ESP
Kouba
Worksheet 2

1. Evaluate the following sums.

a. $\sum_{i=1}^{300} \pi$

b. $\sum_{i=1}^{200} (3i - 2)$

c. $\sum_{i=1}^{500} (5i^2 - i + 1)$

d. $\sum_{i=1}^{1000} (i + 1) i$

e. $\sum_{i=253}^{684} (i - 1)^2$

f. $\sum_{i=1}^{203} [(i + 1)^3 - i^3]$

2. Estimate the area of the region between the graphs of $y = e^x$ and $y = \sqrt{x}$ over the interval $[0, 4]$. Use rectangles determined by the midpoints of four equal subdivisions.

3. Estimate the volume of a hemisphere of radius four feet. Use appropriate cylinders determined by the midpoints of four equal subdivisions.

4. Use rectangles to estimate the area below the graph of $f(x) = \ln x$ and above the interval $[1, e^2]$. Let the partition of $[1, e^2]$ be $x_0 = 1$, $x_1 = 2$, $x_2 = 2.6$, $x_3 = 4$, $x_4 = 6.5$, and $x_5 = e^2$. Let the rectangles be determined by the sampling points $c_1 = 3/2$, $c_2 = 2.1$, $c_3 = 3$, $c_4 = 6$, and $c_5 = 6.9$.

5. Evaluate $\sum_{i=1}^{n} f(c_i)(x_i - x_{i-1})$ for each of the following.

a. $f(x) = \ln x$ on $[1, e^2]$
partition : $x_0 = 1$, $x_1 = 3$, $x_2 = 6$, $x_3 = e^2$
sampling points : c_i is midpoint of subdivision $[x_{i-1}, x_i]$
for $i = 1, 2, 3$.

b. $f(x) = e^{x^2}$ on $[-1, 1]$
partition : $x_0 = -1$, $x_1 = -1/2$, $x_2 = 0$, $x_3 = 1/2$, $x_4 = 1$
sampling points : $c_i = x_i$ for $i = 1, 2, 3, 4$

c. $f(x) = \tan x$ on $[-\pi/4, 0]$
partition : $x_0 = -\pi/4$, $x_1 = -\pi/6$, $x_2 = -\pi/12$, $x_3 = 0$
Sampling points : $c_i = x_{i-1}$ for $i = 1, 2, 3$

6. Use rectangles, determined by the right-hand endpoint of n equal
subintervals, to estimate the area under the graph of $y = 1/2\ x^2 + x$
above the interval [0, 4].

a. $n = 2$ b. $n = 4$
c. $n = 20$ d. $n = 100$
e. What is the limit of the estimates as n approaches infinity ?

DEFINITION : The definite integral of f over the interval [a, b] is

$$\int_a^b f(x)\ dx = \lim_{mesh \to 0} \sum_{i=1}^{n} f(c_i)(x_i - x_{i-1}) .$$

7. Determine the mesh of each of the following intervals and partitions.

a. [0, 2], partitioned into ten equal subdivisions
b. [-4, 2], partitioned into five equal subdivisions
c. [-3, 2], partition : $x_0 = -3$, $x_1 = -2.6$, $x_2 = -1$, $x_3 = 0$,
$x_4 = 1.8$, $x_5 = 2$.

8. Use equal subintervals and the limit definition of a definite integral to
evaluate each of the following.

a. $\int_0^1 7\ dx$

b. $\int_0^2 (3x - 1)\ dx$

c. $\int_{-2}^1 (x^2 + x)\ dx$