Section 5.4

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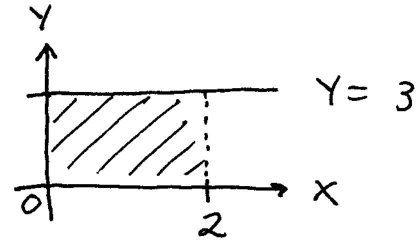
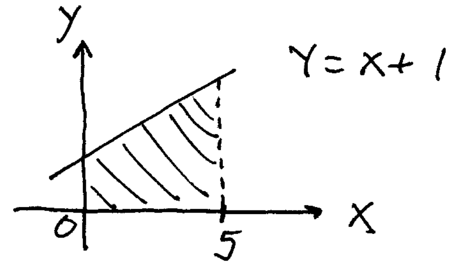
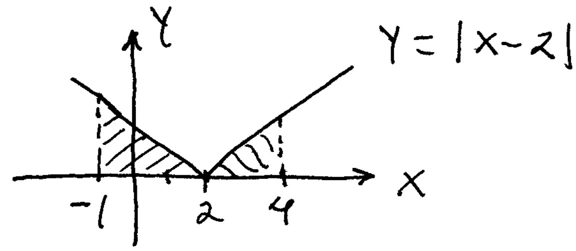
1.) $\int_0^2 3\,dx$



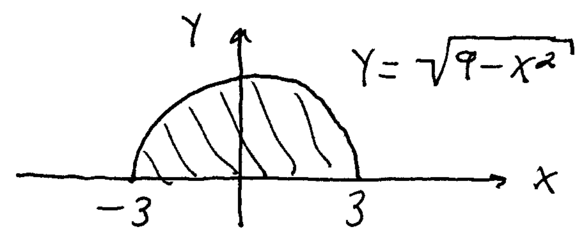
3.) $\int_0^5 (x+1)\,dx$



6.) $\int_{-1}^4 |x-2|\,dx$



7.) $\int_{-3}^3 \sqrt{9-x^2}\,dx$



10.) a.) $\int_0^5 2g(x)\,dx = 2\int_0^5 g(x)\,dx = 2(3) = 6$

b.) $\int_5^0 f(x)\,dx = -\int_0^5 f(x)\,dx = -(8) = -8$

c.) $\int_5^5 f(x)\,dx = 0$

d.) $\int_0^5 (f(x) - f(x))\,dx = \int_0^5 0\,dx = 0$

11.) $\text{Area} = \int_0^1 (x - x^2)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1$

$= \left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0) = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

13.) $\text{Area} = \int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = -x^{-1}\Big|_1^2$

$= \frac{-1}{x}\Big|_1^2 = \frac{-1}{2} - \frac{-1}{1} = 1 - \frac{1}{2} = \frac{1}{2}$

15.) $\text{Area} = \int_0^4 3e^{-x/2}\,dx = 3\int_0^4 e^{-\frac{1}{2}x}\,dx$

$= 3 \cdot \frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\Big|_0^4 = -6 \cdot \frac{1}{e^{\frac{1}{2}x}}\Big|_0^4 = -6 \cdot \frac{1}{e^2} - -6 \cdot \frac{1}{1}$

$= 6 - \frac{6}{e^2}$

18.) $\text{Area} = \int_2^4 \frac{x-2}{x}\,dx = \int_2^4 \left(\frac{x}{x} - \frac{2}{x}\right)dx$

$= \int_2^4 \left(1 - 2 \cdot \frac{1}{x}\right)dx = \left(x - 2\ln|x|\right)\Big|_2^4$

$= (4 - 2\ln 4) - (2 - 2\ln 2) = 4 - \ln 4^2 - 2 + \ln 2^2$

$= 2 + \ln 4 - \ln 16 = 2 + \ln(4/16) = 2 + \ln(1/4)$

$= 2 + \cancelto{0}{\ln 1} - \ln 4 = 2 - \ln 4$

19.) $\int_0^1 2x\,dx = x^2\Big|_0^1 = 1^2 - 0^2 = 1$

22.) $\int_2^5 (-3x+4)\,dx = \left(\frac{-3}{2}x^2 + 4x\right)\Big|_2^5$

$= \left(\frac{-75}{2} + 20\right) - (-6+8) = \frac{-75}{2} + \frac{36}{2} = \frac{-39}{2}$

23.) $\int_{-1}^1 (2t-1)^2\,dt = \int_{-1}^1 (4t^2 - 4t + 1)\,dt$

$= \left(4\cdot\frac{t^3}{3} - 4\cdot\frac{t^2}{2} + t\right)\Big|_{-1}^1 = \left(\frac{4}{3} - \cancel{2} + 1\right) - \left(\frac{-4}{3} - \cancel{2} - 1\right) = \frac{8}{3} + 2$
$= \frac{14}{3}$

28.) $\int_1^4 \sqrt{\frac{2}{x}}\,dx = \int_1^4 \frac{\sqrt{2}}{\sqrt{x}}\,dx = \int_1^4 \sqrt{2}\cdot x^{-1/2}\,dx$

$= \sqrt{2}\cdot\frac{x^{1/2}}{\frac{1}{2}}\Big|_1^4 = 2\sqrt{2}\left(\sqrt{4} - \sqrt{1}\right) = 2\sqrt{2}$

31.) $\int_{-1}^0 \left(t^{1/3} - t^{2/3}\right)dt = \left(\frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3}\right)\Big|_{-1}^0$

$= \left(\frac{3}{4}(0)^{4/3} - \frac{3}{5}(0)^{5/3}\right) - \left(\frac{3}{4}(-1)^{4/3} - \frac{3}{5}(-1)^{5/3}\right) = 0 - \left(3/4 + 3/5\right) = \frac{-27}{20}$

34.) $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx$ (Let $u = 1+2x^2 \rightarrow du = 4x\,dx$
$\rightarrow \frac{1}{4}\,du = x\,dx$)

$= \int \frac{1}{\sqrt{u}}\cdot\frac{1}{4}\,du$

$= \frac{1}{4}\int u^{-1/2}\,du = \frac{1}{4}\cdot 2u^{1/2}\Big|_{x=0}^{x=2} = \frac{1}{2}(1+2x^2)^{1/2}\Big|_0^2 = \frac{1}{2}\sqrt{9} - \frac{1}{2} = 1$

37.) $\int_1^3 \frac{e^{3/x}}{x^2}\,dx$ (Let $u = \frac{3}{x} \rightarrow du = \frac{-3}{x^2}\,dx$
$\rightarrow \frac{-1}{3}\,du = \frac{1}{x^2}\,dx$)

$= \int e^u \cdot \frac{-1}{3}\,du$

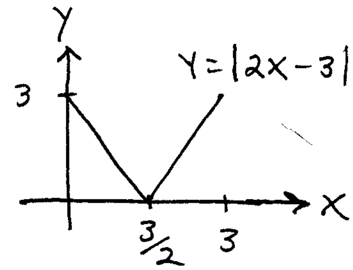
$= \frac{-1}{3}e^u\Big| = \frac{-1}{3}e^{3/x}\Big|_1^3 = \frac{-1}{3}(e - e^3)$

40.) $\int_0^1 \frac{e^{-x}}{\sqrt{e^{-x}+1}}\,dx$ (Let $u = e^{-x} + 1 \rightarrow$
$du = -e^{-x}\,dx \rightarrow -du = e^{-x}\,dx$)

$= -\int_{x=0}^{x=1} \frac{1}{\sqrt{u}}\,du = -\int_{x=0}^{x=1} u^{-1/2}\,du = -\frac{u^{1/2}}{1/2}\Big|_{x=0}^{x=1}$

$$= -2\sqrt{e^{-x}+1}\,\Big|_0^1 = -2\sqrt{e^{-1}+1} - -2\sqrt{1+1}$$

$$= 2\sqrt{2} - 2\sqrt{e^{-1}+1}$$



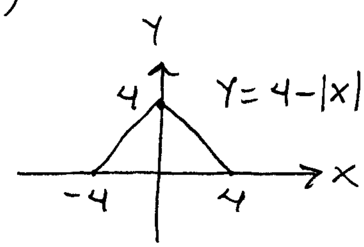
44.) $\int_0^3 |2x-3|\,dx$

$$= \int_0^{3/2} |2x-3|\,dx + \int_{3/2}^3 |2x-3|\,dx$$

$$= \int_0^{3/2} -(2x-3)\,dx + \int_{3/2}^3 (2x-3)\,dx$$

$$= (-x^2+3x)\Big|_0^{3/2} + (x^2-3x)\Big|_{3/2}^3 = \left(-\tfrac{9}{4}+\tfrac{9}{2}\right) - (0+0)$$

$$+ (9-9) - \left(\tfrac{9}{4}-\tfrac{9}{2}\right) = \frac{-18}{4} + \frac{18}{2} = \frac{9}{2}$$



46.) $\int_{-4}^4 (4-|x|)\,dx$

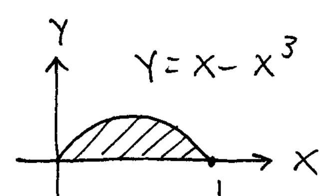
$$= \int_{-4}^0 (4-|x|)\,dx + \int_0^4 (4-|x|)\,dx$$

$$= \int_{-4}^0 4-(-x)\,dx + \int_0^4 (4-x)\,dx = \left(4x+\tfrac{x^2}{2}\right)\Big|_{-4}^0 + \left(4x-\tfrac{x^2}{2}\right)\Big|_0^4$$

$$= (0+0) - (-16+8) + (16-8) - (0-0) = 16$$

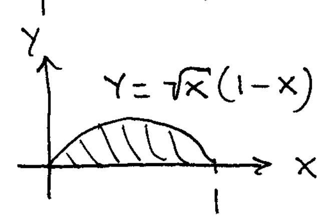
50.) $\int_1^2 \frac{(2+\ln x)^3}{x}\,dx = \frac{(2+\ln x)^4}{4}\Big|_1^2$

$$= \frac{(2+\ln 2)^4}{4} - \frac{(2+\overset{0}{\ln 1})^4}{4} = \frac{(2+\ln 2)^4}{4} - 4$$



53.) $\int_0^1 (x-x^3)\,dx$

$$= \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$



54.) $\int_0^1 \sqrt{x}(1-x)\,dx$

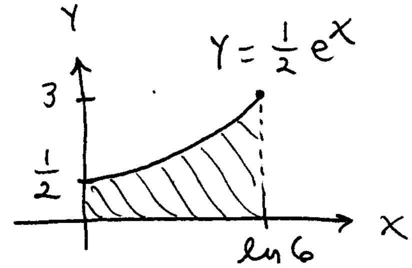
$$= \int_0^1 (x^{1/2} - x^{3/2})\,dx$$

$$= \left(\tfrac{2}{3}x^{3/2} - \tfrac{2}{5}x^{5/2}\right)\Big|_0^1 = \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$$

56.) $\displaystyle\int_0^{\ln 6} \frac{e^x}{2}\,dx$

$\displaystyle = \frac{1}{2}e^x \Big|_0^{\ln 6} = \frac{1}{2}\left(e^{\ln 6} - e^0\right)$

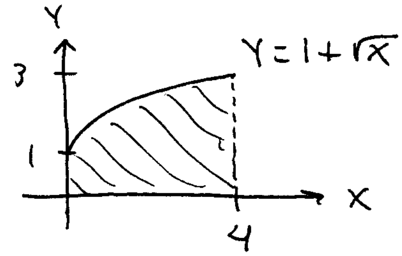
$\displaystyle = \frac{1}{2}(6-1) = \frac{5}{2}$



58.) $\displaystyle \text{Area} = \int_0^4 (1+\sqrt{x})\,dx$

$\displaystyle = \left(x + \frac{2}{3}x^{3/2}\right)\Big|_0^4 = \left(4 + \frac{2}{3}(4)^{3/2}\right) - (0+0)$

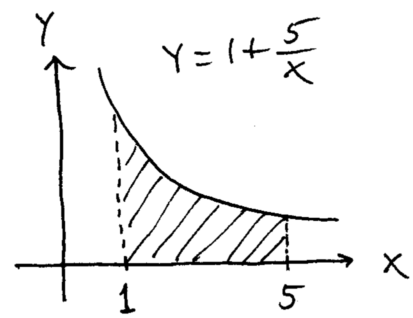
$\displaystyle = 4 + \frac{2}{3}(8) = \frac{12}{3} + \frac{16}{3} = \frac{28}{3}$



59.) $\displaystyle \text{Area} = \int_1^5 \left(1 + \frac{5}{x}\right)dx$

$\displaystyle = (x + 5\ln|x|)\Big|_1^5$

$\displaystyle = (5 + 5\ln 5) - (1 + 5\overset{0}{\cancel{\ln 1}}) = 4 + 5\ln 5$



63.) $\displaystyle \text{AVE} = \frac{1}{10-0}\int_0^{10} 5e^{0.2(x-10)}\,dx = \frac{1}{10}\int_0^{10} 5e^{\frac{1}{5}x-2}\,dx$

$\displaystyle = \frac{1}{10}\cdot 5\ \frac{e^{\frac{1}{5}x-2}}{\frac{1}{5}}\Big|_0^{10} = \frac{5}{2}\left(e^0 - e^{-2}\right) = \frac{5}{2}\left(1 - e^{-2}\right)$

65.) $\displaystyle \text{AVE} = \frac{1}{2-0}\int_0^2 x\sqrt{4-x^2}\,dx$

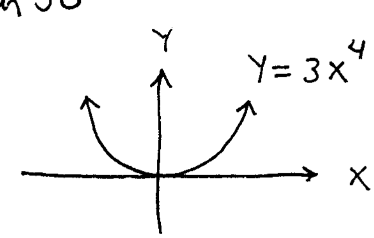
$\displaystyle = \frac{1}{2}\cdot\frac{-1}{2}\cdot\frac{(4-x^2)^{3/2}}{3/2}\Big|_0^2 = -\frac{1}{6}\left((0)^{3/2} - (4)^{3/2}\right) = \frac{-1}{6}(-8) = \frac{4}{3}$

67.) $\displaystyle \text{AVE} = \frac{1}{7-0}\int_0^7 \frac{5x}{x^2+1}\,dx$

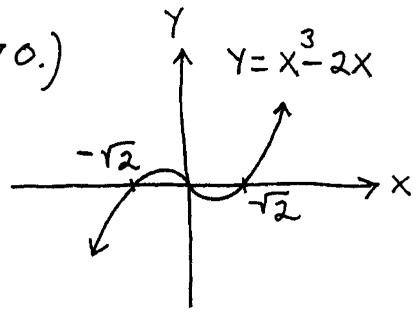
$\displaystyle = \frac{1}{7}\cdot 5\cdot\frac{1}{2}\ln|x^2+1|\Big|_0^7 = \frac{5}{14}\left(\ln 50 - \overset{0}{\cancel{\ln 1}}\right) = \frac{5}{14}\ln 50$

69.) $f(x) = 3x^4$ is <u>even</u> since

$f(-x) = 3(-x)^4 = 3x^4 = f(x)$



70.)



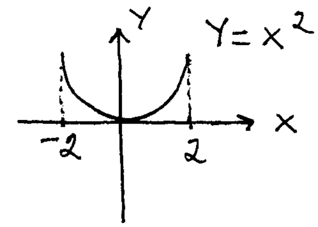
$f(x)=x^3-2x$ is odd since

$f(-x)=(-x)^3-2(-x)$

$= -x^3+2x$

$= -(x^3-2x) = -f(x)$

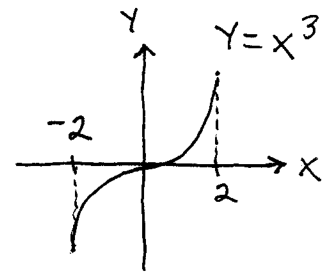
73.) a.) $\int_{-2}^{0} x^2\,dx = \int_{0}^{2} x^2\,dx = 8/3$



b.) $\int_{-2}^{2} x^2\,dx = \int_{-2}^{0} x^2\,dx + \int_{0}^{2} x^2\,dx = \frac{8}{3}+\frac{8}{3} = \frac{16}{3}$

c.) $\int_{0}^{2} -x^2\,dx = -\int_{0}^{2} x^2\,dx = -8/3$

74.) a.) $\int_{-2}^{0} x^3\,dx = -\int_{0}^{2} x^3\,dx = -4$



b.) $\int_{-2}^{2} x^3\,dx = \int_{-2}^{0} x^3\,dx + \int_{0}^{2} x^3\,dx = -4+4=0$

c.) $\int_{0}^{2} 3x^3\,dx = 3\int_{0}^{2} x^3\,dx = 3(4) = 12$

90.) The loss of value over the 1st 3 years is

$V(0)-V(3) = -\int_{0}^{3} V'(t)\,dt = -\int_{0}^{3} 10{,}000\,(t-6)\,dt$

$= -10{,}000\left(\frac{t^2}{2}-6t\right)\Big|_{0}^{3} = -10{,}000\left(\frac{9}{2}-18\right) = \$135{,}000$

91.) $A = Pe^{rt} = 2250\,e^{0.12t}$ so

$AVE = \frac{1}{5-0}\int_{0}^{5} 2250\,e^{0.12t}\,dt = 450\cdot\frac{e^{0.12t}}{0.12}\Big|_{0}^{5}$

$= 3750\,(e^{0.6}-e^{0}) = \$3082.95$

Handout 6

1.) $\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos 2x + 1 = 2\cos^2 x$

$\Rightarrow \dfrac{\cos 2x + 1}{2} = \cos^2 x$ then

$$\int \cos^2 x \, dx = \int \frac{\cos 2x + 1}{2} \, dx$$

$$= \int \left(\tfrac{1}{2} \cos 2x + \tfrac{1}{2} \right) dx = \tfrac{1}{4} \cdot \sin 2x + \tfrac{1}{2} x + c$$

2.) $\int \cos^2 3x \, dx = \int \dfrac{\cos 6x + 1}{2} \, dx$

$$= \int \left(\tfrac{1}{2} \cos 6x + \tfrac{1}{2} \right) dx = \tfrac{1}{12} \cdot \sin 6x + \tfrac{1}{2} x + c$$

3.) $\int (1 + \sin x + \cos x)^2 \, dx$

$$= \int (1 + 2\sin x + 2\cos x + 2\sin x \cos x + \underbrace{\cos^2 x + \sin^2 x}_{1}) \, dx$$

$$= x + {}^{-}2\cos x + 2\sin x + (\sin x)^2 + x + c$$

4.) $\int 2x \sec(x^2+1) \tan(x^2+1) \, dx$ (Let $u = x^2 + 1 \to du = 2x\,dx$)

$$= \int \sec u \tan u \, du = \sec u + c = \sec(x^2+1) + c$$

5.) $\int (x \sec^2 x + \tan x) \sec^2(x \tan x) \, dx$ (Let $u = x \tan x$ $\to du = (x\sec^2 x + \tan x)\,dx$)

$$= \int \sec^2 u \, du$$

$$= \tan u + c = \tan(x \tan x) + c$$

6.) $\int (1 - \sin x) \sin(x + \cos x) \, dx$ (Let $u = x + \cos x$ $\to du = (1 - \sin x)\,dx$)

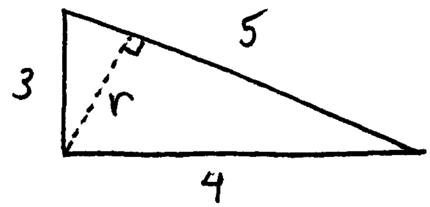
$$= \int \sin u \, du$$

$$= -\cos u + c = -\cos(x + \cos x) + c$$

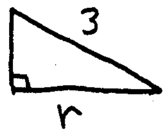
7.) $\int \dfrac{3x^2 \sin x - x^3 \cos x}{\sin^2 x} \, dx = \dfrac{x^3}{\sin x} + c$

quotient rule

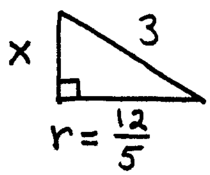
$\boxed{SA10}$: c.)



Solve for r: by similar triangles

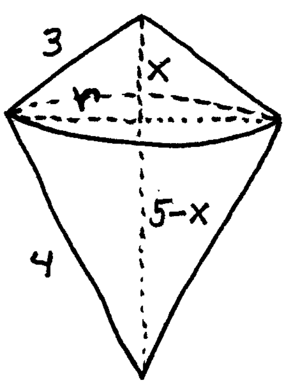


$\frac{3}{r} = \frac{5}{4} \rightarrow 12 = 5r \rightarrow r = 12/5$ then:



$x^2 + \left(\frac{12}{5}\right)^2 = 3^2 \rightarrow x^2 = 9 - \frac{144}{25} = \frac{81}{25} \rightarrow$

$x = \frac{9}{5}$ then:



volume $V = \frac{1}{3}\pi r^2(x) + \frac{1}{3}\pi r^2(5-x)$

$= \frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{9}{5}\right) + \frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{16}{5}\right)$

$= \frac{1}{3}\pi\left(\frac{12}{5}\right)^2 \cdot \left[\frac{9}{5} + \frac{16}{5}\right]$

$= \frac{1}{3}\pi\left(\frac{12}{5}\right)^2 \cdot 5$

$= \frac{48}{5}\pi$