

ESP
Kouba
Worksheet 11

1.) Integrate using any method.

a.) $\int \sqrt{1-x^2} \, dx$ b.) $\int \sqrt{4-x^2} \, dx$ c.) $\int \sqrt{1-9x^2} \, dx$

d.) $\int \sqrt{4-9x^2} \, dx$ e.) $\int \sqrt{x^2-1} \, dx$ f.) $\int x\sqrt{x^2-9} \, dx$

g.) $\int \sec \theta \, d\theta$ h.) $\int \sec^3 \theta \, d\theta$ i.) $\int \sec^5 \theta \, d\theta$

j.) $\int x^2\sqrt{x^2-9} \, dx$ k.) $\int \sqrt{x^2+1} \, dx$ l.) $\int \frac{\sqrt{4x^2+1}}{x} \, dx$

m.) $\int \frac{3x}{\sqrt{4x^2+7}} \, dx$ n.) $\int \frac{x^3}{\sqrt{4x^2+7}} \, dx$ o.) $\int \frac{5}{3+\sqrt{x}} \, dx$

p.) $\int \frac{\sqrt{x}}{\sqrt{x}-4} \, dx$ q.) $\int \frac{x^{1/4}}{1+\sqrt{x}} \, dx$ r.) $\int \frac{\sqrt{x+1}}{x+2} \, dx$

s.) $\int \frac{\sqrt{x}}{1+\sqrt{1+\sqrt{x}}} \, dx$

2.) Consider the definite integral $\int_0^2 \frac{x}{1+x^3} \, dx$.

a.) Use the trapezoidal method with $n = 4$ to estimate the value of this integral.

b.) Use the Simpson's method with $n = 4$ to estimate the value of this integral.

c.) Compute the exact value of this integral.

3.) a.) Use the trapezoidal method with $n=6$ to estimate $\int_0^3 \frac{x}{1+x^4} dx$.

b.) Use Simpson's method with $n=6$ to estimate $\int_0^3 \frac{x}{1+x^4} dx$.

c.) Evaluate $\int_0^3 \frac{x}{1+x^4} dx$.

4.) Consider the definite integral $\int_0^1 f(x) dx$, where $f(x) = e^{-x^2}$.

a.) Compute $f''(x)$ and $f^{(4)}(x)$.

b.) Determine good estimates for

$$M_2 = \max_{0 \leq x \leq 1} |f''(x)| \text{ and } M_4 = \max_{0 \leq x \leq 1} |f^{(4)}(x)|.$$

c.) i.) What should n be in order that the trapezoidal method estimate $\int_0^1 e^{-x^2} dx$ with an error of at most .00001?

ii.) What should n be in order that Simpson's method estimate $\int_0^1 e^{-x^2} dx$ with an error of at most .00001?

5.) A population of mosquitoes is growing exponentially. Initially, there are 1000 insects, and after 3 days there are 1250 insects.

a.) How many insects will there be after 2 weeks?

b.) When will the population reach 10,000 insects?

6.) An elk herd in Yellowstone National Park increases in size at a rate proportional to the number of elk present. In January 1988 there were 300 elk, and one year later the size of the herd had increased by 12%. How many elk do you expect to be in the herd in January 1993?

7.) Assume that a fungus grows at a rate proportional to the square of the weight of the fungus. Initially, the fungus weighs 3 ounces, and after 4 days it weighs 5 ounces.

a.) How much will the fungus weigh in 9 days?

b.) When will the fungus weigh 3 lbs.?

8.) The force due to water pressure on a submerged flat plate lying horizontally is defined to be the weight of the column of water directly above the plate. For example, a flat rectangular plate 2 ft. by 3 ft. submerged in 15 ft. of water is under

$$(2)(3)(15)(62.4) = 5616 \text{ lbs.}$$

of force.

a.) How much water force is on a flat circular plate lying horizontally in 100 ft. of water, if the radius of the plate is 5 ft.?

b.) How much water force is on one side of a flat rectangular plate 2 ft. by 3 ft., which is submerged in 15 ft. of water and is resting vertically on its 2 ft. edge?

9.) For each part find all functions $f(x)$ which satisfy the given equation.

a.) $f'(x) = x^2 + 1$

b.) $f'(x) = e^x \cdot f(x)$

c.) $f'(x) = [f(x)]^2$

d.) $f'(x) = \frac{[f(x)]^2 + 1}{2f(x)}$