ESP
Kouba
Worksheet 2

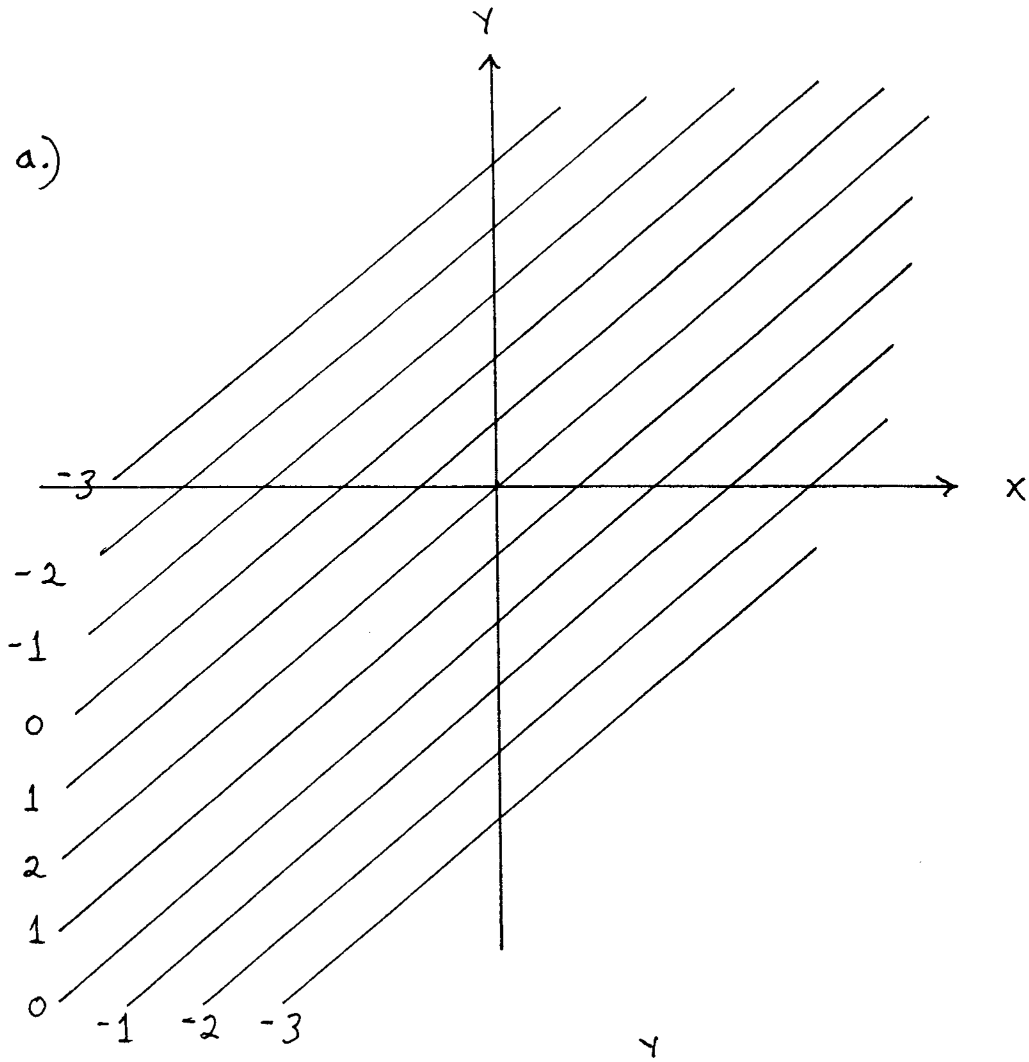
1.) Sketch the level curves for each of the following equations using these values of $z: -3, -2, -1, 0, 1, 2, 3$.

a.) $z = y$
b.) $z = 1 - x - y$
c.) $z^2 = x^2 + y^2$
d.) $x^2 + y^2 + z^2 = 3^2$
e.) $8x^2 + 5y^2 + z^2 = 3^2$
f.) $z = x^2 + y^2$
g.) $z = y^2 - x^2$
h.) $z = 6x/y$

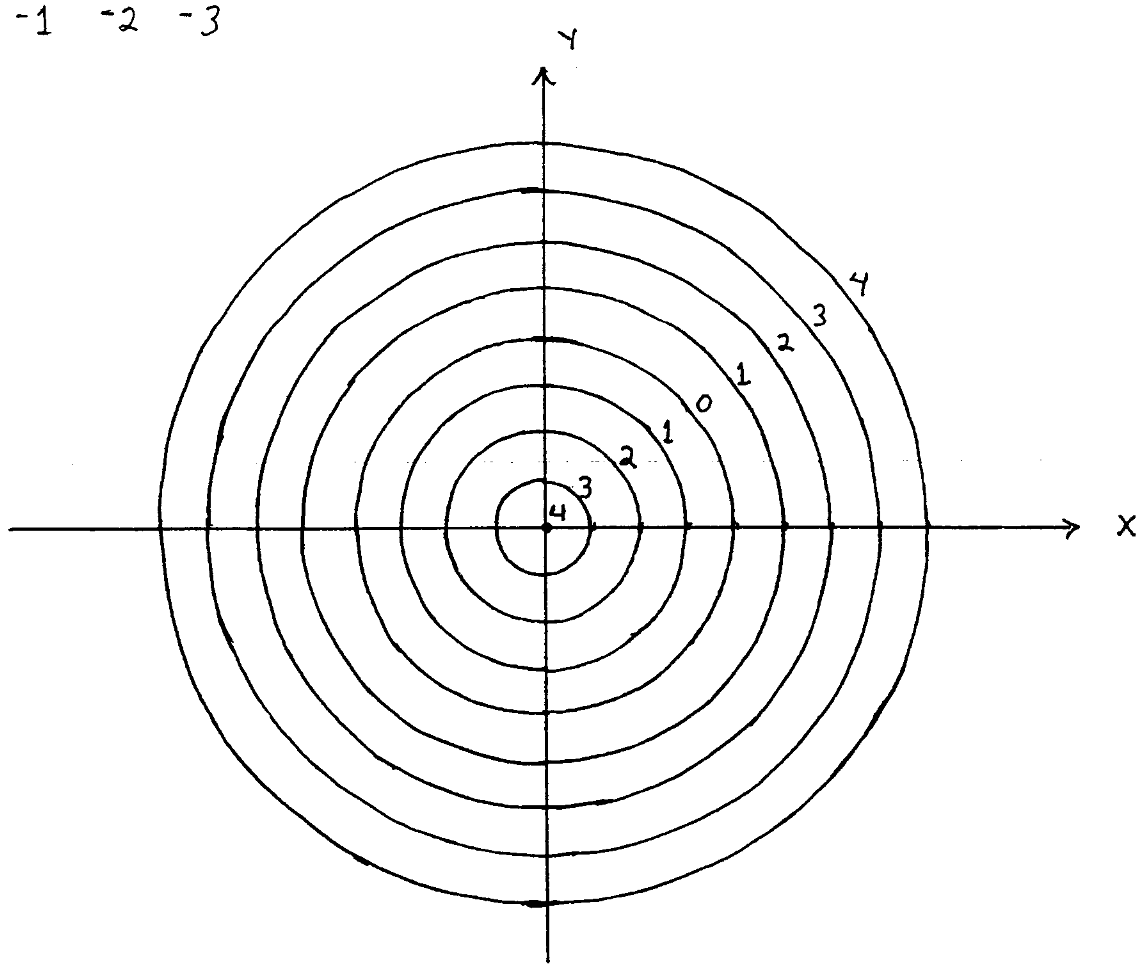
2.) Describe in words (or sketch in three-dimensional space) each of the surfaces in problem 1.).

3.) Describe what the surfaces might be whose level curves are given below (The number next to each level curve refers to the z-value.).

a.)



b.)



4.) Determine an equation for the sphere with center $(1, 2, 3)$ and which is tangent to the plane $z = 1 - y$.

5.) Compute z_x and z_y for each of the following.

a.) $z = xy^2 + \ln x + e^y + 3$

b.) $z = xe^y \arctan x$

c.) $z = \sqrt{x - y^2}$

d.) $z = \dfrac{x^3}{y^2} + \sin(xy)$

e.) $z = \dfrac{x}{x^2 + y^2}$

f.) $z = \left[e^{x^2 - y} + \tan(3y) \right]^5$

g.) $z = y^{1 + x^3}$

6.) Show that $z = \ln(1 + x^2 + y^2)$ satisfies the equation

$$z_{xy} + z_x \cdot z_y = 0.$$

7.) Determine functions z whose partial derivatives are given, or state that this is impossible.

a.) $z_x = 2x$, $z_y = 3y^2 + 1$

b.) $z_x = xy^2 - y$, $z_y = x^2 y - x$

c.) $z_x = 4x^3 y^5 - 1$, $z_y = 5x^4 y^4$

d.) $z_x = ye^x \cos(xy) + e^x \sin(xy)$, $z_y = xe^x \sin(xy)$

e.) $z_x = e^x y - 1$, $z_y = e^x - x$

8.) Plane A, parallel to the xz-plane, and plane B, parallel to the yz-plane, pass through the surface determined by the equation $z = xy^2 - x^3 + 7$. Both planes include the point $(1,0,6)$, which lies on the surface. Determine the slope of the line tangent to the surface at the point $(1,0,6)$ if the line lies in

a.) plane A.

b.) plane B.