ESP
Kouba
Worksheet 5 Solutions

1.) a.) $F'(x) = \frac{1}{\sqrt{1-(e^{x^2})^2}} \cdot e^{x^2} \cdot 2x$

b.) $F'(x) = \frac{1}{1+\int_0^x \sqrt{t^3+5}\,dt} \cdot \sqrt{x^3+5}$

c.) $F'(x) = \frac{\ln(\tan x)\cdot \sec^2(\ln x)\cdot \frac{1}{x} - \tan(\ln x)\cdot \frac{1}{\tan x}\cdot \sec^2 x}{[\ln(\tan x)]^2}$

d.) $F'(x) = 0$

e.) $F'(x) = e^{1-\sin x} \cdot \cos x$

f.) $F'(x) = \cos\left(\left(\sqrt{x-1}\right)^2+1\right)\cdot \frac{1}{2\sqrt{x-1}} = \cos x \cdot \frac{1}{2\sqrt{x-1}}$

g.) $F'(x) = D\left\{\int_{\sqrt{x-1}}^{3} \cos(t^2+1)\,dt\right\}$

$= D\left\{-\int_3^{\sqrt{x-1}} \cos(t^2+1)\,dt\right\} = -\cos x \cdot \frac{1}{2\sqrt{x-1}}$

h.) $F(x) = \int_{\frac{1}{x}}^{x^3} (3-t^5)^{100}\,dt = \int_{\frac{1}{x}}^{0} (3-t^5)^{100}\,dt + \int_0^{x^3} (3-t^5)^{100}\,dt$

$= -\int_0^{\frac{1}{x}} (3-t^5)^{100}\,dt + \int_0^{x^3} (3-t^5)^{100}\,dt \Rightarrow$

$F'(x) = -\left(3-\left(\frac{1}{x}\right)^5\right)^{100}\cdot\frac{-1}{x^2} + \left(3-(x^3)^5\right)^{100}\cdot(3x^2)$

i.) $F'(x) = x^5\cdot\frac{x^2}{x^2+1} + 5x^4\cdot\int_0^x \frac{t^2}{t^2+1}\,dt$

2.) a.) $F'(x) = e^{x^2-7} \cdot 2x = 0$

$-\quad 0 \quad +$ $\quad F'$

$x=0$

global min.

b.) $F'(x) = (x-1)(2-x)^6 = 0$

$-\quad 0 \quad + \quad 0 \quad +$ $\quad F'$

$x=0 \qquad x=1 \qquad x=2$

global min.

3.) a.) $\int_1^2 \left(2 \cdot x^{-2} + \frac{1}{2}x^2\right) dx = \left(-2 \cdot x^{-1} + \frac{1}{6}x^3\right)\Big|_1^2$

$= \left(-\frac{2}{x} + \frac{x^3}{6}\right) = \frac{13}{6}$

b.) $\int_{-1}^{0} (1+x)^2 \, dx = \frac{1}{3}(1+x)^3 \Big|_{-1}^{0} = \frac{1}{3}$

c.) $\int_{-1}^{0} (1+x)^{200} \, dx = \frac{1}{201}(1+x)^{201} \Big|_{-1}^{0} = \frac{1}{201}$

d.) $\int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \, dx = \int_0^{\frac{\pi}{6}} \tan x \cdot \sec x \, dx$

$= \sec x \Big|_0^{\frac{\pi}{6}} = \sec\left(\frac{\pi}{6}\right) - \sec(0) = \frac{2}{\sqrt{3}} - 1$

e.) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x\right) dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(1 + 2(\sin x) \cdot \cos x\right) dx = \left(x + \sin^2 x\right)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \left(\frac{\pi}{3} + \frac{3}{4}\right) - \left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{12} + \frac{1}{4}$

f.) $\int_0^1 x^2(1+x^3)^{\frac{1}{10}}\,dx = \frac{1}{3}\cdot\int_0^1 3x^2(1+x^3)^{\frac{1}{10}}\,dx$

$= \frac{1}{3}\cdot\frac{10}{11}(1+x^3)^{\frac{11}{10}}\Big|_0^1 = \frac{10}{33}\left(2^{\frac{11}{10}}-1\right)$.

g.) $\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1}{2}(1+\cos 2x) \Rightarrow$

$\int_0^{\pi}\cos^2 x\,dx = \int_0^{\pi}\frac{1}{2}\cdot(1+\cos 2x)\,dx$

$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right)\Big|_0^{\pi} = \frac{1}{2}(\pi+0) - \frac{1}{2}(0+0) = \frac{\pi}{2}$

h.) $\int_0^{\frac{\pi}{20}} \sec^2(5x)\,dx = \frac{1}{5}\cdot\tan(5x)\Big|_0^{\frac{\pi}{20}}$

$= \frac{1}{5}\tan\left(\frac{\pi}{4}\right) - \frac{1}{5}\tan(0) = \frac{1}{5}$

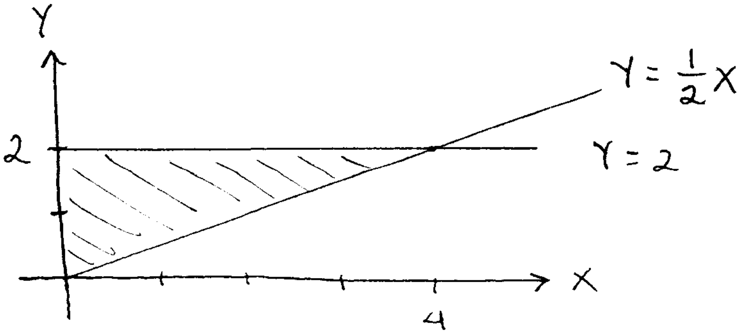
i.) $\int_0^1 2x\cdot\sec^2(x^2)\,dx = \tan(x^2)\Big|_0^1$

$= \tan 1 - \tan 0 = \tan 1$.

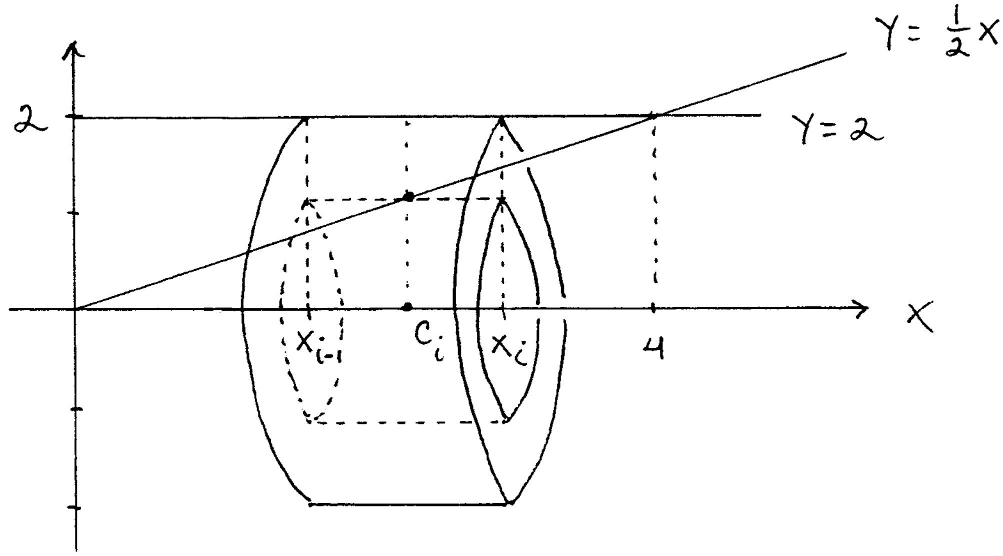
j.) $D(x\cdot\sin x) = x\cos x + \sin x$ so

$\int_0^{\frac{\pi}{2}}(x\cos x + \sin x)\,dx = x\sin x\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2}$.

4.) a.)



b.) i.)



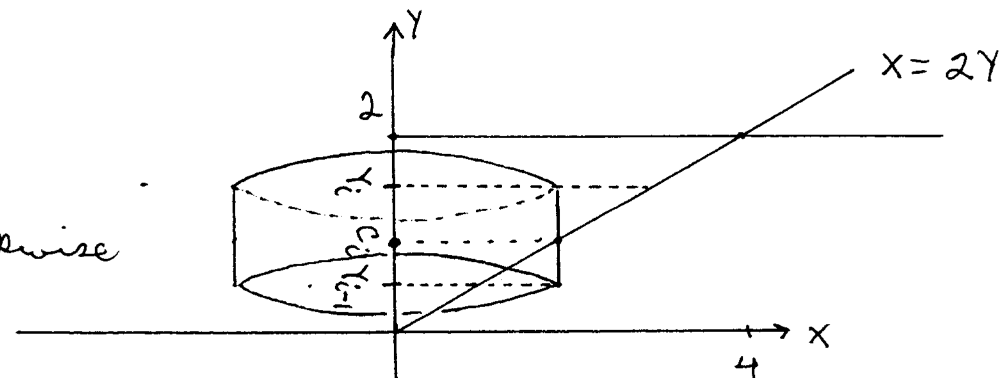
$$\text{Volume} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} \left[\pi(2)^2 (x_i - x_{i-1}) - \pi\left(\tfrac{1}{2}c_i\right)^2 \cdot (x_i - x_{i-1})\right]$$

$$= \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} \pi\left[(2)^2 - \left(\tfrac{1}{2}c_i\right)^2\right] \cdot (x_i - x_{i-1})$$

$$= \int_0^4 \pi\left[(2)^2 - \left(\tfrac{1}{2}x\right)^2\right] \, dx$$

ii.)

Turn $90°$ counter-clockwise $\Rightarrow$

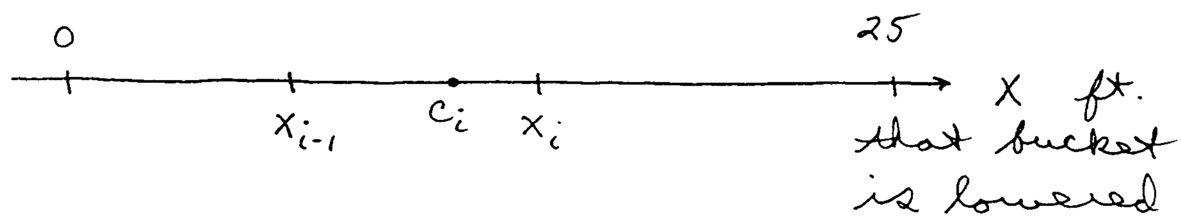


$$\text{Volume} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} \pi(2c_i)^2 \cdot (y_i - y_{i-1})$$

$$= \int_0^2 \pi(2y)^2 \, dy.$$

5.) a.) (100 lbs.)(25 ft.) = 2500 ft.-lbs.

b.)



conversion : $1 \text{ ft.}^3 = (12)^3 \text{ in.}^3 = 1728 \text{ in.}^3 = 62.4 \text{ lbs.}$

so $4 \text{ in.}^3 = .144 \text{ lbs.}$;

after being lowered c_i feet, the bucket of H_2O weighs $100 - .144\, c_i$ lbs. so

$$\text{Work} = \lim_{\text{mesh} \to 0} \sum_{i=1}^{n} (100 - .144\, c_i) \cdot (x_i - x_{i-1})$$

$$= \int_0^{25} (100 - .144\, x)\, dx \quad .$$

6.) a.) $\text{AVE} = \frac{1}{1-0} \int_0^1 x^3\, dx = \frac{1}{4} x^4 \Big|_0^1 = \frac{1}{4}$.

b.) $\text{AVE} = \frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} \sin x\, dx = \frac{2}{\pi} (-\cos x) \Big|_0^{\frac{\pi}{2}}$

$= \frac{2}{\pi} (0 - (-1)) = \frac{2}{\pi}$.

c.) $\text{AVE} = \frac{1}{\sqrt{\ln 5} - \sqrt{\ln 2}} \int_{\sqrt{\ln 2}}^{\sqrt{\ln 5}} 2x e^{x^2}\, dx$

$= \frac{1}{\sqrt{\ln 5} - \sqrt{\ln 2}} \; e^{x^2} \Big|_{\sqrt{\ln 2}}^{\sqrt{\ln 5}} = \frac{3}{\sqrt{\ln 5} - \sqrt{\ln 2}}$.

d.) $\text{AVE} = \frac{1}{\frac{1}{2} - 0} \int_0^{\frac{1}{2}} (x \sec^2 x + \tan x)\, dx$

$= 2\, (x \tan x) \Big|_0^{\frac{1}{2}} = \tan \frac{1}{2}$.