Math 16B
Kouba
Absolute Error

Ex: What should n be so that S_n, Simpson's Rule with n parts, estimates the exact value of $\int_0^2 \ln(x+3)\,dx$ with Absolute Error at most 0.0001?

$$f(x) = \ln(x+3) \xrightarrow{D} f'(x) = \frac{1}{x+3} = (x+3)^{-1}$$

$$\xrightarrow{D} f''(x) = -(x+3)^{-2} \xrightarrow{D} f'''(x) = 2(x+3)^{-3}$$

$$\to f^{(4)}(x) = -6(x+3)^{-4} = \frac{-6}{(x+3)^4} \ ;$$

$$\max_{0 \le x \le 2} |f^{(4)}(x)| = \max_{0 \le x \le 2} \left| \frac{-6}{(x+3)^4} \right|$$

$$= \frac{6}{(0+3)^4} = \frac{6}{81} = \frac{2}{27} \ ; \text{ then}$$

$$h = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n} \text{ and}$$

$$|E_n| \le (2-0) \cdot \frac{\left(\frac{2}{n}\right)^4}{180} \cdot \left\{ \frac{2}{27} \right\}$$

$$= \frac{64}{(180)(27)n^4} = \frac{16}{(45)(27)n^4} = \underline{\underline{\frac{16}{1215 n^4} \le 0.0001}}$$

$$\to n^4 \ge \frac{16}{1215(0.0001)} \approx 131.7$$

$$\to n \ge (131.7)^{1/4} \approx 3.38, \text{ choose } \boxed{n = 4}\ !$$