Math 21B
Kouba
Discussion Sheet 3

1.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ b.) $\displaystyle\int \frac{e^x}{\sqrt{1-(e^x)^2}}\,dx$ c.) $\displaystyle\int \frac{e^x}{1+e^x}\,dx$ d.) $\displaystyle\int \frac{1}{1+e^x}\,dx$

e.) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$ f.) $\displaystyle\int \cos^2 x\,dx$ g.) $\displaystyle\int \cot x\,dx$ h.) $\displaystyle\int \cot^2 x\ \csc^2 x\,dx$

i.) $\displaystyle\int \frac{x^2+5x+6}{x^2}\,dx$ j.) $\displaystyle\int \frac{x^2+5x+6}{x+1}\,dx$ k.) $\displaystyle\int (x^2+1)(x^3+3x)^{10}\,dx$

l.) $\displaystyle\int \frac{x+6}{(x+5)^2}\,dx$ m.) $\displaystyle\int \frac{(\ln x)^4}{x}\,dx$ n.) $\displaystyle\int \sec^2(3x)\ 2^{\tan(3x)}\,dx$

2.) Use any method to compute the area of the region bounded by the graphs of

a.) $y=x, y=0$, and $y=4-x$. b.) $y=\sqrt{1-x^2}$ and $y=0$.

3.) Assume that snow is falling at the rate of $t+\sqrt{t}$ in./hr. at time t hours. SET UP a definite integral and compute the total amount of snowfall between $t=0$ and $t=4$ hours.

4.) Use the limit definition of the definite integral to evaluate $\displaystyle\int_1^5 \frac{1}{x^2}\,dx$; use an arbitrary partition $1=x_0<x_1<x_2<\dots<x_{n-1}<x_n=5$ of the interval $[1,\ 5]$ and use sampling points $c_i=\sqrt{x_{i-1}x_i}$ for $i=1,2,3,\dots,n$.

5.) Determine an equation of the line tangent to the graph of
$F(x)=3+2x+x\displaystyle\int_1^x \arctan t\,dt$ at $x=1$.

6.) Each of the following limits is equal to a definite integral. Determine a definite integral for each. Do not evaluate the definite integral.

a.) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} 3\left(1+\frac{2i}{n}\right)^{-4}\left(\frac{2}{n}\right)$ b.) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \ln\left(3+\frac{2i}{n}\right)\left(\frac{8}{n}\right)$

c.) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \frac{i}{in+n^2}$ d.) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \frac{(n+2i-1)^2}{n^3}$

7.) Assume that f is an odd function and $\displaystyle\int_{-2}^{1} f(x)\,dx=3$. What is the value of

$$\int_{-1}^{-2} f(x)\,dx \quad ?$$

8.) A thin rod lies along the x-axis between $x = 1$ and $x = \ln 5$. Its density at x cm. is given by $e^{-x}(1 - e^{-x})^5$ gm./cm. SET UP a definite integral and compute the exact mass of the rod.

9.) Find all values of c guaranteed by the Mean Value Theorem for Integrals for

$$f(x) = \begin{cases} x + 1, & \text{if } -1 \leq x < 0 \\ 1 - x^2, & \text{if } 0 \leq x \leq 1 \end{cases} .$$

10.) The total distance s (in miles) traveled by a hiker at time t (in hours) is $s(t) = t + \ln(1 + (\frac{1}{2})t)$. Find the hiker's average hiking speed between $t = 0$ hrs. and $t = 4$ hrs.

11.) Determine a function having the following properties :

$$f''(x) = 1 + e^{\frac{x}{2}}, f'(0) = -1, \text{ and } f(0) = 3$$

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

12.) Two bicyclists are twelve miles apart. They begin riding toward each other, one pedaling at 4 mph and the other at 2 mph. At the same time a bumblebee begins flying back and forth between the riders at a constant speed of 10 mph. How far does the bumblebee travel by the time the riders meet ?