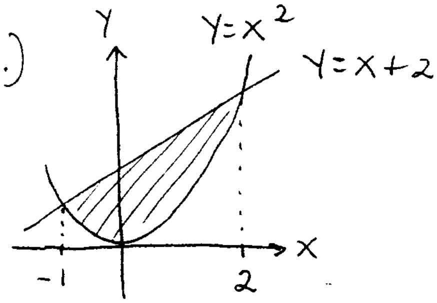
ESP

Kouba

Worksheet 12 Solutions

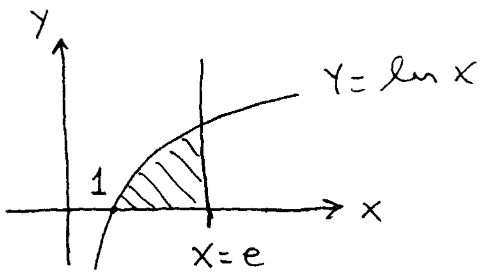
1.) a.)



$$\text{Area} = \int_{-1}^{2} (x+2-x^2)\, dx$$

$$= \left(\tfrac{1}{2}x^2 + 2x - \tfrac{1}{3}x^3\right)\Big|_{-1}^{2} = 9/2$$

b.)

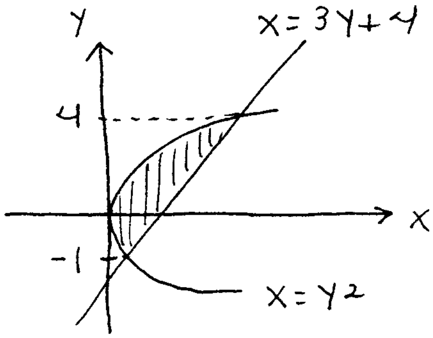


$$\text{Area} = \int_{1}^{e} \ln x\, dx$$

$$= (x\ln x - x)\Big|_{1}^{e}$$

$$= (e-e)-(0-1) = 1$$

c.)

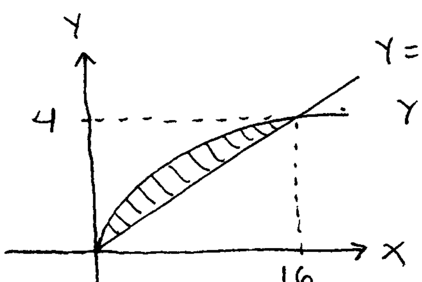


$$\text{Area} = \int_{-1}^{4} (3y+4-y^2)\, dy$$

$$= \left(\tfrac{3}{2}y^2 + 4y - \tfrac{y^3}{3}\right)\Big|_{-1}^{4}$$

$$= \frac{125}{6}$$

2.)



$y = \frac{1}{4}x$ or $x = 4y$

$y = \sqrt{x}$ or $x = y^2$

a.) $$\text{Vol} = \pi\int_0^{16} (\sqrt{x})^2\, dx - \pi\int_0^{16} \left(\tfrac{1}{4}x\right)^2 dx$$

b.) $$\text{Vol} = \pi\int_0^{16} (\sqrt{x}+1)^2\, dx - \pi\int_0^{16} \left(\tfrac{1}{4}x+1\right)^2 dx$$

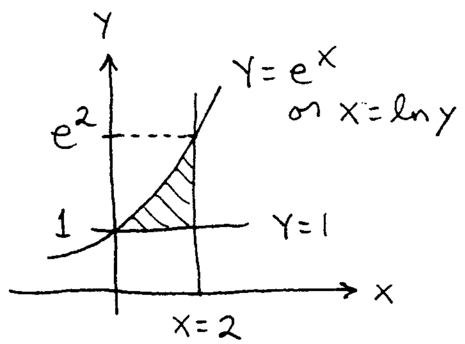
c.) $$\text{Vol} = \pi\int_0^{16} \left(4-\tfrac{1}{4}x\right)^2 dx - \pi\int_0^{16} (4-\sqrt{x})^2\, dx$$

d.) $$\text{Vol} = \pi\int_0^{4} (4y)^2\, dy - \pi\int_0^{4} (y^2)^2\, dy$$

e.) $\text{Vol} = \pi \int_0^4 (4y+5)^2\,dy - \pi \int_0^4 (y^2+5)^2\,dy$

f.) $\text{Vol} = \pi \int_0^4 (20-y^2)^2\,dy - \pi \int_0^4 (20-4y)^2\,dy$

3.)



a.) $\text{Vol} = \pi \int_0^2 (e^x)^2\,dx - \pi \int_0^2 (1)^2\,dx$

b.) $\text{Vol} = \pi \int_1^{e^2} (2)^2\,dy - \pi \int_1^{e^2} (\ln y)^2\,dy$

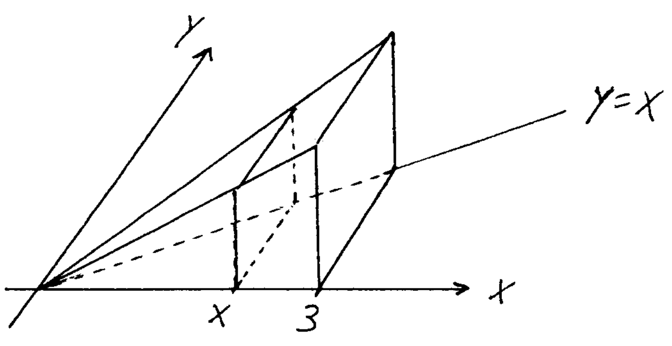
c.) $\text{Vol} = \pi \int_0^2 (e^x - 1)^2\,dx$

d.) $\text{Vol} = \pi \int_1^{e^2} (3-\ln y)^2\,dy - \pi \int_1^{e^2} (1)^2\,dy$

e.) $\text{Vol} = \pi \int_0^2 (9)^2\,dx - \pi \int_0^2 (10-e^x)^2\,dx$

f.) $\text{Vol} = \pi \int_1^{e^2} (4)^2\,dy - \pi \int_1^{e^2} (\ln y + 2)^2\,dy$

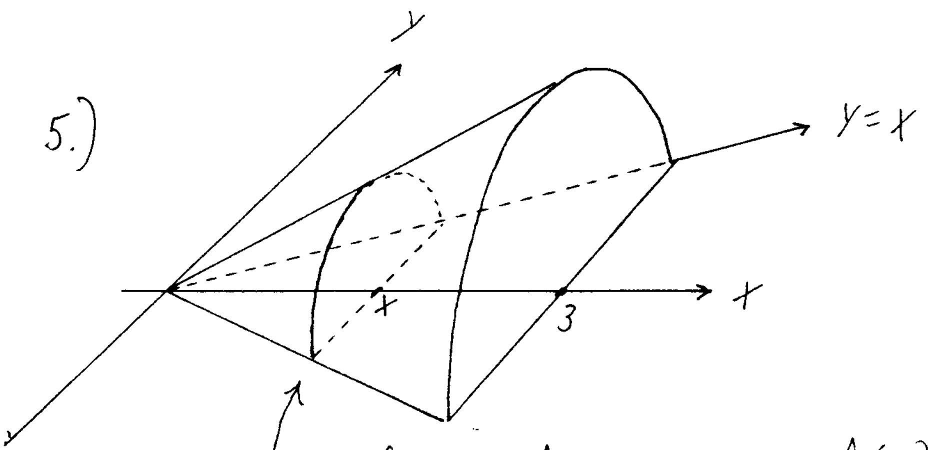
4.)



slice has area $A(x) = x^2$ so

$$\text{Vol} = \int_0^3 A(x)\,dx = \int_0^3 x^2\,dx = \left.\frac{x^3}{3}\right|_0^3 = 9$$

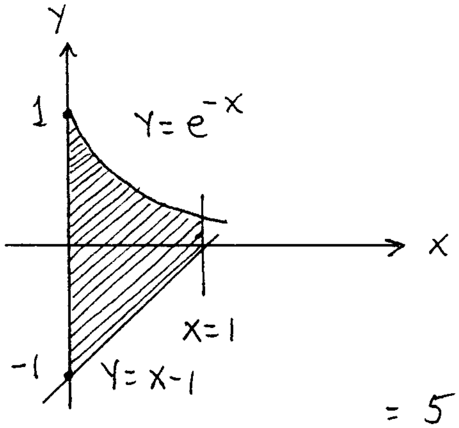
5.)



slice has area $A(x) = \frac{1}{2}\pi x^2$ so

$$\text{Vol} = \int_0^3 A(x)\,dx = \int_0^3 \frac{1}{2}\pi x^2\,dx = \frac{\pi}{2}\cdot\frac{x^3}{3}\Big|_0^3 = \frac{9}{2}\pi$$

6.)



a.) $A(x) = 5\left(e^{-x} - (x-1)\right)$
$= 5\left(e^{-x} - x + 1\right)$ so

$$\text{Vol} = \int_0^1 A(x)\,dx = 5\int_0^1 \left(e^{-x} - x + 1\right)dx$$

$$= 5\left(-e^{-x} - \frac{x^2}{2} + x\right)\Big|_0^1 = \frac{15}{2} - \frac{5}{e}$$

b.) $A(x) = \left(e^{-x} - (x-1)\right)^2 = \left(e^{-x} - x + 1\right)^2 = e^{-2x} - 2xe^{-x} + 2e^{-x} + x^2 - 2x + 1$

so $\text{Vol} = \int_0^1 A(x)\,dx = \int_0^1 \left(e^{-2x} - 2xe^{-x} + 2e^{-x} + x^2 - 2x + 1\right)dx$

$$= \left(\frac{-1}{2}e^{-2x} - 2\left(-xe^{-x} - e^{-x}\right) + -2e^{-x} + \frac{x^3}{3} - x^2 + x\right)\Big|_0^1 = -\frac{1}{2}e^{-2} + 2e^{-1} + \frac{5}{6}.$$

7.) a.) $\text{Vol} = 2\pi\int_0^4 y\cdot(4y - y^2)\,dy$

b.) $\text{Vol} = 2\pi\int_0^4 (y+1)(4y - y^2)\,dy$

c.) $\text{Vol} = 2\pi\int_0^4 (4-y)(4y - y^2)\,dy$

d.) $\text{Vol} = 2\pi \int_0^{16} x \left(\sqrt{x} - \frac{1}{4}x\right) dx$

e.) $\text{Vol} = 2\pi \int_0^{16} (x+5)\left(\sqrt{x} - \frac{1}{4}x\right) dx$

f.) $\text{Vol} = 2\pi \int_0^{16} (20-x)\left(\sqrt{x} - \frac{1}{4}x\right) dx$

8.) a.) $\int \frac{x}{x^2+4}\, dx = \frac{1}{2} \ln|x^2+4| + c$

b.) $\int \frac{x}{x^2-4}\, dx = \int \left[\frac{A}{x-2} + \frac{B}{x+2}\right] dx$

$= \int \left[\frac{\frac{1}{2}}{x-2} + \frac{\frac{1}{2}}{x+2}\right] dx = \frac{1}{2} \ln|x-2| + \frac{1}{2}\ln|x+2| + c$

c.) $\int \frac{x^2}{x^2-4}\, dx = \int \frac{x^2-4+4}{x^2-4}\, dx = \int \left[1 + \frac{4}{x^2-4}\right] dx$

$= x + \int \left(\frac{A}{x-2} + \frac{B}{x+2}\right) dx = x + \int \left(\frac{1}{x-2} + \frac{-1}{x+2}\right) dx$

$= x + \ln|x-2| - \ln|x+2| + c$

d.) $\int \frac{1}{1+\sqrt{x}}\, dx$ (let $u = \sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}}\, dx \rightarrow$
$2u\, du = dx$)

$= \int \frac{2u}{1+u}\, du = 2\int \frac{u+1-1}{u+1}\, du = 2\int \left[1 - \frac{1}{u+1}\right] du$

$= 2\left[u - \ln|u+1|\right] + c = 2\left[\sqrt{x} - \ln|\sqrt{x}+1|\right] + c$

e.) $\int \frac{1}{\sqrt{x}(1+x)}\, dx$ (let $u = \sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}}\, dx \rightarrow$
$2\, du = \frac{1}{\sqrt{x}}\, dx$)

$= \int \frac{1}{(1+(\sqrt{x})^2)} \cdot \frac{1}{\sqrt{x}}\, dx = 2\int \frac{1}{1+u^2}\, du = 2 \arctan u + c$
$= 2\arctan\sqrt{x} + c$