Section 5.3

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1.) $\int 2\cdot e^{2x}\,dx = 2\cdot \frac{e^{2x}}{2} + c = e^{2x} + c$

3.) $\int e^{4x}\,dx = \frac{1}{4}e^{4x} + c$

6.) $\int 3x\,e^{\frac{1}{2}x^2}\,dx$ (Let $u = \frac{1}{2}x^2 \rightarrow du = x\,dx$)

$= \int 3\cdot e^u\,du = 3e^u + c = 3e^{\frac{1}{2}x^2} + c$

10.) $\int 3(x-4)e^{x^2-8x}\,dx$ (Let $u = x^2-8x \rightarrow du = (2x-8)\,dx$

$= \int 3e^u\cdot\frac{1}{2}\,du$ $\rightarrow du = 2(x-4)\,dx \rightarrow \frac{1}{2}du = (x-4)\,dx$)

$= \frac{3}{2}e^u + c = \frac{3}{2}e^{x^2-8x} + c$

11.) $\int 5e^{2-x}\,dx = 5\cdot\frac{e^{2-x}}{-1} + c = -5e^{2-x} + c$

14.) $\int \frac{1}{x-5}\,dx = \ln|x-5| + c$

15.) $\int \frac{1}{3-2x}\,dx = \frac{-1}{2}\ln|3-2x| + c$

20.) $\int \frac{x^2}{3-x^3}\,dx$ (Let $u = 3-x^3 \rightarrow du = -3x^2\,dx$

$= \int \frac{1}{u}\cdot\frac{-1}{3}\,du$ $\rightarrow \frac{-1}{3}du = x^2\,dx$)

$= \frac{-1}{3}\ln|u| + c = \frac{-1}{3}\ln|3-x^3| + c$

24.) $\int \frac{x^2+2x+3}{x^3+3x^2+9x+1}\,dx$ (Let $u = x^3+3x^2+9x+1 \rightarrow$

$= \int \frac{1}{u}\cdot\frac{1}{3}\,du$ $du = (3x^2+6x+9)\,dx = 3(x^2+2x+3)\,dx \rightarrow$

$\frac{1}{3}du = (x^2+2x+3)\,dx$)

$= \frac{1}{3}\ln|u| + c$

$= \frac{1}{3}\ln|x^3+3x^2+9x+1| + c$

25.) $\int \frac{1}{x \ln x}\,dx$ (Let $u = \ln x \to du = \frac{1}{x}\,dx$)

$= \int \frac{1}{u}\,du = \ln|u| + c = \ln|\ln x| + c$

26.) $\int \frac{1}{x(\ln x)^2}\,dx$ (Let $u = \ln x \to du = \frac{1}{x}\,dx$)

$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du = -u^{-1} + c$

$= \frac{-1}{u} + c = \frac{-1}{\ln x} + c$

30.) $\int \frac{1}{x^3} e^{1/4x^2}\,dx$ (Let $u = \frac{1}{4x^2} = \frac{1}{4}x^{-2} \to du = \frac{-1}{2}x^{-3}\,dx$

$= \int e^u \cdot -2\,du$ $\to -2\,du = \frac{1}{x^3}\,dx$)

$= -2e^u + c = -2e^{\frac{1}{4x^2}} + c$

31.) $\int \frac{1}{\sqrt{x}} e^{\sqrt{x}}\,dx$ (Let $u = \sqrt{x} \to du = \frac{1}{2\sqrt{x}}\,dx$

$= \int e^u \cdot 2\,du$ $\to 2\,du = \frac{1}{\sqrt{x}}\,dx$)

$= 2e^u + c = 2e^{\sqrt{x}} + c$

34.) $\int (e^x - e^{-x})^2\,dx = \int \left((e^x)^2 - 2e^x e^{-x} + (e^{-x})^2\right) dx$

$= \int (e^{2x} - 2\cancel{e^0}^{1} + e^{-2x})\,dx = \frac{1}{2}e^{2x} - 2x + \frac{-1}{2}e^{-2x} + c$

36.) $\int \frac{3e^x}{2+e^x}\,dx$ (Let $u = 2 + e^x \to du = e^x\,dx$)

$= \int \frac{3}{u}\,du = 3\ln|u| + c = 3\ln|2+e^x| + c$

38.) $\int \frac{-e^{3x}}{2-e^{3x}}\,dx$ (Let $u = 2 - e^{3x} \to du = -3e^{3x}\,dx$

$= \int \frac{-1}{u} \cdot \frac{-1}{3}\,du$ $\to \frac{-1}{3}\,du = e^{3x}\,dx$)

$= \frac{1}{3}\ln|u| + c = \frac{1}{3}\ln|2-e^{3x}| + c$

39.) $\int \frac{e^{2x}+2e^{x}+1}{e^{x}}\,dx = \int\left(\frac{e^{2x}}{e^{x}}+\frac{2e^{x}}{e^{x}}+\frac{1}{e^{x}}\right)dx$

$= \int\left(e^{x}+2+e^{-x}\right)dx = e^{x}+2x-e^{-x}+c$

41.) $\int e^{x}\sqrt{1-e^{x}}\,dx$ (Let $u=1-e^{x} \to du=-e^{x}dx$

$= \int \sqrt{u}\cdot -du$ $\qquad \to -du = e^{x}dx$)

$= -\frac{u^{3/2}}{3/2}+c = -\frac{2}{3}(1-e^{x})^{3/2}+c$

42.) $\int \frac{2(e^{x}-e^{-x})}{(e^{x}+e^{-x})^{2}}\,dx$ (Let $u=e^{x}+e^{-x} \to du=(e^{x}-e^{-x})dx$)

$= \int \frac{2}{u^{2}}\,du = 2\int u^{-2}\,du = 2\cdot\frac{u^{-1}}{-1}+c = -2(e^{x}+e^{-x})^{-1}+c$

44.) $\int \frac{1}{\sqrt{x+1}}\,dx = \int (x+1)^{-1/2}\,dx = \frac{(x+1)^{1/2}}{1/2}+c$

47.) $\int \frac{x^{3}-8x}{2x^{2}}\,dx = \int\left(\frac{1}{2}x-\frac{4}{x}\right)dx$

$= \frac{1}{2}\cdot\frac{x^{2}}{2}-4\cdot\ln|x|+c$

49.) $\int \frac{2}{1+e^{-x}}\,dx = \int \frac{2}{1+e^{-x}}\cdot\frac{e^{x}}{e^{x}}\,dx = \int \frac{2e^{x}}{e^{x}+e^{-x}e^{x}}\,dx$

$= \int \frac{2e^{x}}{e^{x}+e^{0}}\,dx = 2\int \frac{e^{x}}{e^{x}+1}\,dx = 2\ln|e^{x}+1|+c$

51.) $\int \frac{x^{2}+2x+5}{x-1}\,dx$

$$\begin{array}{r|l} & x+3 \\ \hline x-1 & x^{2}+2x+5 \\ & x^{2}-x \\ \hline & 3x+5 \\ & 3x-3 \\ \hline & 8 \end{array}$$

$= \int\left[x+3+\frac{8}{x-1}\right]dx$

$= \frac{x^{2}}{2}+3x+8\ln|x-1|+c$

52.) $\int \frac{x-3}{x+3}\,dx$ (Let $u=x+3$, $du=1\,dx$, and $x=u-3$)

$= \int \frac{(u-3)-3}{u}\,du = \int \frac{u-6}{u}\,du = \int\left(\frac{u}{u}-\frac{6}{u}\right)du$

$$= \int\left(1-\frac{6}{u}\right)du = u - 6\ln|u| + c = (x+3) - 6\ln|x+3| + c$$

53.) $$\int \frac{1+e^{-x}}{1+xe^{-x}}\,dx = \int \frac{1+e^{-x}}{1+xe^{-x}}\cdot\frac{e^x}{e^x}\,dx$$

$$= \int \frac{e^x + e^{-x}e^x}{e^x + xe^{-x}e^x}\,dx = \int \frac{e^x+e^0}{e^x+xe^0}\,dx = \int\frac{e^x+1}{e^x+x}\,dx = \ln|e^x+x| + c$$

57.) $$\frac{dP}{dt} = \frac{3000}{1+0.25t} \rightarrow P = \int \frac{3000}{1+0.25t}\,dt$$

(Let $u = 1+0.25t \rightarrow du = 0.25\,dt \rightarrow 4\,du = dt$) $\rightarrow$

$$P = \int \frac{3000}{u}\cdot 4\,du = 12{,}000 \ln|u| + c$$

$$= 12{,}000 \ln|1+0.25t| + C \quad \text{and}$$

$t=0,\ P=1000$ so $1000 = 12{,}000\, \overset{0}{\cancel{\ln 1}} + c \rightarrow C = 1000 \rightarrow$

a.) $P = 12{,}000 \ln|1+0.25t| + 1000$

b.) $t = 3$ days $\rightarrow P \approx 7715$ bacteria

c.) $P = 12{,}000$ bacteria $\rightarrow$

$12{,}000 = 12{,}000 \ln|1+0.25t| + 1000 \rightarrow \frac{11{,}000}{12{,}000} = \ln(1+0.25t)$

$\rightarrow 1+0.25t = e^{\frac{11}{12}} \longrightarrow t \approx 6$ days

58.) $\frac{dP}{dt} = -125e^{\frac{-t}{20}}$ and $P = 2500$ when $t = 0$:

so $P = \int -125 e^{\frac{-t}{20}}\,dt = -125\cdot\frac{e^{\frac{-t}{20}}}{\frac{-1}{20}} + c$

$= 2500e^{\frac{-t}{20}} + c$; then $t=0,\ P=2500 \longrightarrow$

$2500 = 2500\cdot e^0 + c \rightarrow c = 0 \longrightarrow$

a.) $P = 2500\,e^{\frac{-t}{20}}$

b.) If $t = 15$ days, then $P = 1181$ trout.

c.) If $P = 0$ then $0 = 2500 e^{\frac{-t}{20}} \rightarrow$
$t \rightarrow \infty$!

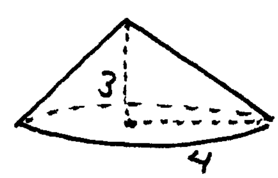
<u>note</u>: If
$t = 150$: $P \approx 1.38$
$t = 155$: $P \approx 1.08$
$t = 156$: $P \approx 1.02$
$t = 157$: $P \approx 0.97 < 1$,

so the last trout dies
on day 157.

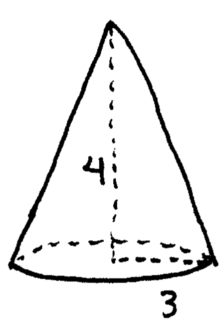
63.) $(\ln x)^{1/2} = \frac{1}{2}(\ln x)$, FALSE :
let $x = e \rightarrow (\ln e)^{1/2} = 1^{1/2} = 1 \neq \frac{1}{2}\ln e = \frac{1}{2}(1) = \frac{1}{2}$

64.) $\int \ln x = \frac{1}{x} + C$, FALSE :
$D\left(\frac{1}{x} + C\right) = \frac{-1}{x^2} + 0 = \frac{-1}{x^2} \neq \ln x$

$\boxed{\text{SA10}}$: a.) $V = \frac{1}{3}\pi(4)^2(3) = 16\pi$



b.)



$V = \frac{1}{3}\pi(3)^2(4)$
$= 12\pi$

Section 8.5

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13.) $\int \tan 3x\, dx$ (Let $u = 3x \xrightarrow{D} du = 3\,dx$
$\rightarrow \frac{1}{3} du = dx$)

$= \frac{1}{3}\int \tan u\, du = \frac{1}{3} \ln|\sec u| + c$

$= \frac{1}{3} \ln|\sec 3x| + c$

15.) $\int \tan^3 x \cdot \sec^2 x\, dx$ (Let $u = \tan x \xrightarrow{D}$
$du = \sec^2 x\, dx$)

$= \int u^3\, du = \frac{1}{4} u^4 + c = \frac{1}{4}(\tan x)^4 + c$

17.) $\int \cot \pi x\, dx$ (Let $u = \pi x \xrightarrow{D} du = \pi\, dx$
$\rightarrow \frac{1}{\pi} du = dx$)

$= \frac{1}{\pi}\int \cot u\, du = \frac{1}{\pi} \ln|\sin u| + c$

$= \frac{1}{\pi} \ln|\sin \pi x| + c$

19.) $\int \csc 2x\, dx$ (Let $u = 2x \xrightarrow{D} du = 2\,dx$
$\rightarrow \frac{1}{2} du = dx$)

$= \frac{1}{2}\int \csc u\, du = \frac{1}{2} \ln|\csc u - \cot u| + c$

$= \frac{1}{2} \ln|\csc 2x - \cot 2x| + c$

20.) $\int \sec\left(\frac{x}{2}\right) dx$ (Let $u = \frac{x}{2} \xrightarrow{D} du = \frac{1}{2} dx$
$\rightarrow 2\, du = dx$)

$= 2\int \sec u \, du = 2 \ln|\sec u + \tan u| + c$

$= 2 \ln\left|\sec\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right)\right| + c$

21.) $\int \frac{\sec^2 2x}{\tan 2x} dx$ (Let $u = \tan 2x \xrightarrow{D}$

$du = \sec^2 2x \cdot 2\, dx \rightarrow \frac{1}{2} du = \sec^2 2x \, dx$)

$= \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + c = \frac{1}{2} \ln|\tan 2x| + c$

23.) $\int \frac{\sec x \tan x}{\sec x - 1} dx$ (Let $u = \sec x - 1 \xrightarrow{D}$

$du = \sec x \tan x \, dx$)

$= \int \frac{1}{u} du = \ln|u| + c = \ln|\sec x - 1| + c$

24.) $\int \frac{\cos t}{1 + \sin t} dt$ (Let $u = 1 + \sin t \xrightarrow{D}$

$du = \cos t \, dt$)

$= \int \frac{1}{u} du = \ln|u| + c = \ln|1 + \sin t| + c$

25.) $\int \frac{\sin x}{1 + \cos x} dx$ (Let $u = 1 + \cos x \xrightarrow{D}$

$du = -\sin x \, dx \rightarrow -du = \sin x \, dx$)

$= -\int \frac{1}{u} du = -\ln|u| + c$

$= -\ln|1 + \cos x| + c$

27.) $\int \frac{\csc^2 x}{\cot^3 x}\,dx$ (Let $u = \cot x \xrightarrow{D}$

$du = -\csc^2 x\,dx \rightarrow -du = \csc^2 x\,dx$)

$= -\int \frac{1}{u^3}\,du = -\int u^{-3}\,du = -\frac{u^{-2}}{-2} + c$

$= \frac{1}{2}(\cot x)^{-2} + c$

28.) $\int \frac{1-\cos\theta}{\theta - \sin\theta}\,d\theta$ (Let $u = \theta - \sin\theta \xrightarrow{D}$

$du = (1-\cos\theta)\,d\theta$)

$= \int \frac{1}{u}\,du = \ln|u| + c = \ln|\theta - \sin\theta| + c$

30.) $\int e^{-x}\tan e^{-x}\,dx$ (Let $u = e^{-x} \xrightarrow{D}$

$du = -e^{-x}\,dx \rightarrow -du = e^{-x}\,dx$)

$= -\int \tan u\,du = -\ln|\sec u| + c$

$= -\ln|\sec e^{-x}| + c$