$ so $m = 2 \rightarrow$

$Y = mx + b \rightarrow 0 = 2(-1) + b \rightarrow b = 2$ and

$$Y = 2X + 2 .$$

3.)



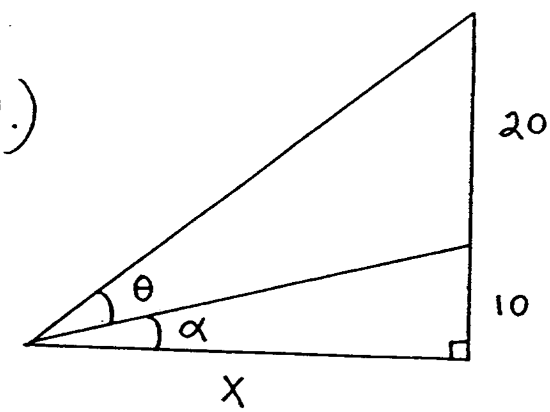
$L^2 = X^2 + (10 - X)^2$ so

$L = \sqrt{X^2 + (10 - X)^2}$ and

area $A = \frac{1}{2}(10 - X)X$ and

perimeter $P = (10 - X) + X + L = 10 + \sqrt{X^2 + (10 - X)^2}$.

4.)



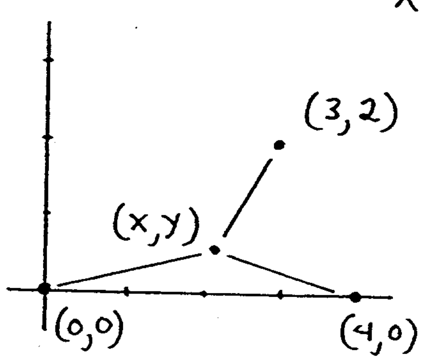
$\tan \alpha = \frac{10}{X} \rightarrow$

$\alpha = \arctan \frac{10}{X}$;

$\tan(\theta + \alpha) = \frac{30}{X} \rightarrow$

$\theta + \alpha = \arctan \frac{30}{X}$ or $\theta = \arctan \frac{30}{X} - \arctan \frac{10}{X}$.

5.)



The three equal distances are

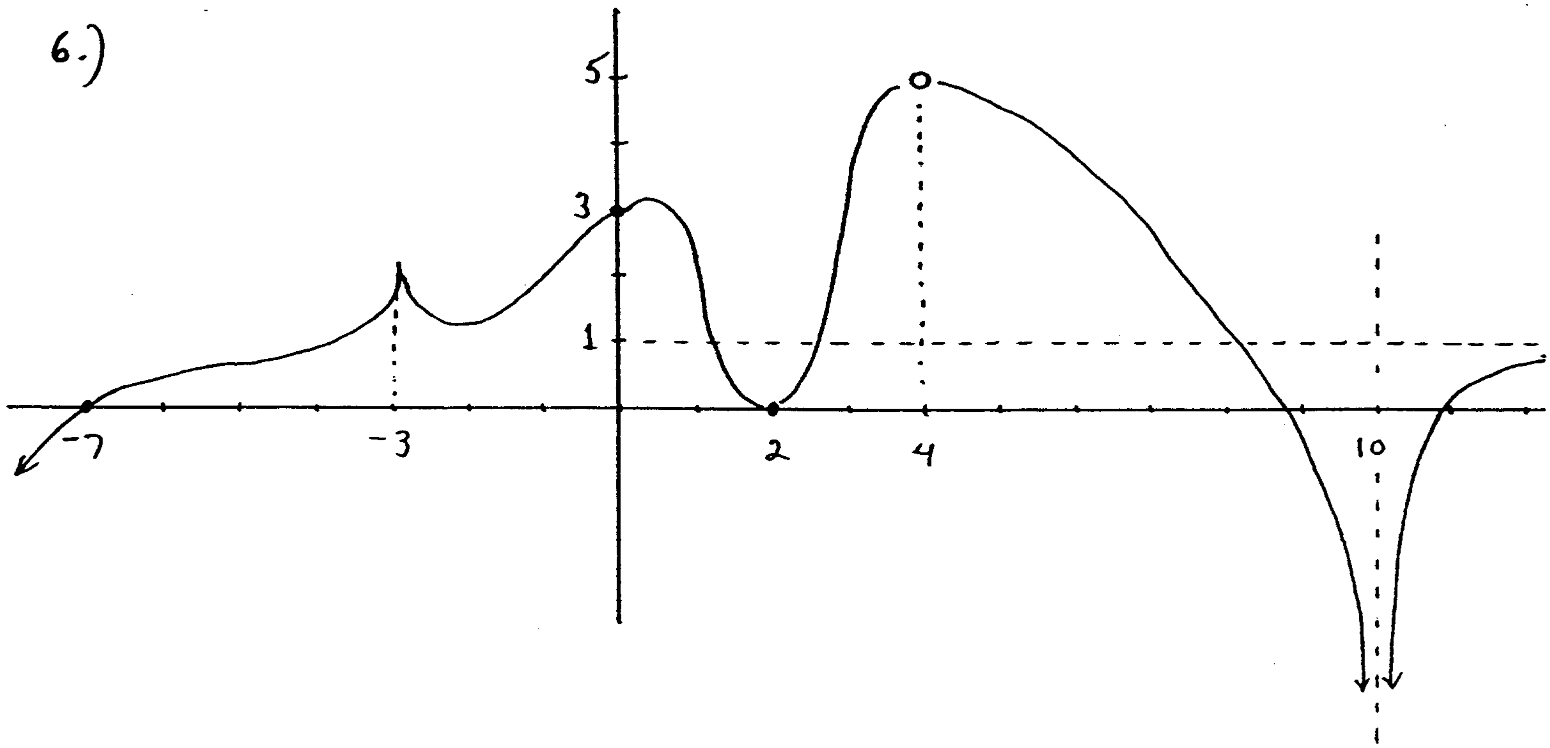
$\sqrt{(X-0)^2 + (Y-0)^2}$, $\sqrt{(X-3)^2 + (Y-2)^2}$,

and $\sqrt{(X-4)^2 + (Y-0)^2} \rightarrow$

$$\left.\begin{array}{l} (X-0)^2 + (Y-0)^2 = (X-4)^2 + (Y-0)^2 \\ (X-0)^2 + (Y-0)^2 = (X-3)^2 + (Y-2)^2 \end{array}\right\} \left\{\begin{array}{l} \cancel{X^2} + \cancel{Y^2} = \cancel{X^2} - 8X + 16 + \cancel{Y^2} \\ \cancel{X^2} + \cancel{Y^2} = \cancel{X^2} - 6X + 9 + \cancel{Y^2} - 4Y + 4 \end{array}\right\}$$

$$\left.\begin{array}{l} 8X = 16 \\ 6X + 4Y = 13 \end{array}\right\} \quad X = 2 \text{ and } Y = \frac{1}{4} .$$

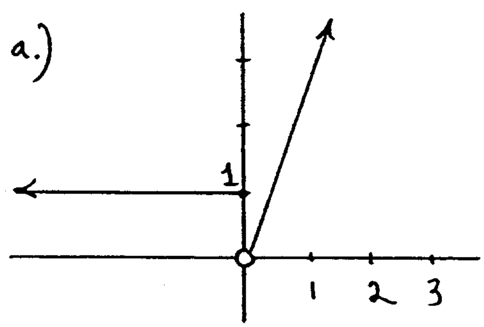
6.)



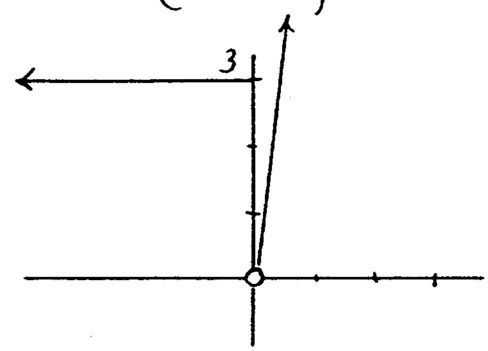
7.)

$$g(x) = f(f(x)) = \begin{cases} f(1) & , x \le 0 \\ f(3x) & , x > 0 \end{cases} = \begin{cases} 3 & , x \le 0 \\ 9x & , x > 0 \end{cases}$$

a.)



b.)



8.) a.) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{7-7}{h} = \lim_{h \to 0} 0 = 0$

b.) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\{3(x+h) + \frac{2}{3}\} - \{3x + \frac{2}{3}\}}{h}$

$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}} = 3$

c.) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\{(x+h) + (x+h+3)^2\} - \{x + (x+3)^2\}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x} + h + \cancel{x^2} + h^2 + 2xh + \cancel{6x} + 6h + \cancel{9} - \cancel{x} - \cancel{x^2} - \cancel{6x} - \cancel{9}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(1 + h + 2x + 6)}{\cancel{h}} = 2x + 7$$

d.) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\{(x+h)^2 + \frac{5}{x+h}\} - \{x^2 + \frac{5}{x}\}}{h}$$

$$= \lim_{h\to 0} \left\{ \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} + \frac{\frac{5}{x+h} - \frac{5}{x}}{h} \right\}$$

$$= \lim_{h\to 0} \left\{ \frac{\cancel{h}(2x+h)}{\cancel{h}} + \frac{\cancel{5x} - \cancel{5x} - 5h}{(x+h)x\ h} \right\}$$

$$= \lim_{h\to 0} \left\{ (2x+h) - \frac{5}{(x+h)x} \right\} = 2x - \frac{5}{x^2}$$

e.) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$$

$$= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2)(\sin x \cos h + \cos x \sin h) - x^2 \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\begin{array}{c} x^2 \sin x \cosh + x^2 \cos x \sinh + 2xh \sin x \cosh \\ + 2xh \cos x \sinh + h^2 \sin x \cosh + h^2 \cos x \sinh - x^2 \sin x \end{array}}{h}$$

$$= \lim_{h \to 0} \left\{ x^2 \sin x \frac{(\cosh - 1)}{h} + x^2 \cos x \cdot \frac{\sinh}{h} + \frac{\cancel{h}\,(2x \sin x \cosh + 2x \cos x \sinh + h \sin x \cosh + h \cos x \sinh)}{\cancel{h}} \right\}$$

$$= x^2 \sin x \cdot (0) + x^2 \cos x \cdot (1) + 2x \sin x \cdot (\,)$$
$$+ 2x \cos x \cdot (0) + 0 + 0 = x^2 \cos x + 2x \sin x$$

f.) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{\sqrt{1+\sqrt{x+h}} - \sqrt{1+\sqrt{x}}}{h}$

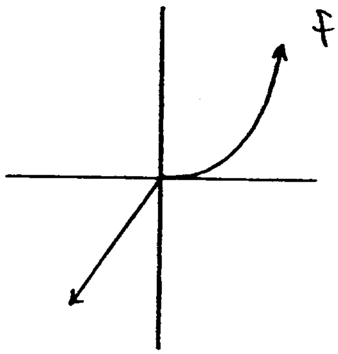
$$= \lim_{h \to 0} \frac{\sqrt{1+\sqrt{x+h}} - \sqrt{1+\sqrt{x}}}{h} \cdot \frac{\sqrt{1+\sqrt{x+h}} + \sqrt{1+\sqrt{x}}}{\sqrt{1+\sqrt{x+h}} + \sqrt{1+\sqrt{x}}}$$

$$= \lim_{h \to 0} \frac{(\cancel{1} + \sqrt{x+h}) - (\cancel{1} + \sqrt{x})}{h\left(\sqrt{1+\sqrt{x+h}} + \sqrt{1+\sqrt{x}}\right)} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{(\cancel{x} + h) - \cancel{x}}{h\left(\sqrt{1+\sqrt{x+h}} + \sqrt{1+\sqrt{x}}\right)\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\left(\sqrt{1+\sqrt{x+h}} + \sqrt{1+\sqrt{x}}\right)\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{1}{2\sqrt{1+\sqrt{x}} \cdot 2\sqrt{x}}$$

9.) $f(x) = \begin{cases} x^2 & \text{for } x > 0 \\ x & \text{for } x \leq 0 \end{cases}$, $g(x) = \begin{cases} x^4 & \text{for } x > 0 \\ x^2 & \text{for } x \leq 0 \end{cases}$

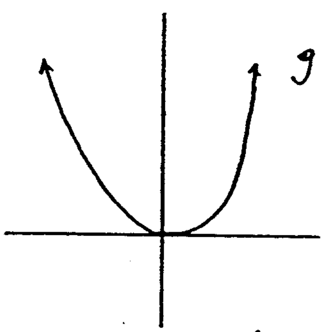
a.) 

b.) Yes, since $\lim\limits_{x \to 0} f(x) = f(0)$

c.) No, since

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

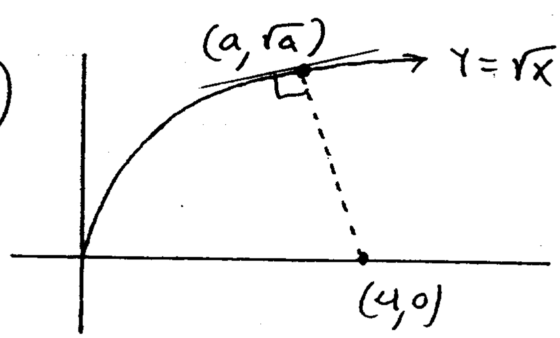
$$= \lim_{h \to 0} \frac{f(h)}{h} \text{ does not exist.}$$

d.) 

e.) Yes, since $\lim\limits_{x \to 0} g(x) = g(0)$

f.) Yes, since

$$g'(0) = \lim_{h \to 0} \frac{g(0+h) - g(0)}{h} = \lim_{h \to 0} \frac{g(h)}{h} = 0 .$$

10.) 

Slope of tangent line at $x = a$ is $f'(a) = \frac{1}{2\sqrt{a}}$ so

path of projectile has slope

i.) $-2\sqrt{a}$

and ii.) $\frac{\sqrt{a} - 0}{a - 4}$ $\Bigg\} \Rightarrow \frac{\sqrt{a}}{a-4} = -2\sqrt{a} \Rightarrow$

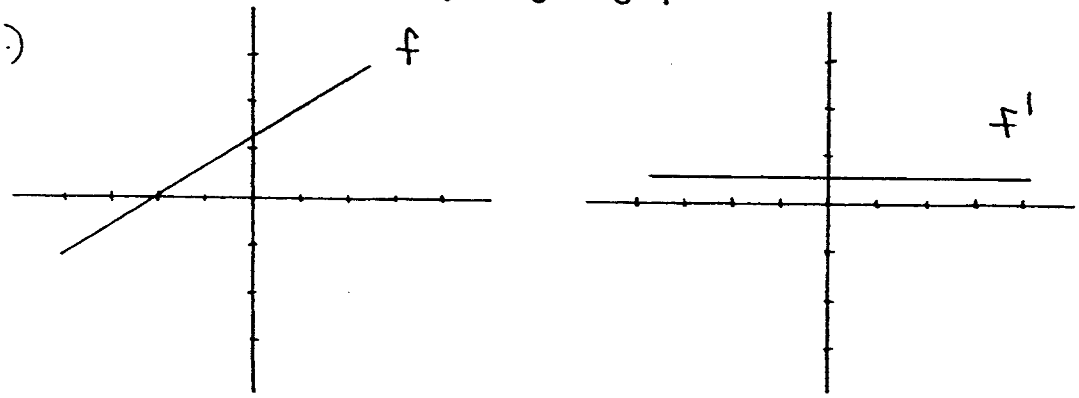
$\sqrt{a} = -2a^{3/2} + 8\sqrt{a} \Rightarrow 2a^{3/2} - 7\sqrt{a} = 0 \Rightarrow$

$\sqrt{a}\,(2a-7) = 0 \Longrightarrow a = 0$ or $a = \frac{7}{2}$ so

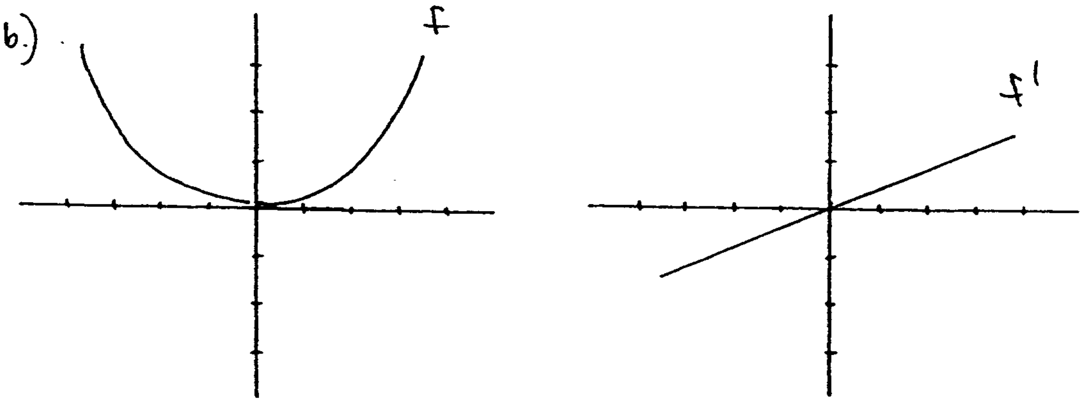
points are $(0,0)$ and $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$.

11.) Sketch a rough graph of f ' by using the graph of f .

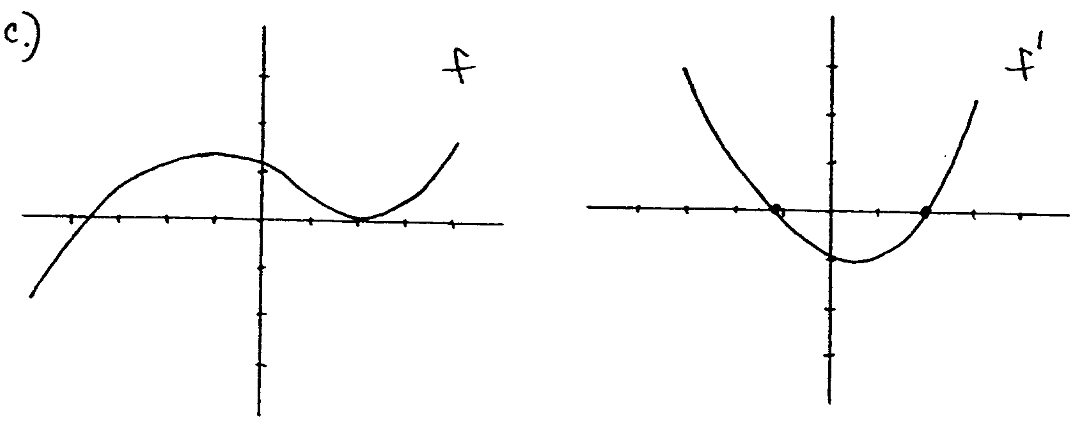
a.)



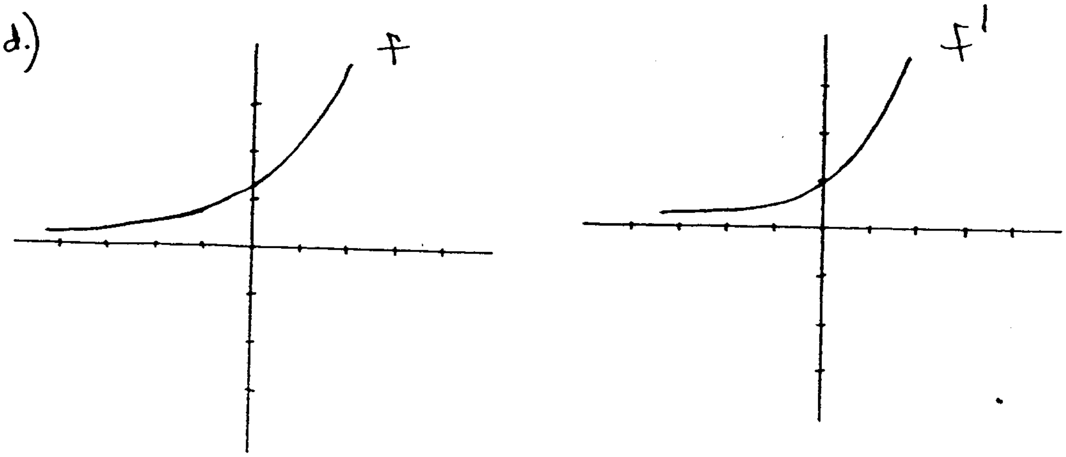
b.)



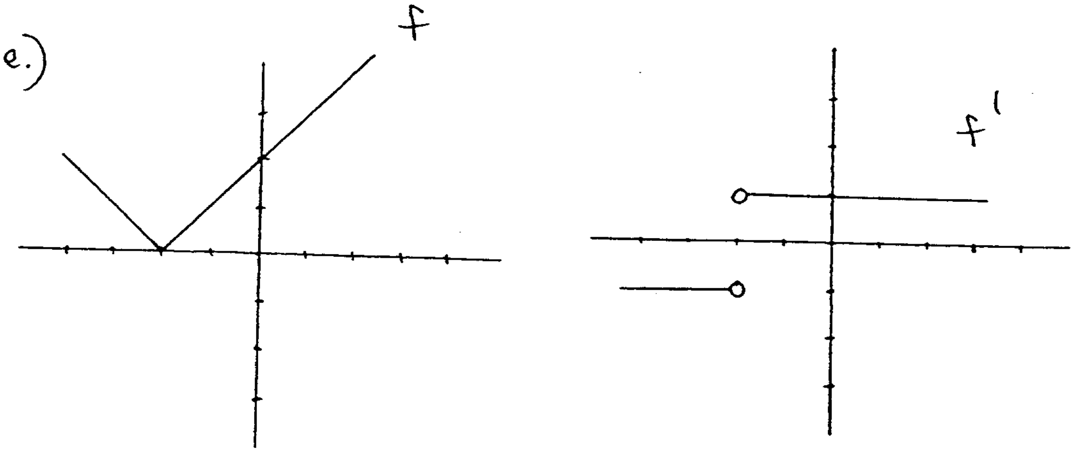
c.)



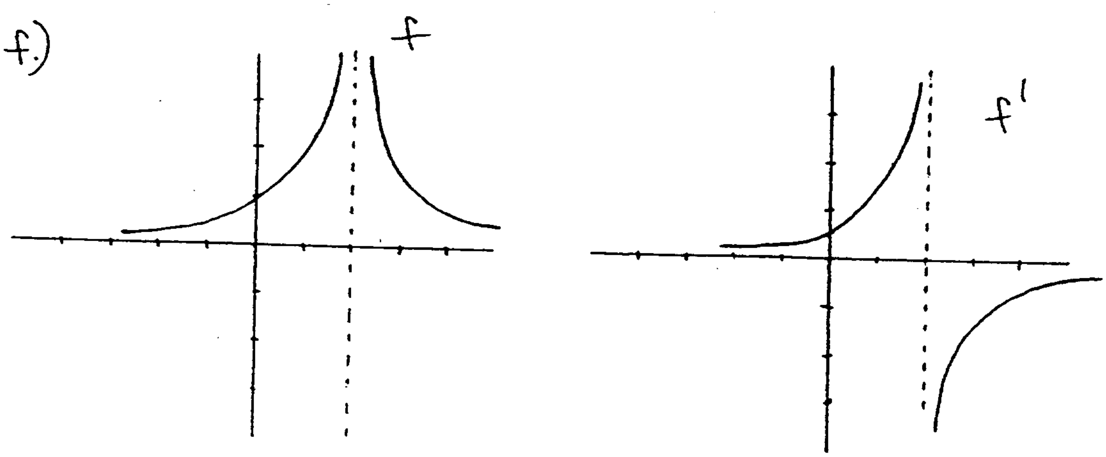
d.)



e.)



f.)



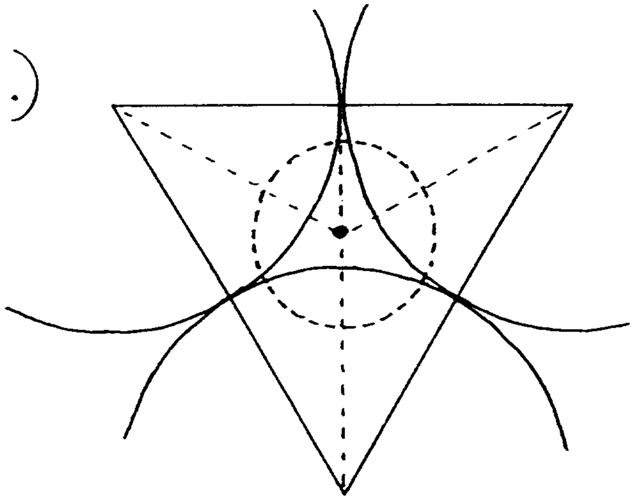
12.) $X^2 - XY + Y^2 \geq XY \quad \longleftrightarrow \quad X^2 - 2XY + Y^2 \geq 0$

$\longleftrightarrow \quad (X-Y)^2 \geq 0$

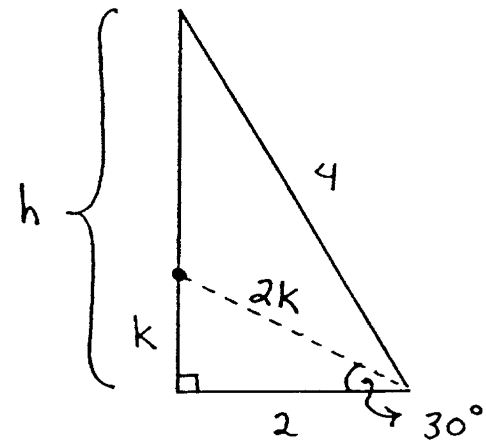
for all values of X and Y.

TRUE !

13.)



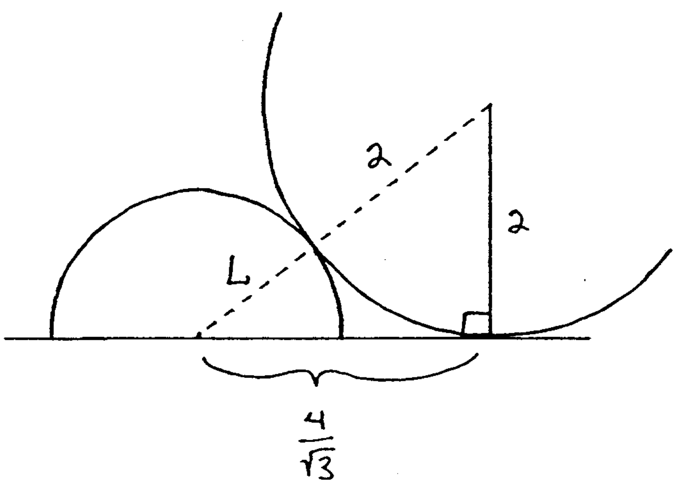
Top View



$$h^2 + 2^2 = 4^2 \rightarrow h^2 = 12 \rightarrow h = 2\sqrt{3} \quad ,$$

$$k^2 + 2^2 = (2k)^2 \rightarrow k^2 + 4 = 4k^2 \rightarrow 4 = 3k^2 \rightarrow$$

$$k = \frac{2}{\sqrt{3}} \quad \text{then} \quad h - k = 2\sqrt{3} - \frac{2}{\sqrt{3}} = \frac{4}{\sqrt{3}} \quad ;$$



Side View

$$(L+2)^2 = \left(\frac{4}{\sqrt{3}}\right)^2 + 2^2 \rightarrow$$

$$(L+2)^2 = \frac{16}{3} + \frac{12}{3} \rightarrow$$

$$L + 2 = \sqrt{\frac{28}{3}} \rightarrow$$

$$L = \sqrt{\frac{28}{3}} - 2 \approx 1.055$$