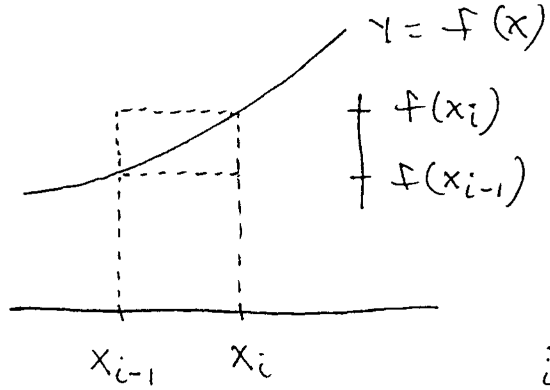
ESP
Kouba
Worksheet 3 Solutions

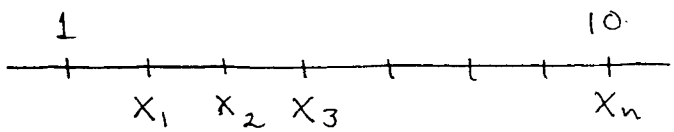
1.)



Since $f'(x) > 0$ the graph of f is increasing. Thus $f(x_i) > f(x_{i-1})$ and

$$\sum_{i=1}^{n} f(x_i)(x_i - x_{i-1}) > \sum_{i=1}^{n} f(x_{i-1})(x_i - x_{i-1}) .$$

2.) a.)

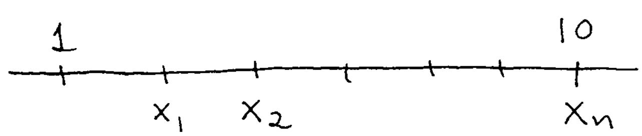


Divide $[1, 10]$ into n equal parts each of length $\frac{9}{n}$. Then $x_i = 1 + \frac{9i}{n}$ and for $f(x) = \pi^3$

$$\sum_{i=1}^{n} f(x_i)\left(\frac{9}{n}\right) = \sum_{i=1}^{n} \pi^3 \cdot \left(\frac{9}{n}\right) = n \cdot \pi^3 \left(\frac{9}{n}\right) = 9\pi^3 \text{ so}$$

$$\int_1^{10} \pi^3 \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \left(\frac{9}{n}\right) = \lim_{n\to\infty} 9\pi^3 = 9\pi^3 .$$

b.)



Divide $[1, 10]$ into n equal parts each of length $\frac{9}{n}$. Then $x_i = 1 + \frac{9i}{n}$ and for $f(x) = x^3$

$$\sum_{i=1}^{n} f(x_i) \cdot \frac{9}{n} = \sum_{i=1}^{n} f\left(1 + \frac{9i}{n}\right) \cdot \frac{9}{n} = \sum_{i=1}^{n} \left(1 + \frac{9i}{n}\right)^3 \cdot \frac{9}{n}$$

$$= \sum_{i=1}^{n} \left(1 + \frac{27i}{n} + \frac{243 i^2}{n^2} + \frac{729 i^3}{n^3}\right) \cdot \frac{9}{n}$$

$$= \left(\sum_{i=1}^{n} \frac{9}{n}\right) + \frac{243}{n^2}\left(\sum_{i=1}^{n} i\right) + \frac{2187}{n^3}\left(\sum_{i=1}^{n} i^2\right) + \frac{6561}{n^4}\left(\sum_{i=1}^{n} i^3\right)$$

$$= n\cdot\left(\frac{9}{n}\right) + \frac{243}{n^2}\cdot\frac{n(n+1)}{2} + \frac{2187}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{6561}{n^4}\cdot\frac{n^2(n+1)^2}{4}$$

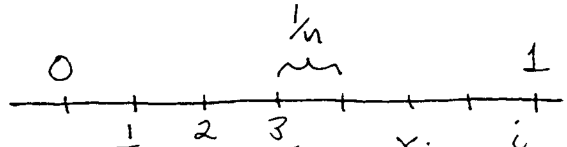
$$= 9 + \frac{243}{2}\left(1+\frac{1}{n}\right) + \frac{729}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right) + \frac{6561}{4}\left(1+\frac{2}{n}+\frac{1}{n^2}\right) \quad \text{so}$$

$$\int_1^{10} x^3\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\frac{9}{n}$$

$$= 9 + \frac{243}{2}(1) + \frac{729}{2}(2) + \frac{6561}{4}(1) = \frac{9999}{4}.$$

3.) a.) <u>length</u> of ith piece of rod
b.) <u>density</u> of wire at point c_i
c.) estimate for <u>mass</u> of ith piece of rod
d.) estimate for <u>mass</u> of entire rod
e.) exact <u>mass</u> of entire rod
f.) exact <u>mass</u> of entire rod

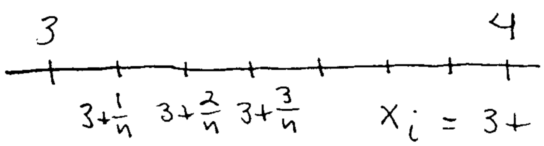
4.) a.)



$x_i = \frac{i}{n}$ $\qquad f(x) = \sin x$ so

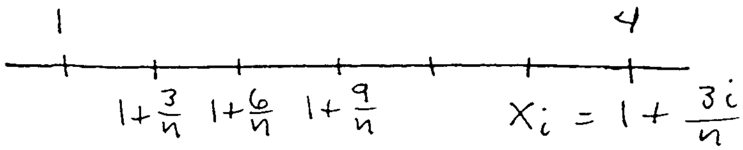
$$\lim_{n\to\infty}\sum_{i=1}^{n}\sin\left(\frac{i}{n}\right)\cdot\frac{1}{n} = \int_0^1 \sin x\,dx.$$

b.)



$x_i = 3+\frac{i}{n}$ $\qquad f(x) = \sin x$ so

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sin\left(3+\frac{i}{n}\right)\cdot\frac{1}{n} = \int_3^4 \sin x\,dx.$$

c.) 

$f(x) = \dfrac{x^4}{7+x}$ so

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(1+\frac{3i}{n}\right)^4}{8+\frac{3i}{n}} \cdot \frac{3}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(1+\frac{3i}{n}\right)^4}{7+\left(1+\frac{3i}{n}\right)} \cdot \frac{3}{n}$$

$$= \int_1^4 \frac{x^4}{7+x}\,dx$$

d.) $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{3n+4i} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{3+\left(\frac{4i}{n}\right)} \cdot \frac{4}{n}$$

$$= \int_0^4 \frac{1}{3+x}\,dx$$

5.) We know f is continuous on $[1,3]$ and differentiable on $(1,3)$ so by MVT there is a number c, $1<c<3$, satisfying

$$f'(c) = \frac{f(3)-f(1)}{3-1},$$ i.e.,

$$5f(c) = \frac{f(3)-f(1)}{2},$$ i.e.,

$$10f(c) = f(3)-f(1).$$