

ESP

Kouba

Worksheet 12

1.) Determine whether the following sequences converge or diverge.

a.) $a_n = \frac{e^n}{n+e^n}$

b.) $a_n = \frac{\ln(3n)}{\ln(4n)}$

c.) $a_n = \left[1 + \left(\frac{3}{2}\right)^n\right]^{1/n}$

d.) $a_n = (2^n + 3^n)^{1/n}$

2.) Determine whether the following series converge or diverge.

a.) $\sum_{n=1}^{\infty} \frac{\ln(n+3)}{n}$

b.) $\sum_{n=1}^{\infty} \frac{\cos^4 n}{n^3}$

c.) $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

d.) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{3/4}}$

e.) $\sum_{n=1}^{\infty} \frac{n}{(n^4+10)^{1/3}}$

f.) $\sum_{n=10}^{\infty} \frac{2^n}{4^n - 100}$

g.) $\sum_{n=2}^{\infty} \frac{1}{n \ln n^2}$

h.) $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n^2}$

i.) $\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n-1)}$

3.) Give an example of two convergent series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that $\sum_{n=1}^{\infty} a_n b_n$

a.) converges

b.) diverges

4.) Write each of the following complex numbers in the form $a+bi$.

a.) $(8-3i)-(7-6i)$ b.) $(4+3i)(3-2i)$ c.) $\frac{2+3i}{4+3i}$

5.) Write each of the following complex numbers in polar form.

a.) $\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ b.) i c.) -5 d.) $3-3i$

6.) Simplify each of the following

a.) $\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^4$ b.) $(\sqrt{3}+i)^{10}$

c.) $\frac{[2(\cos 60^\circ + i \sin 60^\circ)]^3}{16(\cos 135^\circ + i \sin 135^\circ)^4}$

7.) Solve $z^2 - 2z + 2 = 0$.

8.) Find all solutions to $z^4 = 16\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$.

9.) Find all cube roots of 8.

10.) Solve for z .

a.) $e^z = i$

b.) $e^z = -5 + 5\sqrt{3}i$