Section 5.7

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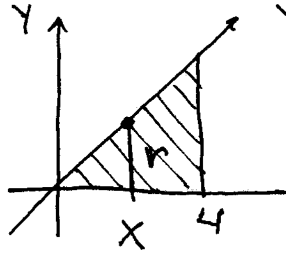
1.) $\text{Vol.} = \pi \int_0^2 (\sqrt{4-x^2})^2 dx = \pi \int_0^2 (4-x^2) dx$

$= \pi (4x - \frac{1}{3}x^3)\Big|_0^2 = \pi(8 - \frac{8}{3}) = \frac{16}{3}\pi$

2.) $\text{Vol.} = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx$

$= \pi \cdot \frac{1}{5} x^5 \Big|_0^1 = \frac{\pi}{5}$

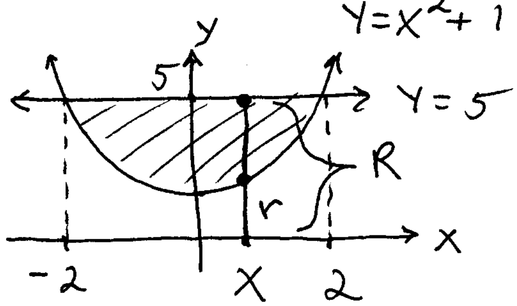
6.)



$\text{Vol.} = \pi \int_0^4 (x)^2 dx$

$= \pi \cdot \frac{1}{3} x^3 \Big|_0^4 = \frac{64}{3}\pi$

8.)



$x^2 + 1 = 5 \rightarrow$

$x^2 = 4 \rightarrow$

$x = 2,\ x = -2;$

$\text{Vol.} = \pi \int_{-2}^{2} R^2 dx - \pi \int_{-2}^{2} r^2 dx$

$= \pi \int_{-2}^{2} (R^2 - r^2) dx$

$= \pi \int_{-2}^{2} ((5)^2 - (x^2+1)^2) dx$

$= \pi \int_{-2}^{2} (25 - x^4 - 2x^2 - 1) dx$

$= \pi \int_{-2}^{2} (24 - x^4 - 2x^2) dx$

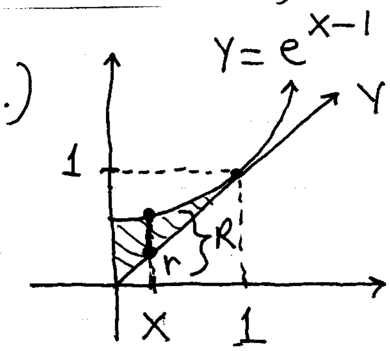
$= \pi (24x - \frac{1}{5}x^5 - \frac{2}{3}x^3)\Big|_{-2}^{2}$

$$= \pi\left(48 - \frac{32}{5} - \frac{16}{3}\right) - \pi\left(-48 + \frac{32}{5} + \frac{16}{3}\right)$$

$$= \pi\left(96 - \frac{64}{5} - \frac{32}{3}\right) = \pi\left(\frac{1440}{15} - \frac{192}{15} - \frac{160}{15}\right)$$

$$= \pi\left(\frac{1088}{15}\right)$$

10.)



$$\text{Vol.} = \pi\int_0^1 R^2\,dx - \pi\int_0^1 r^2\,dx$$

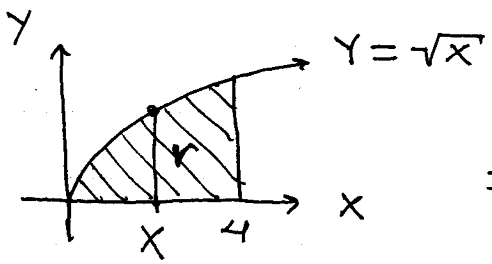
$$= \pi\int_0^1 (R^2 - r^2)\,dx$$

$$= \pi\int_0^1 \left((e^{x-1})^2 - (x)^2\right) dx$$

$$= \pi\int_0^1 (e^{2x-2} - x^2)\,dx = \pi\left(\frac{1}{2}e^{2x-2} - \frac{1}{3}x^3\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{2}\cancel{e^0}^{\,1} - \frac{1}{3}\right) - \pi\left(\frac{1}{2}e^{-2} - 0\right) = \pi\left(\frac{1}{6} - \frac{e^{-2}}{2}\right)$$

12.)

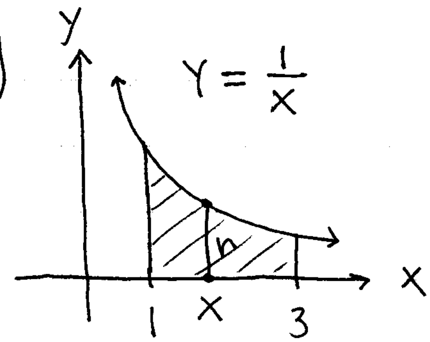


$$\text{Vol.} = \pi\int_0^4 (\sqrt{x})^2\,dx$$

$$= \pi\int_0^4 x\,dx = \pi \cdot \frac{x^2}{2}\Big|_0^4$$

$$= 8\pi$$

14.)



$$\text{Vol.} = \pi\int_1^3 \left(\frac{1}{x}\right)^2 dx$$

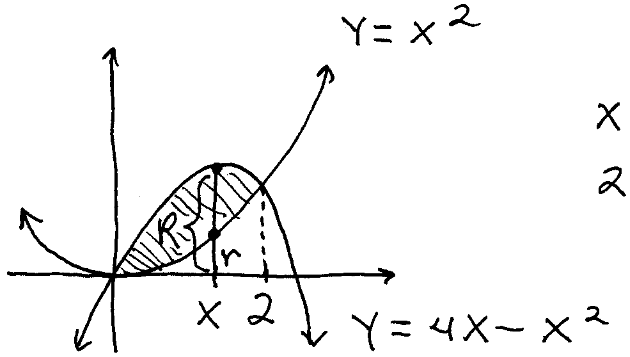
$$= \pi\int_1^3 \frac{1}{x^2}\,dx$$

$$= \pi\int_1^3 x^{-2}\,dx$$

$$= \pi \cdot \frac{x^{-1}}{-1}\Big|_1^3 = \pi \cdot \frac{-1}{x}\Big|_1^3 = \pi\left(\frac{-1}{3} - -1\right)$$

$$= \frac{2}{3}\pi$$

16.)



$x^2 = 4x - x^2 \rightarrow$

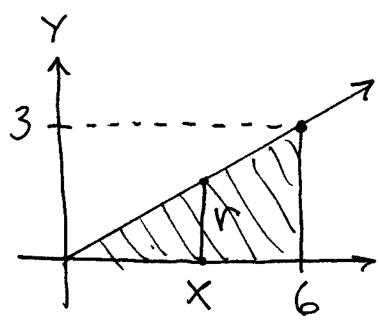
$2x^2 - 4x = 0 \rightarrow$

$2x(x-2) = 0$

$x = 0 \quad x = 2$

$$\text{Vol.} = \pi \int_0^2 R^2\,dx - \pi \int_0^2 r^2\,dx$$
$$= \pi \int_0^2 (R^2 - r^2)\,dx$$
$$= \pi \int_0^2 \left[(4x - x^2)^2 - (x^2)^2\right] dx$$
$$= \pi \int_0^2 \left[16x^2 - 8x^3 + \cancel{x^4} - \cancel{x^4}\right] dx$$
$$= \pi \int_0^2 \left[16x^2 - 8x^3\right] dx$$
$$= \pi \left(\frac{16}{3}x^3 - 2x^4\right)\Big|_0^2$$
$$= \pi \left(\frac{128}{3} - 32\right)$$
$$= \pi \left(\frac{128}{3} - \frac{96}{3}\right) = \frac{32}{3}\pi$$

25.)

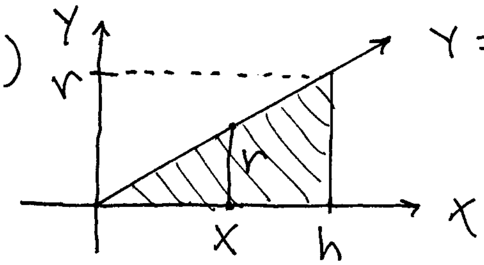


$Y = \frac{1}{2}x$

$$\text{Vol.} = \pi \int_0^6 \left(\tfrac{1}{2}x\right)^2 dx$$
$$= \pi \int_0^6 \frac{1}{4}x^2\,dx$$

$$= \pi \cdot \frac{1}{12}x^3\Big|_0^6 = \frac{216}{12}\pi = 18\pi$$

27.)



$Y = \frac{r}{h}x$

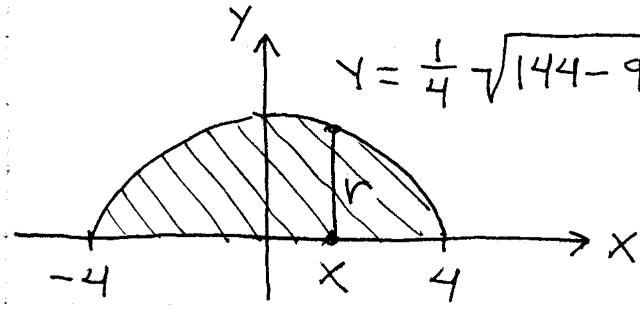
$$\text{Vol.} = \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$= \pi \int_0^h \frac{r^2}{h^2} x^2\,dx = \pi \frac{r^2}{h^2} \cdot \frac{1}{3} x^3 \Big|_0^h$$

$$= \pi \cdot \frac{r^2}{h^2} \cdot \frac{1}{3} \cdot h^3 = \boxed{\frac{1}{3}\pi r^2 h}$$

30.) $9x^2 + 16y^2 = 144 \rightarrow 16y^2 = 144 - 9x^2$

$\rightarrow y^2 = \frac{1}{16}(144 - 9x^2) \rightarrow y = \frac{1}{4}\sqrt{144 - 9x^2}$



$$\text{Vol} = \pi \int_{-4}^{4} \left(\frac{1}{4}\sqrt{144 - 9x^2}\right)^2 dx$$

(Use symmetry.)

$$= \pi \cdot 2 \int_0^4 \frac{1}{16}(144 - 9x^2)\,dx$$

$$= \frac{\pi}{8}(144x - 3x^3)\Big|_0^4 = \frac{\pi}{8}(576 - 192)$$

$$= \frac{384}{8}\pi = 48\pi$$

31.) $\text{Vol.} = \pi \int_0^2 \left(\frac{1}{8}x^2 \sqrt{2-x}\right)^2 dx$

$$= \pi \int_0^2 \frac{1}{64} \cdot x^4 (2 - x)\,dx$$

$$= \frac{\pi}{64} \int_0^2 (2x^4 - x^5)\,dx$$

$$= \frac{\pi}{64} \cdot \left(\frac{2}{5}x^5 - \frac{1}{6}x^6\right)\Big|_0^2$$

$$= \frac{\pi}{64}\left(\frac{64}{5} - \frac{64}{6}\right) = \pi\left(\frac{1}{5} - \frac{1}{6}\right) = \frac{\pi}{30}$$

Applications of Definite Integrals

1.) a.) $t=0: \quad \frac{15}{2(0)+3} = 5$ ft./yr.

b.) $t=9: \quad \frac{15}{2(9)+3} = \frac{15}{21} = \frac{5}{7}$ ft./yr.

c.) TOTAL HEIGHT $= \int_0^{12} \underbrace{\frac{15}{2t+3}}_{(\text{ft./yr.})} \cdot \underbrace{dt}_{(\text{yr.})}$

$= 15 \cdot \frac{1}{2} \ln(2t+3) \Big|_0^{12}$

$= \frac{15}{2} \cdot (\ln 27 - \ln 3) = \frac{15}{2} \ln\left(\frac{27}{3}\right)$

$= \frac{15}{2} \ln 9 \approx 16.48$ ft.

2.) a.) $t=1: \quad (1)\sqrt{1^2+3} = 2$ mph.

b.) $t=3: \quad (3)\sqrt{3^2+3} = 3\sqrt{12} \approx 10.39$ mph.

c.) TOTAL DISTANCE $= \int_0^5 \underbrace{t\sqrt{t^2+3}}_{\left(\frac{\text{miles}}{\text{hr.}}\right)} \cdot \underbrace{dt}_{(\text{hr.})}$

$= \frac{2}{3} \cdot \frac{1}{2} (t^2+3)^{3/2} \Big|_0^5 = \frac{1}{3}(28)^{3/2} - \frac{1}{3}(3)^{3/2}$

≈ 47.66 miles

3.) a.) $t=1$: $\left(\frac{1}{4}\right)(1)^2 = \frac{1}{4}$ gal./hr.

b.) $t=5$: $\left(\frac{1}{4}\right)(5)^2 = 6.25$ gal./hr.

c.) TOTAL LEAKAGE $= \int_0^6 \frac{1}{4}t^2 \, dt$

$$\underbrace{\tfrac{1}{4}t^2}_{(\text{gal./hr.})} \cdot \underbrace{dt}_{(\text{hr.})}$$

$= \frac{1}{4} \cdot \frac{1}{3} t^3 \Big|_0^6 = 18$ gal.

d.) $100 = \int_0^A \frac{1}{4} t^2 \, dt = \frac{1}{12}A^3 - \frac{1}{12}(0)^3 \rightarrow$

$\frac{1}{12}A^3 = 100 \rightarrow A^3 = 1200 \rightarrow$

$A = 1200^{1/3} \approx 10.63$ hr.

4.) a.) $x=0$: $3 + \sin 0 = 3+0 = 3$ grams/cm.

b.) $x = \frac{\pi}{4}$: $3 + \sin\frac{\pi}{2} = 3+1 = 4$ grams/cm.

c.) $x = \frac{7\pi}{4}$: $3 + \sin\frac{7\pi}{2} = 3-1 = 2$ grams/cm.

d.) TOTAL MASS $= \int_0^{2\pi} (3 + \sin 2x) \, dx$

$$\underbrace{(3+\sin 2x)}_{\left(\frac{\text{grams}}{\text{cm.}}\right)} \cdot \underbrace{dx}_{(\text{cm.})}$$

$= \left(3x - \frac{1}{2}\cos 2x\right) \Big|_0^{2\pi}$

$= \left(6\pi - \frac{1}{2}\overset{1}{\cancel{\cos 4\pi}}\right) - \left(0 - \frac{1}{2}\overset{1}{\cancel{\cos 0}}\right)$

$$= 6\pi - \frac{1}{2} + \frac{1}{2} = 6\pi \quad \text{grams}$$

e.) $$\text{MASS} = \int_{\frac{\pi}{2}}^{\pi} (3 + \sin 2x)\, dx$$

$$= \left(3x - \frac{1}{2}\cos 2x\right)\Big|_{\frac{\pi}{2}}^{\pi}$$

$$= \left(3\pi - \frac{1}{2}\overset{1}{\cancel{\cos 2\pi}}\right) - \left(\frac{3\pi}{2} - \frac{1}{2}\overset{-1}{\cancel{\cos \pi}}\right)$$

$$= 3\pi - \frac{1}{2} - \frac{3}{2}\pi - \frac{1}{2}$$

$$= \frac{3}{2}\pi - 1 \quad \text{grams}$$