ESP
Kouba
Worksheet 14

1.) Set up but do not evaluate the integrals which represent the centroids $(\bar{x}, \bar{y})$ of each of the following regions bounded by the graphs of

a.) $y = 2x$, $x = 0$, $y = 3$

b.) $y = x^4$, $y = 0$, $x = 2$

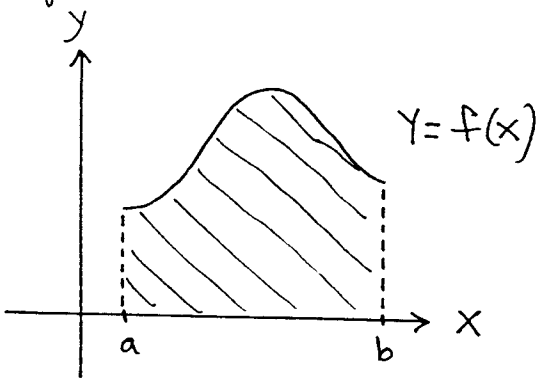
c.) $y = \ln x$, $y = 10 + e^{-x}$, $x = 1$, $x = 2$

d.) $y = x + 3$, $y = 3 - \frac{1}{2}x$, $y = 0$

e.) $y = e^{x^2}$, $x = 0$, $y = 0$, $x = 1$

f.) $y = \sqrt{4 - (x-2)^2}$, $y = 0$

2.) Consider the region R below the graph of $y = f(x)$ and above the x-axis from $x = a$ to $x = b$. Assume that the centroid of R is $(\bar{x}, \bar{y})$. Derive the equation



$$\bar{y} = \frac{\int_a^b \frac{1}{2}[f(x)]^2 \, dx}{\int_a^b f(x) \, dx}$$

3.) The region bounded by the graphs of $y = e^x$, $x = 0$, $x = 1$, and $y = 0$ is revolved about the

line $Y=-5$. Set up but do not evaluate the integrals representing the volume of the resulting solid using

a.) the disc method.
b.) the shell method.

4.) A flat, circular plate of diameter four feet is submerged in a tank full of water to a depth of ten feet. Compute the total force due to water pressure on one side of the plate if the plate

a.) lies flatly at the bottom of the tank?
b.) rests vertically on its edge?

5.) Integrate.

a.) $\int \sqrt{1+\sqrt{x}}\, dx$

b.) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\, dx$

c.) $\int \frac{1}{\sqrt{2-x^2}}\, dx$

d.) $\int \frac{1}{\sqrt{2-x}\sqrt{x-1}}\, dx$

e.) $\int \frac{1}{\sqrt{x}\sqrt{1-x}}\, dx$

f.) $\int \sin(\ln x)\, dx$

g.) $\int (\sec x \tan x)^2 dx$

h.) $\int \frac{x^2}{2x^6+3x^3+1}\, dx$

6.) A flat, circular plate is spinning at the rate of five revolutions per minute around an axis of revolution passing through the center of the circle. The circle's radius is ten feet and the density of the plate x ft. from the center is given by $e^{-x}\sqrt{x^2+1}$ gm./ft.^{2} Set up but do not evaluate the integral which represents the kinetic energy of this spinning plate.

7.) Determine the volume of the solid lying inside the intersection of the two quarter cylinders in the diagram.