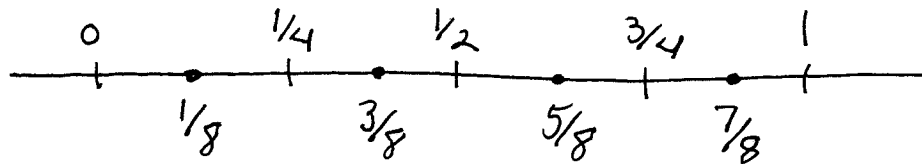


Section 5.6

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$$M_n = \frac{b-a}{n} [f(x_1) + f(x_2) + \dots + f(x_n)]$$

1.) $f(x) = 3 - 2x$ on $[0, 1]$, $n = 4$

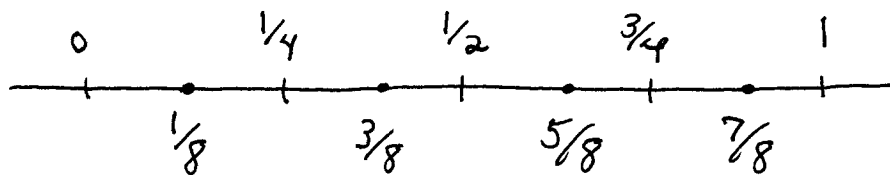


$$M_4 = \frac{1-0}{4} [f(1/8) + f(3/8) + f(5/8) + f(7/8)]$$

$$= \frac{1}{4} \left[\frac{11}{4} + \frac{9}{4} + \frac{7}{4} + \frac{5}{4} \right] = \frac{1}{4} \left[\frac{32}{4} \right] = \textcircled{2}$$

Exact: $\int_0^1 (3-2x) dx = (3x - x^2) \Big|_0^1 = (3-1) - (0-0) = \textcircled{2}$.

3.) $f(x) = \sqrt{x}$ on $[0, 1]$, $n = 4$



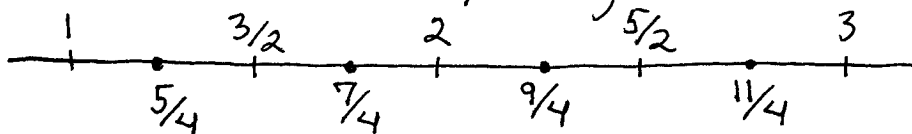
$$M_4 = \frac{1-0}{4} \left[f\left(\frac{1}{8}\right) + f\left(\frac{3}{8}\right) + f\left(\frac{5}{8}\right) + f\left(\frac{7}{8}\right) \right]$$

$$= \frac{1}{4} \left[\sqrt{\frac{1}{8}} + \sqrt{\frac{3}{8}} + \sqrt{\frac{5}{8}} + \sqrt{\frac{7}{8}} \right] \approx \textcircled{0.670}$$

Exact: $\int_0^1 \sqrt{x} dx = \frac{2}{3} x^{3/2} \Big|_0^1 = \frac{2}{3} (1)^{3/2} - \frac{2}{3} (0)^{3/2}$

$$= \frac{2}{3} (1) = \frac{2}{3} \approx \textcircled{0.667}$$

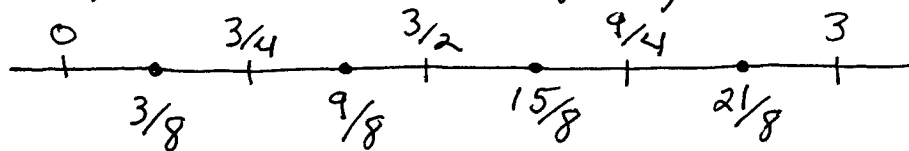
7.) $f(x) = 2x^2$ on $[1, 3]$, $n = 4$



$$\begin{aligned}
 M_4 &= \frac{3-1}{4} [f(\frac{5}{4}) + f(\frac{7}{4}) + f(\frac{9}{4}) + f(\frac{11}{4})] \\
 &= \frac{1}{2} [2(\frac{5}{4})^2 + 2(\frac{7}{4})^2 + 2(\frac{9}{4})^2 + 2(\frac{11}{4})^2] \\
 &= \frac{25}{16} + \frac{49}{16} + \frac{81}{16} + \frac{121}{16} = \frac{276}{16} = \boxed{17.25};
 \end{aligned}$$

$$\begin{aligned}
 \text{Exact: } \int_1^3 2x^2 dx &= \frac{2}{3} x^3 \Big|_1^3 = \frac{2}{3}(27) - \frac{2}{3}(1) \\
 &= \frac{52}{3} \approx \boxed{17.333}
 \end{aligned}$$

12.) $f(x) = 3x^2 - x^3$ on $[0, 3]$, $n = 4$



$$\begin{aligned}
 M_4 &= \frac{3-0}{4} [f(\frac{3}{8}) + f(\frac{9}{8}) + f(\frac{15}{8}) + f(\frac{21}{8})] \\
 &= \frac{3}{4} [3(\frac{3}{8})^2 - (\frac{3}{8})^3 + 3(\frac{9}{8})^2 - (\frac{9}{8})^3 \\
 &\quad + 3(\frac{15}{8})^2 - (\frac{15}{8})^3 + 3(\frac{21}{8})^2 - (\frac{21}{8})^3]
 \end{aligned}$$

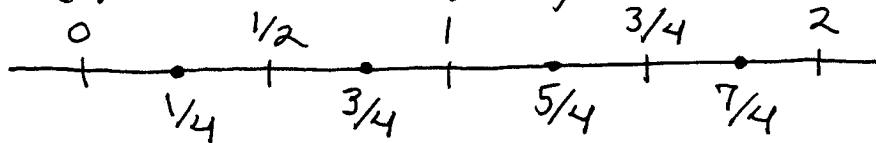
$$\begin{aligned}
 &= \frac{3}{4} [3(\frac{9}{64} + \frac{81}{64} + \frac{225}{64} + \frac{441}{64}) \\
 &\quad - (\frac{27}{512} + \frac{729}{512} + \frac{3375}{512} + \frac{9261}{512})]
 \end{aligned}$$

$$= \frac{9}{4} \left(\frac{756}{64} \right) - \frac{3}{4} \left(\frac{13,392}{512} \right) \approx \boxed{6.961};$$

$$\text{Exact: } \int_0^3 (3x^2 - x^3) dx = (x^3 - \frac{1}{4}x^4) \Big|_0^3$$

$$= 27 - \frac{81}{4} = \frac{27}{4} = \boxed{6.75}$$

18.) $f(y) = 2y$ on $[0, 2]$, $n = 4$

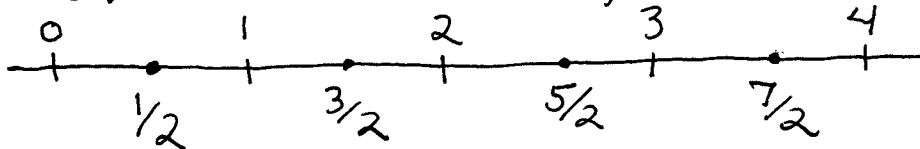


$$M_4 = \frac{2-0}{4} \left[f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) \right]$$

$$= \frac{1}{2} \left[\frac{2}{4} + \frac{6}{4} + \frac{10}{4} + \frac{14}{4} \right] = \frac{1}{2} \left[\frac{32}{4} \right] = \textcircled{4};$$

$$\text{Exact: } \int_0^2 2y \, dy = y^2 \Big|_0^2 = 4 - 0 = \textcircled{4}.$$

19.) $f(y) = y^2 + 1$ on $[0, 4]$, $n = 4$



$$M_4 = \frac{4-0}{4} \left[f\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right) \right]$$

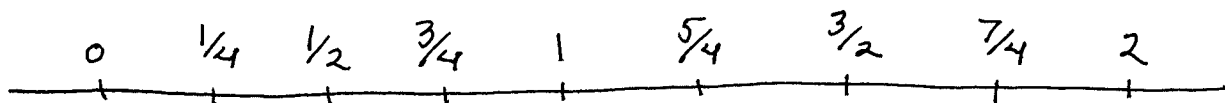
$$= \left[\frac{5}{4} + \frac{13}{4} + \frac{29}{4} + \frac{53}{4} \right] = \frac{100}{4} = \textcircled{25};$$

$$\text{Exact: } \int_0^4 (y^2 + 1) \, dy = \left(\frac{1}{3}y^3 + y \right) \Big|_0^4$$

$$= \frac{64}{3} + 4 = \frac{76}{3} \approx \textcircled{25.333}$$

$$T_n = \frac{b-a}{2n} \left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n) \right]$$

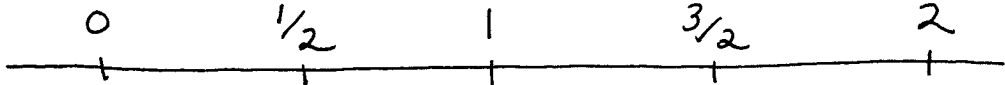
21.) $f(x) = x^3$ on $[0, 2]$, $n = 8$



$$\begin{aligned}
T_8 &= \frac{2-0}{2(8)} \left[f(0) + 2f\left(\frac{1}{4}\right) + 2f\left(\frac{1}{2}\right) + 2f\left(\frac{3}{4}\right) + 2f(1) \right. \\
&\quad \left. + 2f\left(\frac{5}{4}\right) + 2f\left(\frac{3}{2}\right) + 2f\left(\frac{7}{4}\right) + f(2) \right] \\
&= \frac{1}{8} \left[0 + 2 \cdot \frac{1}{64} + 2 \cdot \frac{1}{8} + 2 \cdot \frac{27}{64} + 2(1) + 2 \cdot \frac{125}{64} \right. \\
&\quad \left. + 2 \cdot \frac{27}{8} + 2 \cdot \frac{343}{64} + 8 \right] \\
&= \frac{1}{8} \left[10 + \frac{2+16+54+250+432+686}{64} \right] \\
&= \frac{1}{8} \left[10 + \frac{1440}{64} \right] = \boxed{4.0625} ;
\end{aligned}$$

$$\text{Exact: } \int_0^2 x^3 dx = \frac{1}{4} x^4 \Big|_0^2 = \frac{16}{4} = \boxed{4}$$

23.) $f(x) = \frac{1}{x+1}$ on $[0, 2]$, $n=4$

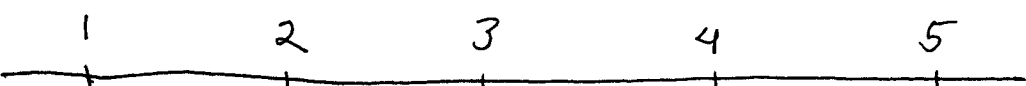


A horizontal number line with tick marks at 0, 1/2, 1, 3/2, and 2.

$$\begin{aligned}
T_4 &= \frac{2-0}{2(4)} \left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2) \right] \\
&= \frac{1}{4} \left[1 + 2 \cdot \frac{2}{3} + 2 \cdot \frac{1}{2} + 2 \cdot \frac{2}{5} + \frac{1}{3} \right] \approx \boxed{1.117} ;
\end{aligned}$$

$$\begin{aligned}
\text{Exact: } \int_0^2 \frac{1}{x+1} dx &= \ln|x+1| \Big|_0^2 \\
&= \ln 3 - \ln 1 \approx \boxed{1.099}
\end{aligned}$$

26.) $f(x) = \frac{\sqrt{x-1}}{x}$ on $[1, 5]$, $n=4$



A horizontal number line with tick marks at 1, 2, 3, 4, and 5.

$$T_4 = \frac{5-1}{2(4)} \cdot [f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)]$$

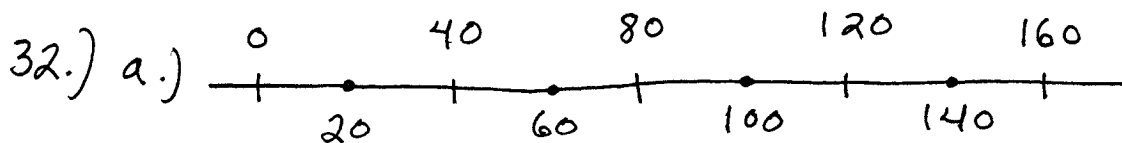
$$= \frac{1}{2} \left[0 + 2 \cdot \frac{1}{2} + 2 \cdot \frac{\sqrt{2}}{3} + 2 \cdot \frac{\sqrt{3}}{4} + \frac{2}{5} \right] \approx 1.604$$

Exact (Using calculator): $\int_1^5 \frac{\sqrt{x-1}}{x} dx \approx 1.786$

$$31.) T_4 = \frac{20-0}{2(4)} \cdot [v(0) + 2 \cdot v(5) + 2 \cdot v(10) + 2 \cdot v(15) + v(20)]$$

$$= \frac{5}{2} \cdot [0.0 + 2(29.3) + 2(51.3) + 2(66.0) + 73.3]$$

$$= \frac{5}{2} \cdot [366.5] = 916.25 \text{ ft.}$$



$$M_4 = \frac{160-0}{4} \cdot [f(20) + f(60) + f(100) + f(140)]$$

$$= 40 [50 + 82 + 73 + 80] = 40(285) = 11,400 \text{ ft.}^2$$

$$b.) T_8 = \frac{160-0}{2(8)} \cdot [f(0) + 2f(20) + 2f(40) + 2f(60)$$

$$+ 2f(80) + 2f(100) + 2f(120) + 2f(140) + f(160)]$$

$$= 10 \cdot [0 + 2(50) + 2(54) + 2(82) + 2(82) + 2(73)$$

$$+ 2(75) + 2(80) + 0]$$

$$= 10 [992] = 9920 \text{ ft.}^2$$

Section 5.5

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51.) If C is cost in millions of \$/yr., then $\int_4^{10} C(t) dt$ is total # of millions of \$ spent from 2004 to 2010:

Projected SAVINGS is

$$\begin{aligned} & \int_4^{10} C_1(t) dt - \int_4^{10} C_2(t) dt = \int_4^{10} (C_1(t) - C_2(t)) dt \\ & = \int_4^{10} [(568.5 + 7.15t) - (525.6 + 6.43t)] dt \\ & = \int_4^{10} [42.9 + 0.72t] dt \\ & = (42.9t + 0.36t^2) \Big|_4^{10} \\ & = (429 + 36) - (171.6 + 5.76) = 287.64 \text{ or} \\ & \$287,640,000 \text{ saved.} \end{aligned}$$

52.) $N_1(t) = 0.1t^2 + 0.5t + 150$ are infected/wk. on week t ; for example, $N(0) = 150$ people/wk., $N(10) = 165$ people/wk., $N(25) = 225$, etc. Thus, $\int_{25}^{50} N_1(t) dt$ is total # infected from
 $\begin{matrix} \uparrow & \uparrow \\ \text{25} & \text{week} \end{matrix}$ people per week week 25 to week 50.

But $N_2(t) = -0.2t^2 + 6t + 200$ is a new infection rate model starting

on week 25; for example $N_2(25) = 225$
people/yr. $N(40) = 120$ people/yr and
 $N(50) = 0$ people/yr. Thus, the total
of people prevented from infection
is

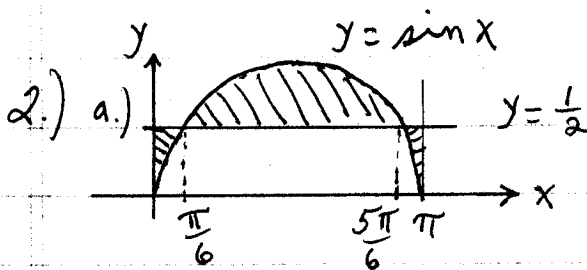
$$\begin{aligned} T &= \int_{25}^{50} (N_1(t) - N_2(t)) dt \\ &= \int_{25}^{50} [(0.1t^2 + 0.5t + 150) - (-0.2t^2 + 6t + 200)] dt \\ &= \int_{25}^{50} [0.3t^2 - 5.5t - 50] dt \\ &= (0.1t^3 - 2.75t^2 - 50t) \Big|_{25}^{50} \\ &= 3125 - (-1406.25) \approx 4531 \text{ people} \end{aligned}$$

Handout 8

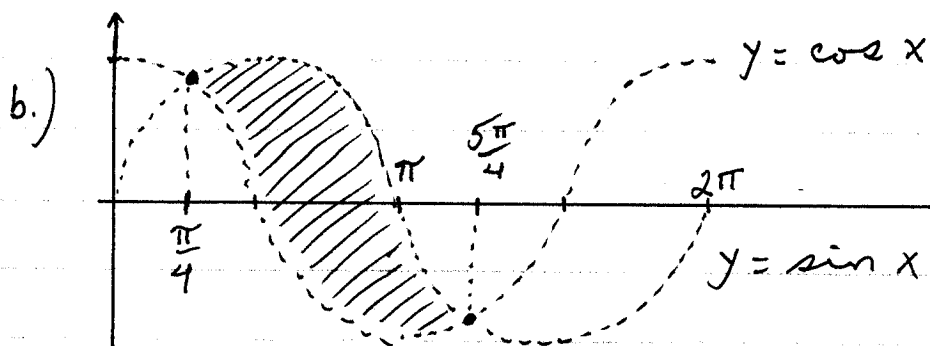
$$\begin{aligned} 1.) \text{ a.) Ave.} &= \frac{1}{\pi-0} \int_0^{\pi} \sin x \, dx \\ &= \frac{1}{\pi} (-\cos x) \Big|_0^{\pi} \\ &= \frac{1}{\pi} (-\cos \pi) - \frac{1}{\pi} (-\cos 0) \\ &= \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi} \end{aligned}$$

$$\begin{aligned} \text{b.) Ave.} &= \frac{1}{2\pi-0} \int_0^{2\pi} \cos x \, dx \\ &= \frac{1}{2\pi} (\sin x) \Big|_0^{2\pi} = \frac{1}{2\pi} \sin 2\pi - \frac{1}{2\pi} \sin 0 = 0 \end{aligned}$$

$$\begin{aligned} \text{c.) Ave.} &= \frac{1}{\frac{\pi}{6}-0} \int_0^{\frac{\pi}{6}} \sec^2 2x \, dx = \frac{6}{\pi} \cdot \frac{1}{2} \tan 2x \Big|_0^{\frac{\pi}{6}} \\ &= \frac{3}{\pi} \tan \frac{\pi}{3} - \frac{3}{\pi} \tan 0 = \frac{3 \cdot \sqrt{3}}{\pi} \end{aligned}$$



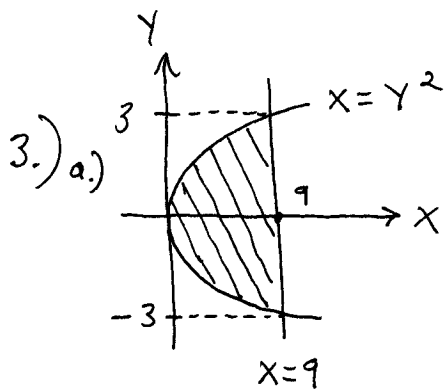
$$\begin{aligned} \text{Area} &= 2 \int_0^{\frac{\pi}{6}} \left[\frac{1}{2} - \sin x \right] dx \\ &\quad + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[\sin x - \frac{1}{2} \right] dx \\ &= 2 \left(\frac{1}{2}x + \cos x \right) \Big|_0^{\frac{\pi}{6}} + \left(-\cos x - \frac{1}{2}x \right) \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \\ &= 2 \left(\frac{\pi}{12} + \frac{\sqrt{3}}{2} \right) - 2(0+1) + \left(\frac{\sqrt{3}}{2} - \frac{5\pi}{12} \right) - \left(-\frac{\sqrt{3}}{2} - \frac{\pi}{12} \right) \\ &= \frac{\pi}{6} + \sqrt{3} - 2 + \sqrt{3} - \frac{\pi}{3} = 2\sqrt{3} - 2 - \frac{\pi}{6} \approx .94 \end{aligned}$$



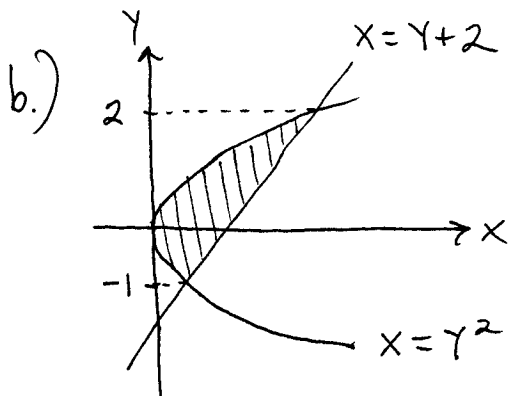
$$\text{Area} = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} [\sin x - \cos x] dx$$

$$= (-\cos x - \sin x) \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) = \boxed{2\sqrt{2}}$$

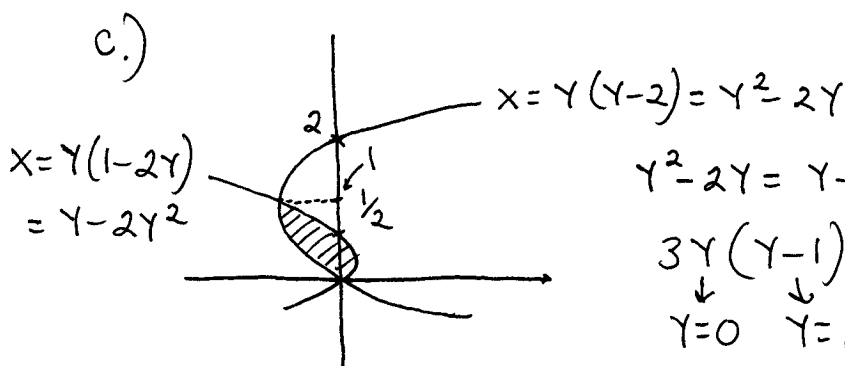


$$\begin{aligned} \text{Area} &= \int_{-3}^3 (9 - y^2) dy \\ &= \left(9y - \frac{1}{3}y^3 \right) \Big|_{-3}^3 \\ &= (27 - 9) - (-27 + 9) = \boxed{36} \end{aligned}$$



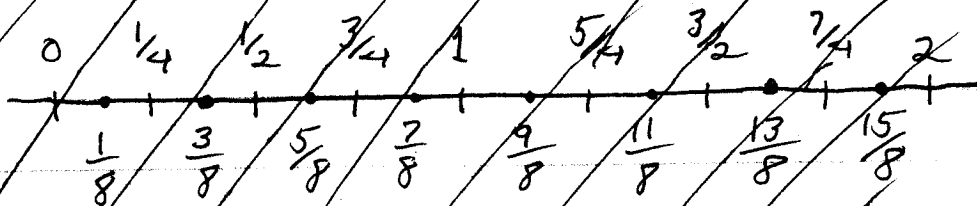
$$\begin{aligned} y + 2 &= y^2 \rightarrow y^2 - y - 2 = 0 \rightarrow \\ (y - 2)(y + 1) &= 0 \\ \downarrow \quad \downarrow \\ y = 2 \quad y = -1 \end{aligned}$$

$$\begin{aligned} \text{Area} &= \int_{-1}^2 [(y + 2) - y^2] dy = \left(\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \right) \Big|_{-1}^2 \\ &= \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right) = \boxed{\frac{9}{2}} \end{aligned}$$



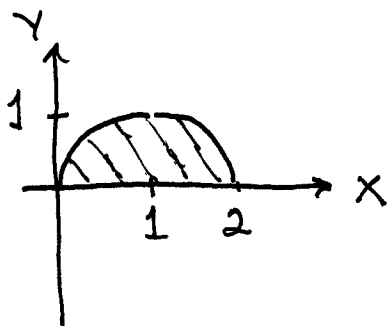
$$\begin{aligned} y^2 - 2y &= y - 2y^2 \rightarrow 3y^2 - 3y = 0 \rightarrow \\ 3y(y - 1) &= 0 \\ \downarrow \quad \downarrow \\ y = 0 \quad y = 1 \end{aligned}$$

$$\begin{aligned} \text{Area} &= \int_0^1 [(y - 2y^2) - (y^2 - 2y)] dy \\ &= \int_0^1 (3y - 3y^2) dy = \left(\frac{3}{2}y^2 - y^3 \right) \Big|_0^1 \\ &= \frac{3}{2} - 1 = \boxed{\frac{1}{2}} \end{aligned}$$



$$M_g = \frac{2-0}{8} \left[\frac{1}{8} \sqrt{\frac{513}{512}} + \frac{3}{8} \sqrt{\frac{539}{512}} + \frac{5}{8} \sqrt{\frac{637}{512}} \right. \\ \left. + \frac{7}{8} \sqrt{\frac{835}{512}} + \frac{9}{8} \sqrt{\frac{1241}{512}} + \frac{11}{8} \sqrt{\frac{1843}{512}} \right. \\ \left. + \frac{13}{8} \sqrt{\frac{2709}{512}} + \frac{15}{8} \sqrt{\frac{3887}{512}} \right] \approx \boxed{3.901}$$

SA II: $y = \sqrt{1 - (x-1)^2} \rightarrow y^2 = 1 - (x-1)^2 \rightarrow$
 $(x-1)^2 + y^2 = 1 \rightarrow$ top half of circle only!



Solid formed by revolving region about the x-axis is a sphere of radius 1:

$$V = \frac{4}{3} \pi (1)^3 = \frac{4}{3} \pi$$