ESP

Worksheet 5½ Solutions

1.) assume $A = Ce^{kt}$; $t = 2$ yrs., $A = 100$ hogs

$\rightarrow \underline{100 = ce^{2k}}$; $t = 6$ yrs., $A = 400$ hogs $\rightarrow$

$\underline{400 = ce^{6k}}$; then $c = \dfrac{100}{e^{2k}}$ and

$c = \dfrac{400}{e^{6k}} \rightarrow \dfrac{100}{e^{2k}} = \dfrac{400}{e^{6k}} \rightarrow$

$\dfrac{e^{6k}}{e^{2k}} = \dfrac{400}{100} \rightarrow e^{6k-2k} = 4 \rightarrow e^{4k} = 4 \rightarrow$

$\ln e^{4k} = \ln 4 \rightarrow 4k = \ln 4 \rightarrow k = \frac{1}{4}\ln 4 \rightarrow$

$c = \dfrac{400}{e^{6k}} = \dfrac{400}{e^{6(\frac{1}{4}\ln 4)}} = \dfrac{400}{e^{\frac{3}{2}\ln 4}} = \dfrac{400}{e^{\ln 4^{3/2}}}$

$= \dfrac{400}{8} = 50$ so $A = 50e^{(\frac{1}{4}\ln 4)t}$

a.) If $t = 0$ then $A = 50e^0 = 50(1)$
$= 50$ hogs

b.) If $t = 10$ yrs. then

$A = 50e^{(\frac{1}{4}\ln 4)10} = 50e^{\ln 4^{5/2}} = 1600$ hogs

2.) assume $A = Ce^{kt}$; $t = 0$ hrs.,

$A = 0.17 \rightarrow \underline{A = 0.17e^{kt}}$; $t = 1$ hr.,

$A = 0.13 \rightarrow 0.13 = 0.17e^{k}$; $\rightarrow$

$\dfrac{0.13}{0.17} = e^{k} \rightarrow \dfrac{13}{17} = e^{k} \rightarrow \ln\left(\frac{13}{17}\right) = \ln e^{k}$

$\rightarrow k = \ln\left(\frac{13}{17}\right) \rightarrow A = 0.17e^{\ln(\frac{13}{17})\cdot t} \rightarrow$

$A = 0.17\left(\frac{13}{17}\right)^{t}$;

if $A = 0.08$ then $0.08 = 0.17\left(\frac{13}{17}\right)^t \rightarrow$

$\frac{0.08}{0.17} = \left(\frac{13}{17}\right)^t \rightarrow \frac{8}{17} = \left(\frac{13}{17}\right)^t \rightarrow$

$\ln\left(\frac{8}{17}\right) = \ln\left(\frac{13}{17}\right)^t = t \cdot \ln\left(\frac{13}{17}\right) \rightarrow$

$t = \frac{\ln(8/17)}{\ln(13/17)} \approx$ 2.81 hrs. $\approx$ 2 hrs., 49 min.

3.) assume $A = Ce^{kt}$; $t = 0$ wks.,

$A = 9$ mg. $\rightarrow \underline{A = 9e^{kt}}$; $t = 36$ wks.,

$A = 5$ mg. $\rightarrow 5 = 9e^{36k} \rightarrow \frac{5}{9} = e^{36k} \rightarrow$

$\ln\left(\frac{5}{9}\right) = \ln e^{36k} \rightarrow \ln\left(\frac{5}{9}\right) = 36k \rightarrow k = \frac{1}{36}\ln\left(\frac{5}{9}\right) \rightarrow$

$A = 9 \cdot e^{\frac{1}{36}\ln(5/9)\cdot t} = 9e^{\ln\left(\frac{5}{9}\right)^{t/36}} \rightarrow$

$A = 9\left(\frac{5}{9}\right)^{t/36}$; 20% of 9 = 1.8 mg. so

$A = 7.2$ mg. $\rightarrow 7.2 = 9\left(\frac{5}{9}\right)^{t/36} \rightarrow$

$\frac{7.2}{9} = \left(\frac{5}{9}\right)^{t/36} \rightarrow \ln\left(\frac{72}{90}\right) = \ln\left(\frac{5}{9}\right)^{t/36} \rightarrow$

$\ln\left(\frac{4}{5}\right) = \frac{t}{36}\ln\left(\frac{5}{9}\right) \rightarrow$

$t = \frac{36\ln(4/5)}{\ln(5/9)} \approx 13.7$ wks.

4.) assume $A = Ce^{kt}$; $t = 0$ yrs. (1875),

$A = 325 \rightarrow \underline{A = 325 e^{kt}}$; $t = 128$ yrs.

(2003), $A = 117 \rightarrow 117 = 325 e^{128k} \rightarrow$

$\frac{117}{325} = e^{128k} \rightarrow \ln\left(\frac{117}{325}\right) = \ln e^{128k} = 128k \rightarrow$

$k = \frac{1}{128} \ln\left(\frac{117}{325}\right) \rightarrow A = 325 e^{\frac{1}{128}\ln\left(\frac{117}{325}\right)t} \rightarrow$

$A = 325 e^{\ln\left(\frac{117}{325}\right) \cdot \frac{t}{128}} \rightarrow A = 325\left(\frac{117}{\cancel{128}}\right)^{t/128}$;

a.) $t = 225$ yrs. $(2100) \rightarrow$

$A = 325\left(\frac{117}{325}\right)^{225/128} \approx 54$ people

b.) $t = -175$ yrs. $(1700) \rightarrow$

$A = 325\left(\frac{117}{325}\right)^{-175/128} \approx 1314$ people

5.) Let I be the allowable amount of iodine 31 and assume $A = Ce^{kt}$;

$t = 0$ days, $A = 7I \rightarrow \underline{A = 7I e^{kt}}$;

$t = 8$ days, $A = \frac{7}{2} I \rightarrow$

$\frac{7}{2} I = 7 I e^{8k} \rightarrow \frac{1}{2} = e^{8k} \rightarrow$

$\ln(1/2) = \ln(e^{8k}) \rightarrow \ln(1/2) = 8k \rightarrow$

$k = \frac{1}{8}\ln\left(\frac{1}{2}\right) \rightarrow A = 7I e^{\frac{1}{8}\ln\left(\frac{1}{2}\right)t} \rightarrow$

$A = 7I e^{\ln\left(\frac{1}{2}\right) \cdot \frac{t}{8}} \rightarrow A = 7I\left(\frac{1}{2}\right)^{t/8}$;

if $A = I$ then $I = 7I\left(\frac{1}{2}\right)^{t/8} \rightarrow$

$\frac{1}{7} = \left(\frac{1}{2}\right)^{t/8} \rightarrow \ln(1/7) = \ln\left(\frac{1}{2}\right)^{t/8} \rightarrow$

$\ln(1/7) = \frac{t}{8}\ln(1/2) \rightarrow$

$$t = \frac{8\ln(1/7)}{\ln(1/2)} \approx 22.5 \text{ days.}$$

6.) Assume $A = Ce^{kt}$, where C is the initial amount; $t = 5$ hrs.,

$A = C + 40\%$ of $C = 1.4C \rightarrow 1.4C = Ce^{5k} \rightarrow$

$1.4 = e^{5k} \rightarrow \ln(1.4) = \ln e^{5k} = 5k \rightarrow$

$k = \frac{1}{5}\ln(1.4) \rightarrow A = Ce^{\frac{1}{5}\ln(1.4)\cdot t} \rightarrow$

$A = Ce^{\ln(1.4)\cdot t/5} \rightarrow A = C(1.4)^{t/5}$;

if $A = 3C$ then $3C = C(1.4)^{t/5} \rightarrow$

$3 = (1.4)^{t/5} \rightarrow \ln 3 = \ln(1.4)^{t/5} \rightarrow$

$\ln 3 = \frac{t}{5}\ln(1.4) \rightarrow t = \frac{5\ln 3}{\ln(1.4)} \approx 16.3$ hrs.

7.) Let C be the initial amounts of U^{235} and U^{238} at the "big bang"; let T years be the age of the universe; then

$$\left.\begin{aligned} &\text{for } U^{235}: \quad A = Ce^{k_1 T} \\ &\text{for } U^{238}: \quad 137.7\,A = Ce^{k_2 T} \end{aligned}\right\}$$

$$\frac{1}{2}C = Ce^{k_1(0.71)} \rightarrow \frac{1}{2} = e^{0.71 k_1} \rightarrow$$

$$\ln(1/2) = 0.71 k_1 \rightarrow k_1 = \ln(1/2)/0.71 \;;$$

and

$$\frac{1}{2}C = Ce^{k_2(4.51)} \rightarrow \frac{1}{2} = e^{4.51 k_2} \rightarrow$$

$$\ln(1/2) = 4.51 k_2 \rightarrow k_2 = \ln(1/2)/4.51 \;;$$

then

$$A = Ce^{\frac{\ln(1/2)}{0.71}T} \longrightarrow \text{(plug in)}$$

$$137.7\,A = 137.7\left(\cancel{C}e^{\frac{\ln(1/2)}{0.71}T}\right) = \cancel{C}e^{\frac{\ln(1/2)}{4.51}T} \rightarrow$$

$$137.7 = \frac{e^{\frac{\ln(1/2)}{4.51}T}}{e^{\frac{\ln(1/2)}{0.71}T}} = e^{\left(\frac{\ln(1/2)}{4.51} - \frac{\ln(1/2)}{0.71}\right)T} \rightarrow$$

$$\ln 137.7 = \left(\frac{\ln(1/2)}{4.51} - \frac{\ln(1/2)}{0.71}\right)T \rightarrow$$

$$T = \frac{\ln 137.7}{\frac{\ln(1/2)}{4.51} - \frac{\ln(1/2)}{0.71}} \approx 5.99 \text{ billion years}$$