ESP
Kouba
Worksheet 3

1. Assume that f is a function satisfying $f'(x) > 0$ for all values of x in the interval $[a, b]$. Let $[a, b]$ be partitioned by $a = x_0 < x_1 < x_2 < \cdots < x_n = b$. Determine which of the following quantities is largest, or state that it is not possible to determine.

$$\sum_{i=1}^{n} f(x_i)(x_i - x_{i-1}) \quad \text{and} \quad \sum_{i=1}^{n} f(x_{i-1})(x_i - x_{i-1})$$

2. Use equal subdivisions and the limit definition of a definite integral to evaluate each of the following.

a. $\displaystyle\int_1^{10} \pi^3 \, dx$

b. $\displaystyle\int_1^{10} x^3 \, dx$

3. Assume that a thin rod lies along the interval $[a, b]$ and has density $f(x)$ grams per centimeter at point x. Let $x_0, x_1, x_2, \ldots, x_n$ be a partiton of $[a, b]$ and $c_1, c_2, c_3, \ldots, c_n$ corresponding sampling numbers with c_i in $[x_{i-1}, x_i]$ for $i = 1, 2, 3, \ldots, n$. What is the physical interpretation of each of the following quatities ?

a. $x_i - x_{i-1}$

b. $f(c_i)$

c. $f(c_i)(x_i - x_{i-1})$

d. $\displaystyle\sum_{i=1}^{n} f(c_i)(x_i - x_{i-1})$

e. $\displaystyle\lim_{\text{mesh} \to 0} \sum_{i=1}^{n} f(c_i)(x_i - x_{i-1})$

f. $\displaystyle\int_a^b f(x) \, dx$

4. Each of the following limits is equal to a definite integral $\displaystyle\int_a^b f(x)\,dx$. Determine the definite integral for each.

a. $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} \sin(i/n) \cdot 1/n$

b. $\lim_{n\to\infty} \sum_{i=1}^{n} \sin(3 + i/n) \cdot 1/n$

c. $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{\left(1+\frac{3i}{n}\right)^4}{8+\frac{3i}{n}} \cdot \dfrac{3}{n}$

d. $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{4}{3n+4i}$

5. Let f be a function which is differentiable for all values of x. In addition, the function satisfies $f'(x) = 5 f(x)$ for all values of x. Show that there exists at least one number c satisfying

$$10 f(c) = f(3) - f(1) .$$