FRS– Combinatorics and Problem Solving
D. A. Kouba
Take Home Problem Set 2

This problem set is due in my office not later than 12 noon on Monday, March 19, 2018. Here are guidelines : Attach your copy of the problem set to your solutions. Write clear, complete, and organized solutions on separate sheets of paper. Write up solutions as if you are teaching someone how to do the problem. Answers without supporting work may receive little or no partial credit. Please do your own work. However, if you are stuck on a problem you may ask me for hints or get hints from classmates. Please do not copy solutions from each other or have someone else do the work for you. Thank you for following these guidelines.

1.) (10 pts.) How many ways can you select two dozen donuts if you choose from among caramel, maple, coconut, chocolate, and glazed donuts and you must have at least one of each kind ?

2.) (10 pts.) Determine the number of positive divisors of 15,435.

3.) (5 pts. each) There are 7 (different) Brazilian and 3 (different) British women. How many distinct ways can the women be seated

a.) in a row of 10 chairs ?
b.) in a row of 15 chairs ?
c.) in a row of 10 chairs if all of the Brazilian women must sit next to each other ?
d.) in a row of 10 chairs if all of the British women must sit next to each other ?
e.) in a row of 10 chairs if no British women can sit next to each other ?

4.) (5 pts. each) One hundred UC Davis students were surveyed about where they buy frozen yogurt. The results were : 3 bought at Sugar Plum only, 7 bought at Yogurt Shack only, 10 bought at Yolo Berry and Yogurt Shack but not Sugar Plum, 12 did not buy frozen yogurt, 33 bought at Yolo Berry and Sugar Plum, 13 bought at Sugar Plum and Yogurt Shack but not Yolo Berry, 25 bought at Yolo Berry, Sugar Plum, and Yogurt Shack.

a.) How many students bought at Sugar Plum ?
b.) How many students bought at Yolo Berry but not Yogurt Shack or Sugar Plum ?
c.) How many bought at Yolo Berry and Yogurt Shack ?
d.) How many bought at Yolo Berry or Sugar Plum ?

5.) (10 pts. each) Evaluate the following sums.

a.) $8 + 11 + 14 + \cdots + 368 + 371 + 374$
b.) $(-1/2)^3 + (-1/2)^4 + (-1/2)^5 + \cdots$
c.) $7 + 10 + 14 + 19 + 25 + \cdots + 5054$

6.) (10 pts.) A Super Ball is thrown straight up 50 feet from ground level. On each successive bounce the ball returns to a height of 90% of the distance that it fell. What is the total distance that the ball will travel through the air before it comes to rest ?

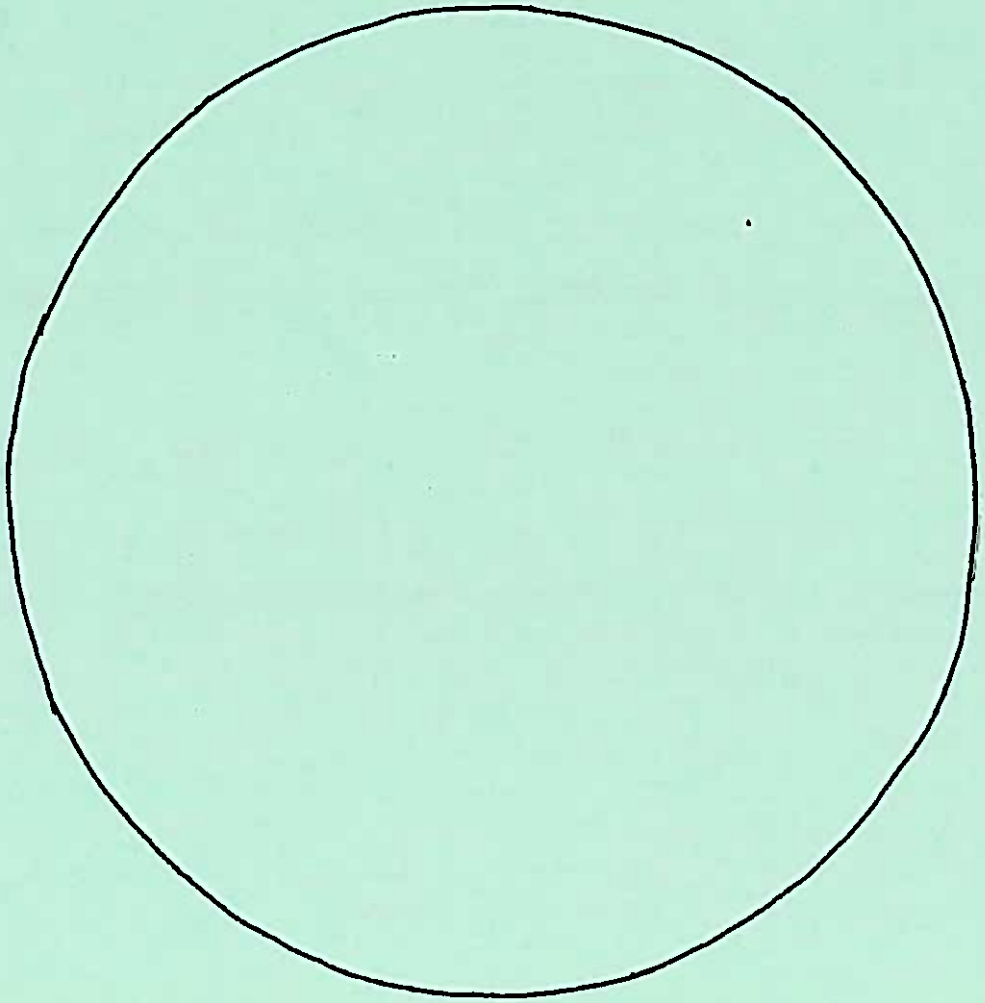
7.) (10 pts.) Write the repeating decimal $0.123412341234\cdots$ as a rational number.

8.) (10 pts. each) Let set $A = P(\phi)$.

a.) List all elements in $P(P(A))$.

b.) How many elements are in $P(P(P(A)))$?

9.) (10 pts.) What is the maximum number of distinct parts (Do not count overlapping parts) that the following circle can be divided using 100 lines ?



10.) (10 pts. OPTIONAL EXTRA CREDIT) Determine the maximum number of distinct rectangles of all sizes (including overlapping rectangles) into which a rectangle can be divided using m vertical lines and n horizontal lines. Use your answer to find the maximum number of rectangles into which a rectangle can be divided using 500 lines.

11.) (10 pts. OPTIONAL EXTRA CREDIT) Determine the exact value of

$$1 + 2(1/2) + 3(1/2)^2 + 4(1/2)^3 + 5(1/2)^4 + \cdots$$