

Math 21B
Kouba
Challenge Sheet 1

1.) Evaluate the following sums.

$$\begin{array}{lll} \text{a.) } \sum_{i=1}^{1000} (i+3)^2 & \text{b.) } \sum_{i=1}^{750} \{(i+1)^3 - i^3\} & \text{c.) } \sum_{i=17}^{83} \left(\frac{1}{i+4} - \frac{1}{i+3} \right) \\ \text{d.) } \sum_{i=8}^{98} \log \left(\frac{i+1}{i+2} \right) & \text{e.) } \sum_{i=1}^{200} \ln(1 + 1/i) & \text{f.) } \sum_{i=734}^{10,125} (3 + \sin i\pi) \end{array}$$

2.) Evaluate the following limit : $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{i^2 + i}$

3.) The speed of a runner at time t seconds is given by $v(t) = t^2 - t + 2$ ft./sec.

a.) Sketch the speed graph for $0 \leq t \leq 5$.

b.) What is the runner's speed when

- i.) $t=0$ seconds ?
- ii.) $t=1$ second ?
- iii.) $t=5$ seconds ?

c.) Estimate the distance the runner travels for $0 \leq t \leq 5$ by using five equal subdivisions of time on the interval $[0, 5]$ and speeds determined by

- i.) left endpoints of the subdivisions.
- ii.) right endpoints of the subdivisions.
- iii.) midpoints of the subdivisions.

d.) Evaluate the following limit : $\lim_{n \rightarrow \infty} \sum_{i=1}^n v(5i/n) \cdot (5/n)$. Explain why the answer represents the exact distance covered by the runner for $0 \leq t \leq 5$.