Math 21B
Kouba
Challenge Sheet 4

1.) Use any method to integrate the following.

a.) $\int x \cdot \sqrt{x^2+25}\ dx$ b.) $\int x \cdot \sqrt{x+25}\ dx$

c.) $\int x^3 \cdot \sqrt{x^2+25}\ dx$ d.) $\int \sqrt{2+x}\ dx$

e.) $\int \sqrt{2+\sqrt{2+\sqrt{x}}}\ dx$ f.) $\int \sqrt{1+\sin x}\ dx$

g.) $\int \frac{\cos^3 x}{\sqrt{1+\sin x}}\ dx$ h.) $\int \frac{1-\tan^2 x}{1+\tan x}\ dx$

i.) $\int \frac{1+\tan^2 x}{1+\tan x}\ dx$ j.) $\int \frac{\tan x+\sec^2 x}{\tan x+e^{-x}}\ dx$

2.) Show that the following equation is true. HINT: Use integration by parts :

$$\int_a^b \left(\int_a^x f(t)\ dt \right)\ dx = \int_a^b (b-x)f(x)\ dx$$

3.) Use any method to integrate the following.

a.) $\int \frac{1}{x^{1/2}+x}\ dx$ b.) $\int \frac{1}{x^{1/n}+x}\ dx$, where n is a positive integer.

4.) Determine the volume of the

a.) sphere of largest volume
b.) cylinder of largest volume

which can be inscribed in a symmetrical pyramid with square base of area 36 square feet and height 4 feet.