ESP
Kouba
Worksheet 7 Solutions

1.) $N$: # of blackbirds
$t$: # of years

$$\frac{dN}{dt} = kN \Rightarrow N = ce^{kt} \text{ and } t=0 \text{ (1988)},$$

$$N = 750 \text{ birds} \Rightarrow N = 750e^{kt}. \text{ also } t=2 \text{ (1990)},$$

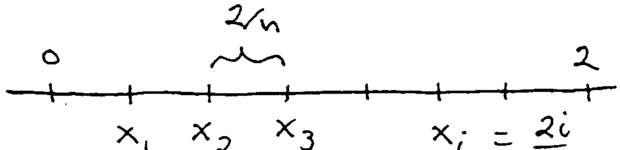
$$N = 1000 \text{ birds} \Rightarrow 1000 = 750e^{2k} \Rightarrow \frac{4}{3} = e^{2k} \Rightarrow$$

$$\ln\left(\frac{4}{3}\right) = 2k \Rightarrow k = \frac{1}{2}\ln\left(\frac{4}{3}\right) = \ln\left(\frac{4}{3}\right)^{1/2} \Rightarrow$$

$$N = 750\left(e^{\ln\left(\frac{4}{3}\right)^{1/2}}\right)^t = 750\left(\frac{4}{3}\right)^{t/2} \text{ or } N = 750\left(\frac{4}{3}\right)^{t/2}.$$

If $t = 7$ (1995), then $N \approx 2053$ birds

2.) a.)

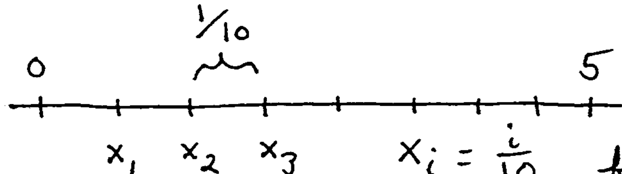



$x_i = \frac{2i}{n}$ for $i = 1, \dots, n$ $\qquad f(x) = \frac{4+x}{7+\sin(1+x)}$

$$\sum_{i=1}^{n} \frac{4+\left(\frac{2i}{n}\right)}{7+\sin\left(1+\left(\frac{2i}{n}\right)\right)} \cdot \frac{2}{n} = \sum_{i=1}^{n} f(x_i)\cdot(x_i - x_{i-1})$$

$$\approx \int_0^2 f(x)\,dx = \int_0^2 \frac{4+x}{7+\sin(1+x)}\,dx.$$

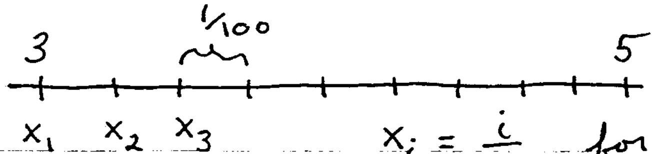
b.)




$x_i = \frac{i}{10}$ for $i = 1, \dots, 50$ $\qquad f(x) = x^4$

$$\sum_{i=1}^{50} \left(\frac{i}{10}\right)^4 \cdot \left(\frac{1}{10}\right) = \sum_{i=1}^{50} f(x_i)\cdot(x_i - x_{i-1}) \approx \int_0^5 f(x)\,dx = \int_0^5 x^4\,dx.$$

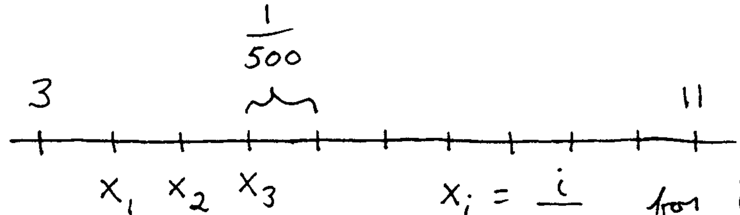
c.)




$x_i = \frac{i}{100}$ for $i = 300, \dots, 499$ $\qquad f(x) = x^2$

$$\sum_{i=300}^{499} \left(\frac{i}{100}\right)^2 \cdot \left(\frac{1}{100}\right) = \sum_{i=300}^{499} f(x_i)\,(x_i - x_{i-1}) \approx \int_3^5 f(x)\,dx$$

$$= \int_3^5 x^2\,dx$$

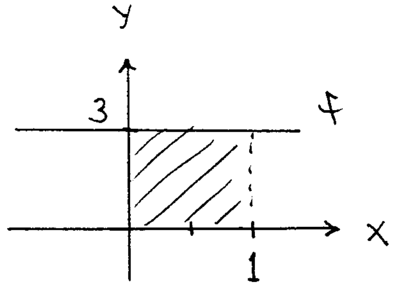
d.) 

$f(x) = x^5 + \sqrt{\frac{1}{x}}$

$x_i = \frac{i}{500}$ for $i = 1501, \ldots, 5500$

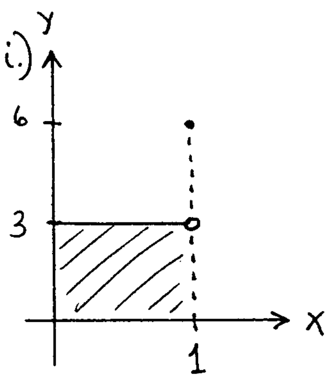
$$\sum_{i=1501}^{5500} \left[ \left(\frac{i}{500}\right)^5 + \sqrt{\frac{500}{i}} \right] \cdot \left(\frac{1}{500}\right) = \sum_{i=1501}^{5500} f(x_i) \cdot (x_i - x_{i-1})$$

$$\approx \int_3^{11} f(x)\, dx = \int_3^{11} \left(x^5 + \sqrt{\frac{1}{x}}\right) dx .$$

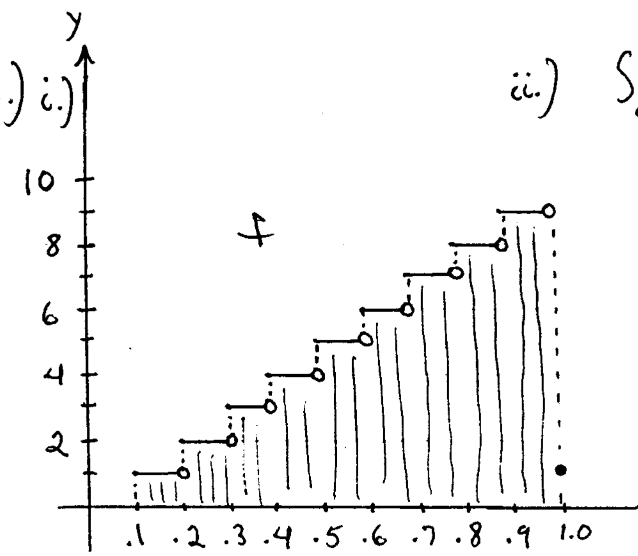
3.) a.) i.) 

ii.) $\int_0^1 f(x)\, dx$

$= \int_0^1 3\, dx = 3x \Big|_0^1 = 3$

b.) i.) 

ii.) $\int_0^1 f(x)\, dx = 3$

c.) i.) 

ii.) $\int_0^1 f(x)\, dx = \frac{1}{10}(0) + \frac{1}{10}(1) + \frac{1}{10}(2)$

$+ \frac{1}{10}(3) + \cdots + \frac{1}{10}(9)$

$= \frac{1}{10}(1 + 2 + 3 + \cdots + 9)$

$= \frac{1}{10}\left(\frac{9\,(9+1)}{2}\right)$

$= \frac{9}{2}$

4.) a.) $Y' = 0$ b.) $Y' = \arctan\sqrt{e^x}\cdot e^x$

c.) $Y = -\int_0^{\sin x}(t^2+5)^{10}\,dt + \int_0^{\cos x}(t^2+5)^{10}\,dt \Rightarrow$

$Y' = -(\sin^2 x+5)^{10}\cdot\cos x + (\cos^2 x+5)^{10}\cdot -\sin x$

d.) $Y^2\cdot 1 + 2YY'x - \sin(3Y)\cdot 3Y' = 2\tan x\cdot\sec^2 x \Rightarrow$

$(2xY - 3\sin(3Y))\cdot Y' = 2\tan x\cdot\sec^2 x - Y^2 \Rightarrow$

$$Y' = \frac{2\tan x\cdot\sec^2 x - Y^2}{2xY - 3\sin(3Y)}$$

e.) $[7+\cos(e^Y)^2]\cdot e^Y\cdot Y' = \frac{1}{Y}Y' - 5x^4 \Rightarrow$

$$Y' = \frac{-5x^4}{[7+\cos(e^{2Y})]\cdot e^y - \frac{1}{y}}$$

5.) a.) $\int\frac{1}{x^2}\,dx = \frac{-1}{x} + c$

b.) $\int\frac{1}{1+x^2}\,dx = \arctan x + c$

c.) $\int\frac{}{1+x^2}\,dx = \frac{1}{2}\ln(x^2+1) + c$

d.) $\int\frac{x^2}{1+x^2}\,dx = \int\frac{1+x^2-1}{1+x^2}\,dx = \int\left[1-\frac{1}{1+x^2}\right]dx$

$= x - \arctan x + c$

$$\begin{array}{r|l} & x \\ x^2+1 & x^3 \\ & \underline{x^3+x} \\ & -x \end{array}$$

e.)

$$\int \frac{x^3}{1+x^2}\,dx = \int \left[x + \frac{-x}{1+x^2}\right] dx = \frac{x^2}{2} - \frac{1}{2}\ln(1+x^2) + c$$

f.) $\int \frac{1}{4+x^2}\,dx = \int \frac{\frac{1}{4}}{1+\left(\frac{x}{2}\right)^2}\,dx$ (Let $u = \frac{x}{2}$, $du = \frac{1}{2}dx \Rightarrow$ $2\,du = dx$) $= \int \frac{\frac{1}{4}}{1+u^2}\,2\,du = \frac{1}{2}\int \frac{1}{1+u^2}\,du$

$= \frac{1}{2}\arctan u + c = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + c$

g.) $\int \frac{1}{(x+1)^2}\,dx = \frac{-1}{x+1} + c$

h.) $\int \frac{x}{(x+1)^2}\,dx$ (Let $u = x+1$, $x = u-1$, $du = dx$)

$= \int \frac{u-1}{u^2}\,du = \int \left[\frac{1}{u} - \frac{1}{u^2}\right] du = \ln|u| + \frac{1}{u} + c$

$= \ln|x+1| + \frac{1}{x+1} + c$.

i.) $\int \frac{x^2}{(x+1)^2}\,dx$ (Let $u = x+1$, $x = u-1$, $du = dx$)

$= \int \frac{(u-1)^2}{u^2}\,du = \int \frac{u^2 - 2u + 1}{u^2}\,du = \int \left[1 - \frac{2}{u} + \frac{1}{u^2}\right] du$

$= u - 2\ln|u| - \frac{1}{u} + c = x+1 - 2\ln|x+1| - \frac{1}{x+1} + c$

j.) $\int \frac{x^2}{(7x-3)^2}\,dx$ (Let $u = 7x-3$, $x = \frac{1}{7}(u+3)$, $du = 7\,dx \Rightarrow$ $\frac{1}{7}du = dx$)

$= \int \frac{\frac{1}{49}(u+3)^2}{u^2} \cdot \frac{1}{7}\,du = \frac{1}{343}\int \frac{u^2+6u+9}{u^2}\,du$

$= \frac{1}{343}\int \left[1 + \frac{6}{u} + \frac{9}{u^2}\right] du = \frac{1}{343}\left(u - 6\ln|u| - \frac{9}{u}\right) + c$

$$= \frac{1}{343}\left(7x-3 - 6\ln|7x-3| - \frac{9}{7x+3}\right)+c$$

k.) $\int \frac{1}{x}\,dx = \ln|x| + c$

l.) $\int \frac{\ln x}{x}\,dx = \frac{1}{2}(\ln x)^2 + c$

m.) $\int \frac{(\ln x)^2}{x}\,dx = \frac{1}{3}(\ln x)^3 + c$

n.) $\int \frac{1}{x \ln x}\,dx = \ln|\ln x| + c$

o.) $\int \frac{(\ln x)^{-1/2}}{x}\,dx = 2(\ln x)^{1/2} + c$

p.) $\int \frac{(7+\ln x)^{-12}}{x}\,dx = \frac{-1}{11}(7+\ln x)^{-11} + c$

q.) $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}\cdot\tan\sqrt{x}\cdot(3+\ln(\tan\sqrt{x}))}\,dx$ (Let $u = 3+\ln(\tan\sqrt{x})$,

$$du = \frac{1}{\tan\sqrt{x}}\cdot\sec^2\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\,dx \Rightarrow$$

$$2\,du = \frac{\sec^2\sqrt{x}}{\sqrt{x}\cdot\tan\sqrt{x}}\,dx\ )$$

$$= \int \frac{1}{u}\,2\,du = 2\ln|u| + c$$

$$= 2\ln|3+\ln(\tan\sqrt{x})| + c$$

r.) $\int x\ln x\,dx$ (Let $u = \ln x$, $dv = x\,dx \Rightarrow$

$du = \frac{1}{x}\,dx$, $v = \frac{x^2}{2}$ )

$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + c$

s.) $\int \sqrt{x}\cdot \ln x^3\,dx = 3\int \sqrt{x}\cdot \ln x\,dx$ ( Let $u = \ln x$, $dv = \sqrt{x}\,dx$, $du = \frac{1}{x}\,dx$, $v = \frac{2}{3}x^{3/2}$ )

$= 3\left[\frac{2}{3}x^{3/2}\cdot \ln x - \frac{2}{3}\int x^{1/2}\,dx\right] = 3\left[\frac{2}{3}x^{3/2}\cdot \ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + c\right]$

t.) $\int \ln x^{1/2}\,dx = \frac{1}{2}\int \ln x\,dx$ (Let $u = \ln x$, $dv = dx$, $du = \frac{1}{x}\,dx$, $v = x$)

$= \frac{1}{2}\left(x\ln x - \int 1\,dx\right) = \frac{1}{2}x\ln x - \frac{1}{2}x + c$

u.) $\int (\ln x)^2\,dx$ (Let $u = (\ln x)^2$, $dv = dx$, $du = 2\frac{\ln x}{x}\,dx$, $v = x$)

$= x(\ln x)^2 - 2\int \ln x\,dx$ (Let $u = \ln x$, $dv = dx \Rightarrow du = \frac{1}{x}\,dx$, $v = x$ )

$= x(\ln x)^2 - 2\left[x\ln x - \int 1\,dx\right]$

$= x(\ln x)^2 - 2\left[x\ln x - x + c\right]$

v.) $\int xe^{-x}\,dx$ ( Let $u = x$, $dv = e^{-x}\,dx \Rightarrow du = dx$, $v = -e^{-x}$ )

$= -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + c$

w.) $\int \arctan x\,dx$ (Let $u = \arctan x$, $dv = dx \Rightarrow$

$du = \frac{1}{1+x^2}\,dx\,, \quad v = x\ )$

$= x \arctan x - \int \frac{x}{1+x^2}\,dx = x \arctan x - \frac{1}{2}\ln|1+x^2| + c$

x.) $\int \sin\sqrt{x}\,dx$ (Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2u}\,dx$

$\Rightarrow \quad 2u\,du = dx$ )

$= \int \sin u \cdot 2u\,du = 2\int u \sin u\,du$

(Let $w = u$, $dv = \sin u\,du \Rightarrow$

$dw = du$, $v = -\cos u$ )

$= 2\left(-u\cos u + \int \cos u\,du\right) = -2u\cos u + 2\sin u + c$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + c$ .

y.) $\int e^{\sqrt{x}}\,dx$ (Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2u}\,dx \Rightarrow$

$2u\,du = dx$ )

$= \int e^u \cdot 2u\,du = 2\int u e^u\,du$ (Let $w = u$, $dv = e^u\,du \Rightarrow$

$dw = du$, $v = e^u$ )

$= 2\left[ue^u - \int e^u\,du\right] = 2\left[ue^u - e^u + c\right]$

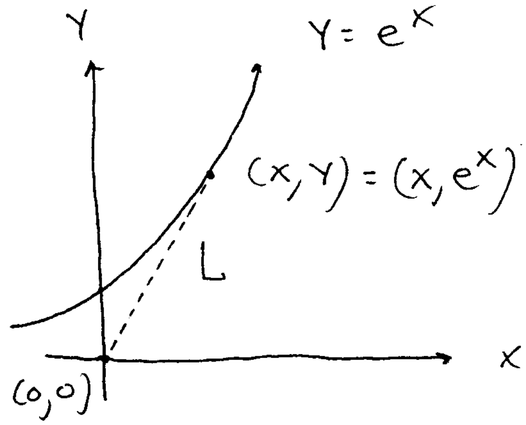
$= 2\left[\sqrt{x}\,e^{\sqrt{x}} - e^{\sqrt{x}} + c\right]$ .

z.) $\int x\sqrt{x+1}\,dx$ (Let $u = x+1$, $x = u-1$, $du = dx$)

$= \int (u-1)\sqrt{u}\,du = \int \left(u^{3/2} - u^{1/2}\right)du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + c$

$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$

6.)




Distance

$$L = \sqrt{x^2 + (e^x)^2} = \sqrt{x^2 + e^{2x}}$$

so

$$S(x) = L^2 = x^2 + e^{2x}$$ so

$$AVE = \frac{1}{2-0} \int_0^2 S(x)\, dx$$

$$= \frac{1}{2} \int_0^2 (x^2 + e^{2x})\, dx$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 + \frac{1}{2}e^{2x}\right)\Big|_0^2$$

$$= \frac{1}{2}\left(\frac{8}{3} + \frac{1}{2}e^4\right) - \frac{1}{2}\left(0 + \frac{1}{2}\right)$$

$$= \frac{13}{12} + \frac{1}{4}e^4$$