ESP
Kouba
Worksheet 4

1. Evaluate each of the following definite integrals.

a. $\int_0^3 7\,dx$

b. $\int_{-1}^2 (x^2 + x)\,dx$

c. $\int_2^3 x(x+7)^2\,dx$

d. $\int_0^1 \frac{x^2+5x+6}{x+3}\,dx$

e. $\int_1^e \frac{x^2 - 1 + x^{-3}}{x^2}\,dx$

f. $\int_0^{\frac{\pi}{2}} \cos 3x\,dx$

g. $\int_0^1 x\sqrt{1+x^2}\,dx$

2. Set up a definite integral which represents the *area* of the region below the graph of $f(x) = e^{x^2}$ and above the x-axis from $x = -1$ to $x = 2$.

3. A wire lies along the x-axis from $x = 1$ to $x = 7$. It's density at point $x$ is given by $f(x) = 1/(1+x^2)$ pounds per inch. Set up a definite integral which represents the *mass* of the wire.

4. A snail's speed at time $t$ (hours) is given by $g(t) = 2t \sin(t^2 + 3)$ inches per hour. Set up a definite integral which represents the total *distance* traveled by the snail during the interval from $t = 0$ to $t = 5$ hours.

5. The region below the graph of $y = x^2$ and above the x-axis from $x = 0$ to $x = 2$ is revolved about the x-axis. Set up a definite integral which represents the *volume* of the resulting solid.

6. Differentiate each of the following functions.

a. $F(x) = \arctan(x^3)$

b. $F(x) = \sin(\ln(3-x))$

c. $F(x) = [e^{\sin^2 x} + e^3]^5$

d. $F(x) = \int_0^X e^{t^2}\,dt$

e. $F(x) = \int_{-1}^{x} e^{t^2}\, dt$

f. $F(x) = \int_{1}^{x^3} \sin(t^{20})\, dt$

7. Assume that f is a continuous function on the interval [a, b], and let

$$F(x) = \int_{a}^{x} f(t)\, dt$$

for x in [a, b].

a. What is F(a) ?

b. What is $F(b) - \left( \int_{a}^{x} f(t)\, dt \right)$ ?

c. Is F a continuous function ? Explain.

8. Evaluate the following limit.

$$\lim_{h \to 0} \frac{\int_{3}^{x+h} \frac{1}{t^5 + 1}\, dt - \int_{3}^{x} \frac{1}{t^5 + 1}\, dt}{h}$$

9. a. Sketch the region below the graph of $y = x$ and above the graph of $y = x^2$ from $x = 0$ to $x = 1$.

b. Set up definite integrals which represent the volume of the solids created by revolving the region around

i. the x-axis .

ii. the line $y = x$ .