ESP
Kouba
Worksheet 5

1. Differentiate each of the following.

a. $F(x) = \arcsin\left(e^{x^2}\right)$

b. $F(x) = \ln\left\{1 + \int_0^x \sqrt{t^3 + 5}\, dt\right\}$

c. $F(x) = \dfrac{\tan(\ln x)}{\ln(\tan x)}$

d. $F(x) = \int_0^3 \cos x^2 \cdot e^{\sin^2 x^2}\, dx$

e. $F(x) = \int_1^{\sin x} e^{1-t}\, dt$

f. $F(x) = \int_3^{\sqrt{x-1}} \cos(t^2 + 1)\, dt$

g. $F(x) = \int_{\sqrt{x-1}}^3 \cos(t^2 + 1)\, dt$

h. $F(x) = \int_{\frac{1}{x}}^{x^3} (3 - t^5)^{100}\, dt$

i. $F(x) = x^5 \cdot \int_0^x \dfrac{t^2}{t^2 + 1}\, dt$

2. Find the x-value(s) for which each of the following functions has a global minimum value.

a. $F(x) = e^{x^2 - 7}$

b. $F(x) = 9 + \int_0^x (t - 1)(2 - t)^6\, dt$ for $x \geq 0$

3. Evaluate the following definite integrals. Think carefully. Nothing sophisticated is needed to solve these problems.

a. $\int_1^2 (2/x^2 + x^2/2)\, dx$

b. $\int_{-1}^0 (1 + x)^2\, dx$

c. $\int_{-1}^0 (1 + x)^{200}\, dx$

d. $\int_0^{\frac{\pi}{6}} \dfrac{\sin x}{\cos^2 x}\, dx$

e. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x + \cos x)^2 \, dx$

f. $\int_0^1 x^2 (1 + x^3)^{1/10} \, dx$

g. $\int_0^{\pi} \cos^2 x \, dx$

h. $\int_0^{\frac{\pi}{20}} \sec^2 5x \, dx$

i. $\int_0^1 2x \sec^2 (x^2) \, dx$

j. $\int_0^{\frac{\pi}{2}} (x \cos x + \sin x) \, dx$

4. a. Make a sketch of the region bounded by the graphs of $y = 1/2\, x$, $y = 2$, and $x = 0$.

b. Set up definite integrals which represent the volumes of the solids formed by revolving the region in part a. around

i. the x-axis
ii. the y-axis

5. A large bucket full of water weighing 100 lbs. is slowly lowered 25 feet by a rope and pulley.

a. How much work is done in lowering the bucket ?

b. How much work is done in lowering the bucket if the bucket is leaking , losing 4 $in.^3$ of water per foot lowered. (Assume that one cubic foot of water weighs 62.4 lbs.)

6. Find the average value of each function over the indicated interval.

a. $f(x) = x^3$ on $[0, 1]$

b. $f(x) = \sin x$ on $[0, \pi/2]$

c. $f(x) = 2x\, e^{x^2}$ on $\left[\sqrt{\ln 2}\,,\ \sqrt{\ln 5}\right]$

d. $f(x) = x \sec^2 x + \tan x$ on $[0, 1/2]$