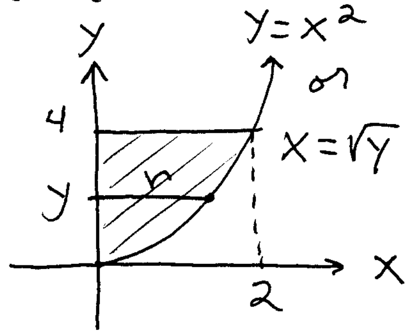
Section 5.7

Page 376

17.)



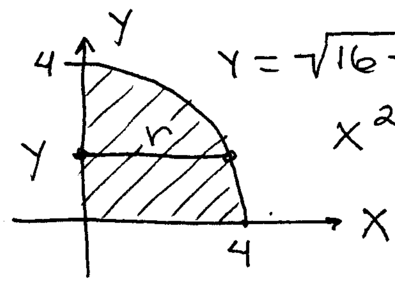
$$\text{Vol.} = \pi \int_0^4 r^2\,dy$$

$$= \pi \int_0^4 (\sqrt{y})^2\,dy$$

$$= \pi \int_0^4 y\,dy$$

$$= \pi \cdot \frac{1}{2} y^2 \Big|_0^4 = 8\pi$$

18.)



$$y = \sqrt{16-x^2} \rightarrow y^2 = 16 - x^2 \rightarrow$$

$$x^2 = 16 - y^2 \rightarrow x = \sqrt{16-y^2} \;;$$

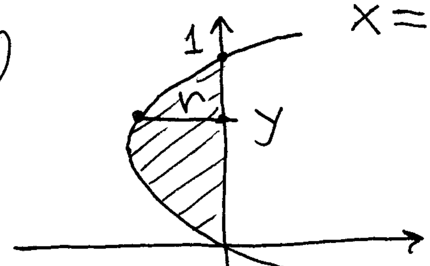
$$\text{Vol.} = \pi \int_0^4 r^2\,dy$$

$$= \pi \int_0^4 (\sqrt{16-y^2})^2\,dy = \pi \int_0^4 (16 - y^2)\,dy$$

$$= \pi \left(16y - \frac{1}{3}y^3\right)\Big|_0^4 = \pi\left(64 - \frac{1}{3}\cdot 64\right)$$

$$= \frac{2}{3}\cdot 64\pi = \frac{128}{3}\pi$$

20.)



$$x = y(y-1) = y^2 - y \;;$$

$$\text{Vol.} = \pi \int_0^1 r^2\,dy$$

$$= \pi \int_0^1 (y^2 - y)^2\,dy$$

$$= \pi \int_0^1 (y^4 - 2y^3 + y^2)\,dy$$

$$= \pi \left[\frac{1}{5}y^5 - \frac{1}{2}y^4 + \frac{1}{3}y^3\right]\Big|_0^1$$

$$= \pi\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right) = \pi\left(\frac{6}{30} - \frac{15}{30} + \frac{10}{30}\right)$$

$$= \frac{\pi}{30}$$

22.) $\text{Vol.} = \pi \int_1^4 r^2\,dy = \pi \int_1^4 (4Y - Y^2)^2\,dy$

$= \pi \int_1^4 (16Y^2 - 8Y^3 + Y^4)\,dy$

$= \pi \left(\frac{16}{3}Y^3 - 2Y^4 + \frac{1}{5}Y^5\right)\Big|_1^4$

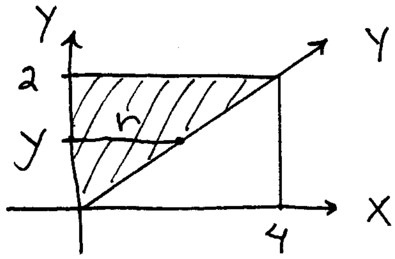
$= \pi\left[\left(\frac{1024}{3} - 512 + \frac{1024}{5}\right) - \left(\frac{16}{3} - 2 + \frac{1}{5}\right)\right]$

$= \pi\left[\frac{1008}{3} - 510 + \frac{1023}{5}\right]$

$= \pi\left[\frac{5040}{15} - \frac{7650}{15} + \frac{3069}{15}\right]$

$= \frac{459}{15}\pi = \frac{153}{5}\pi$

26.)



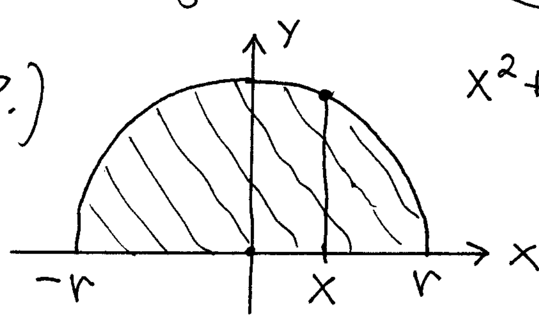
$Y = \frac{1}{2}X$ or $X = 2Y$;

$\text{Vol.} = \pi \int_0^2 r^2\,dy$

$= \pi \int_0^2 (2Y)^2\,dy = \pi \int_0^2 4Y^2\,dy$

$= \pi \cdot \frac{4}{3}Y^3\Big|_0^2 = \frac{32}{3}\pi$

28.)



$X^2 + Y^2 = r^2 \to Y^2 = r^2 - X^2$

$\to Y = \sqrt{r^2 - X^2}$;

$\text{Vol.} = \pi \int_{-r}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx$

$= \pi \int_{-r}^{r} (r^2 - x^2)\,dx$

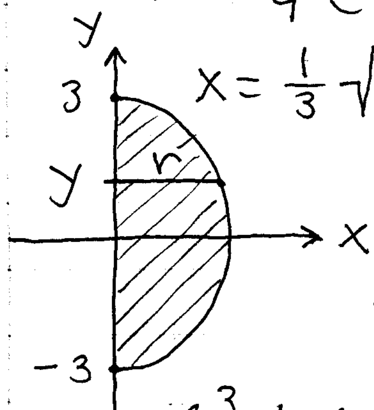
$$= \pi \left(r^2 x - \frac{1}{3}x^3\right)\Big|_{x=-r}^{x=r}$$

$$= \pi\left(r^3 - \frac{1}{3}r^3\right) - \pi\left(-r^3 + \frac{1}{3}r^3\right)$$

$$= \pi\left(\frac{2}{3}r^3\right) - \pi\left(\frac{-2}{3}r^3\right) = \frac{4}{3}\pi r^3$$

29.) $9x^2 + 25Y^2 = 225 \rightarrow 9x^2 = 225 - 25Y^2$

$\rightarrow x^2 = \frac{1}{9}(225 - 25Y^2) \rightarrow x = \frac{1}{3}\sqrt{225 - 25Y^2}$;



$x = \frac{1}{3}\sqrt{225 - 25Y^2}$

$$\text{Vol.} = \pi \int_{-3}^{3} r^2\, dy$$

$$= \pi \int_{-3}^{3} \left(\frac{1}{3}\sqrt{225 - 25Y^2}\right)^2 dy$$

$$= \pi \int_{-3}^{3} \frac{1}{9}(225 - 25Y^2)\, dy$$

$$= \frac{\pi}{9}\left(225Y - \frac{25}{3}Y^3\right)\Big|_{-3}^{3}$$

$$= \frac{\pi}{9}(675 - 225) - \frac{\pi}{9}(-675 + 225)$$

$$= \frac{\pi}{9}(450 + 450) = 100\pi$$

Math 16B
Kouba
Handout 9

1.) Determine the volume of the solid formed by revolving the region bounded by the graphs of the given equations about the x-axis.

a.) $y = \cos((1/2)x), \ y = 0, \ x = 0, \ x = \pi$
b.) $y = 3 + \sin x, \ y = 1, \ x = 0, \ x = \pi$
c.) $y = \sqrt{2} \cos x, \ y = \tan x, \ x = 0, \ x = \pi/4$

2.) Determine the volume of the solid formed by revolving the region bounded by the graphs of the given equations about the y-axis.

a.) $y = x + 1, \ y = 0, \ x = 0$
b.) $y = x^2, \ y = x^3, \ x = 0, \ x = 1$

3.) Determine the volume of the solid formed by revolving the region bounded by the graphs of $y = e^x$, $y = 1$, and $x = \ln 2$ about the given axis. SET UP ONLY.

a.) x-axis
b.) y-axis

4.) Determine the volume of the solid formed by revolving the region bounded by the graphs of $y = 4 - 2x$, $y = 0$, and $x = 0$ about the given axis. SET UP ONLY.

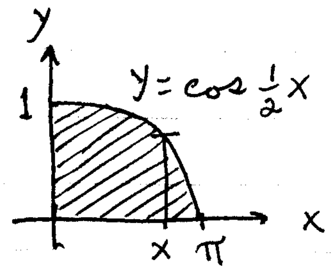
a.) x-axis
b.) y-axis
c.) line $x = 3$
d.) line $y = -2$

5.) Determine the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3$ and $y = 4\sqrt{2x}$ about the given axis. SET UP ONLY.

a.) x-axis
b.) y-axis
c.) line $x = -1$
d.) line $y = 8$

Handout 9

1.) a.)



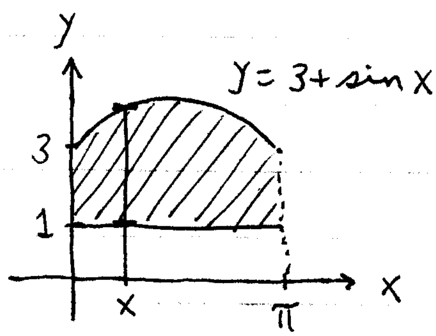
$$\text{Vol} = \pi\int_0^{\pi} \cos^2 \tfrac{1}{2}x \; dx$$

$$= \pi\int_0^{\pi} \frac{1+\cos x}{2}\, dx$$

$$= \pi\int_0^{\pi}\left(\tfrac{1}{2}+\tfrac{1}{2}\cos x\right)dx = \pi\left(\tfrac{1}{2}x+\tfrac{1}{2}\sin x\right)\Big|_0^{\pi}$$

$$= \pi\left(\tfrac{1}{2}\pi+\tfrac{1}{2}\sin\pi\right) - \pi\left(0+\tfrac{1}{2}\sin 0\right) = \frac{\pi^2}{2}$$

b.)



$$\text{Vol.} = \pi\int_0^{\pi}(3+\sin x)^2\,dx$$

$$-\pi\int_0^{\pi}(1)^2\,dx$$

$$= \pi\int_0^{\pi}\left(8+6\sin x+\sin^2 x\right)dx$$

$$= \pi\int_0^{\pi}\left(8+6\sin x+\frac{1-\cos 2x}{2}\right)dx$$

$$= \pi\int_0^{\pi}\left(\tfrac{17}{2}+6\sin x-\tfrac{1}{2}\cos 2x\right)dx$$

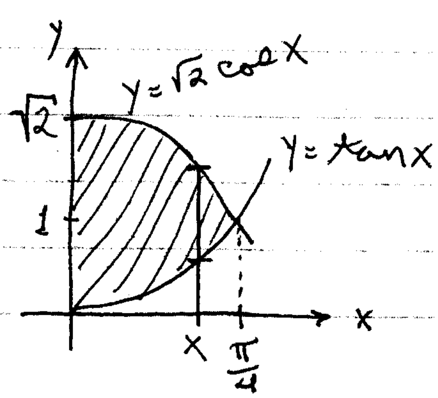
$$= \pi\left(\tfrac{17}{2}x-6\cos x-\tfrac{1}{4}\sin 2x\right)\Big|_0^{\pi}$$

$$= \pi\left(\tfrac{17}{2}\pi-6\overset{-1}{\cancel{\cos}\,\pi}-\tfrac{1}{4}\overset{0}{\cancel{\sin}\,2\pi}\right)$$

$$-\pi\left(0-6\overset{1}{\cancel{\cos}\,0}-\tfrac{1}{4}\overset{0}{\cancel{\sin}\,0}\right)$$

$$= \pi\left(\tfrac{17}{2}\pi+6\right)+\pi\cdot 6 = \frac{17}{2}\pi^2+12\pi$$

c.)



$$\text{Vol.} = \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{2}\cos x\right)^2dx$$

$$-\pi\int_0^{\frac{\pi}{4}}\tan^2 x\;dx$$

$$= \pi \int_0^{\frac{\pi}{4}} 2\cos^2 x \, dx - \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - 1)\, dx$$

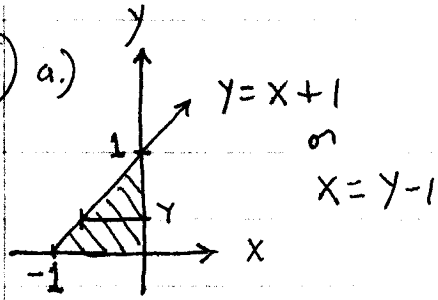
$$= \pi \int_0^{\frac{\pi}{4}} \cancel{2} \cdot \cancel{\tfrac{1}{2}} (1 + \cos 2x)\, dx - \pi (\tan x - x)\Big|_0^{\frac{\pi}{4}}$$

$$= \pi \left(x + \tfrac{1}{2} \sin 2x\right)\Big|_0^{\frac{\pi}{4}} - \pi \left(\cancelto{1}{\tan \tfrac{\pi}{4}} - \tfrac{\pi}{4}\right) + \pi \left(\cancelto{0}{\tan 0} - 0\right)$$

$$= \pi \left(\tfrac{\pi}{4} + \tfrac{1}{2} \cancelto{1}{\sin \tfrac{\pi}{2}}\right) - \pi \left(0 + \tfrac{1}{2} \cancelto{0}{\sin 0}\right)$$

$$- \pi \left(1 - \tfrac{\pi}{4}\right) \quad = \boxed{\frac{\pi^2}{2} - \frac{\pi}{2}}$$

2.) a.)



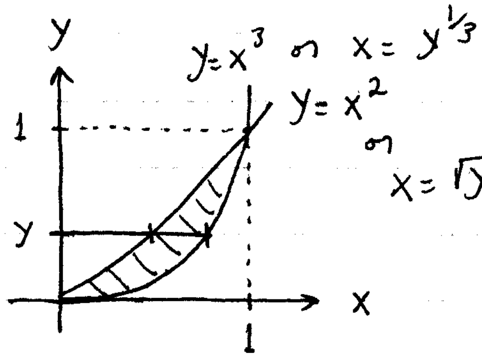
$$\text{Vol.} = \pi \int_0^1 (y-1)^2 \, dy$$

$$= \pi \int_0^1 (y^2 - 2y + 1)\, dy$$

$$= \pi \left(\tfrac{1}{3} y^3 - y^2 + y\right)\Big|_0^1$$

$$= \pi\left(\tfrac{1}{3}\right) - \pi(0) \quad = \tfrac{1}{3}\pi$$

b.)



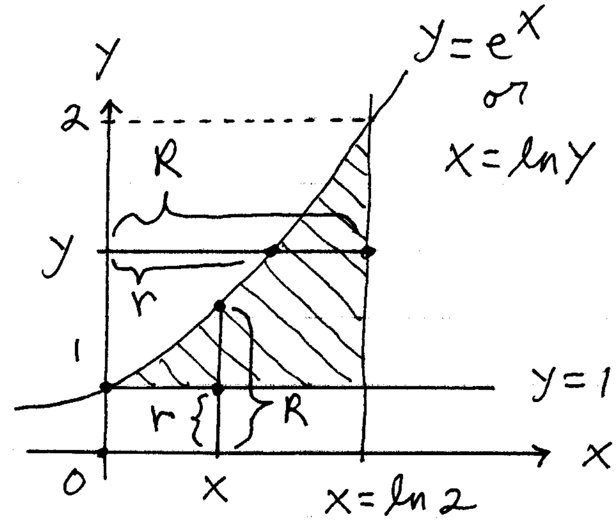
$$\text{Vol.} = \pi \int_0^1 \left(y^{\frac{1}{3}}\right)^2 dy - \pi \int_0^1 (\sqrt{y})^2 \, dy$$

$$= \pi \int_0^1 \left(y^{\frac{2}{3}} - y\right) dy$$

$$= \pi \left(\tfrac{3}{5} y^{5/3} - \tfrac{1}{2} y^2\right)\Big|_0^1 = \pi\left(\tfrac{3}{5} - \tfrac{1}{2}\right) = \frac{\pi}{10}$$

3.) a.) $Vol = \pi \int_0^{\ln 2} R^2 \, dx - \pi \int_0^{\ln 2} r^2 \, dx$

$= \pi \int_0^{\ln 2} (e^x)^2 \, dx - \pi \int_0^{\ln 2} (1)^2 \, dx$



b.) $Vol = \pi \int_1^2 R^2 \, dy - \pi \int_1^2 r^2 \, dy$

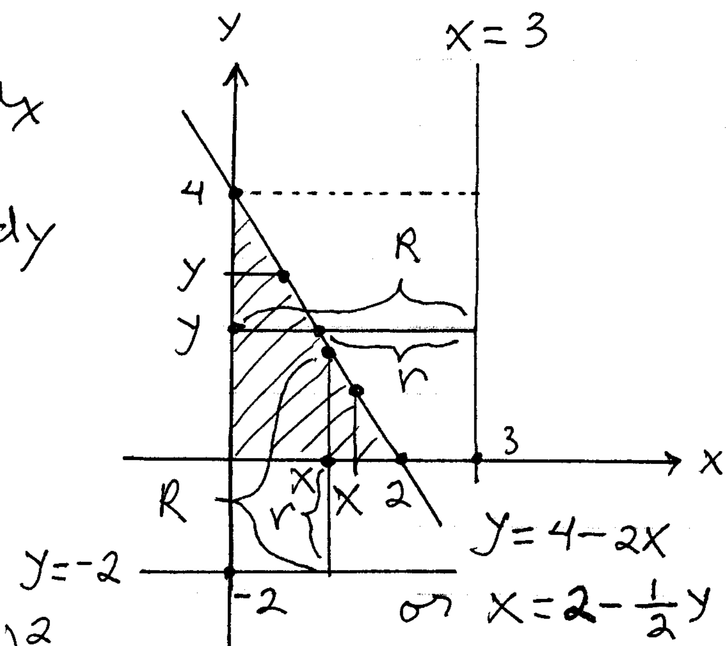
$= \pi \int_1^2 (\ln 2)^2 \, dy - \pi \int_1^2 (\ln y)^2 \, dy$

4.) a.) $Vol = \pi \int_0^2 (4 - 2x)^2 \, dx$

b.) $Vol = \pi \int_0^4 (2 - \frac{1}{2}y)^2 \, dy$

c.) $Vol = \pi \int_0^4 R^2 \, dy - \pi \int_0^4 r^2 \, dy$

$= \pi \int_0^4 (3)^2 \, dy - \pi \int_0^4 \left(3 - (2 - \frac{1}{2}y)\right)^2 \, dy$



d.) $Vol = \pi \int_0^2 R^2 \, dx - \pi \int_0^2 r^2 \, dx$

$= \pi \int_0^2 (2 + (4 - 2x))^2 \, dx - \pi \int_0^2 (2)^2 \, dx$

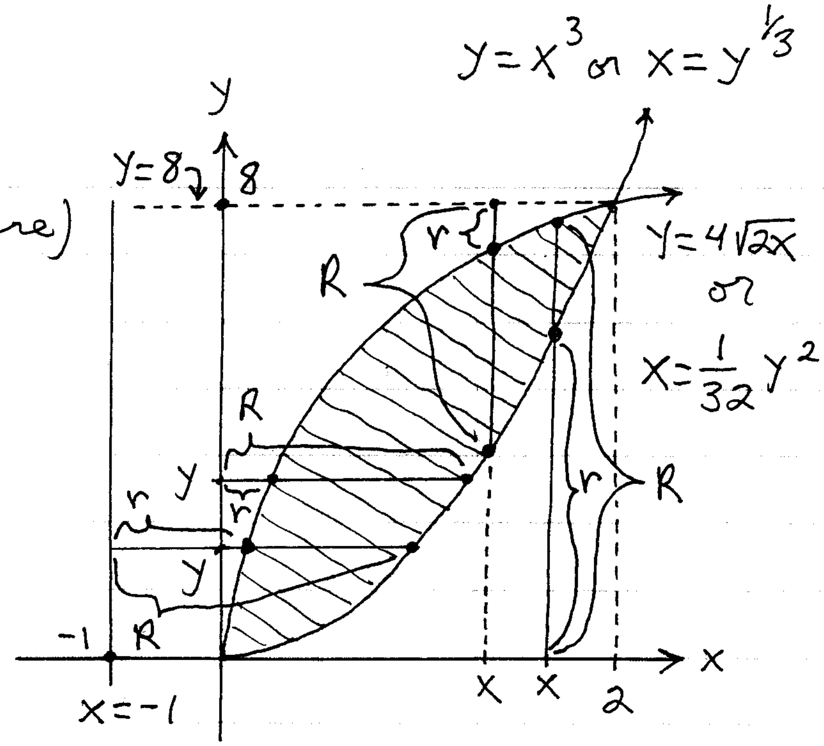
5.) $x^3 = 4\sqrt{2x} \rightarrow$ (square)

$x^6 = 16(2x) \rightarrow$

$x^6 - 32x = 0 \rightarrow$

$x(x^5 - 32) = 0 \rightarrow$

$x = 0$ $\quad$ $x = 2$



a.) $\text{Vol} = \pi \int_0^2 R^2\,dx - \pi \int_0^2 r^2\,dx$

$= \pi \int_0^2 (4\sqrt{2x})^2\,dx - \pi \int_0^2 (x^3)^2\,dx$

b.) $\text{Vol} = \pi \int_0^8 R^2\,dy - \pi \int_0^8 r^2\,dy$

$= \pi \int_0^8 (y^{1/3})^2\,dy - \pi \int_0^8 \left(\frac{1}{32}y^2\right)^2 dy$

c.) $\text{Vol} = \pi \int_0^8 R^2\,dy - \pi \int_0^8 r^2\,dy$

$= \pi \int_0^8 (1 + y^{1/3})^2\,dy - \pi \int_0^8 \left(1 + \frac{1}{32}y^2\right)^2 dy$

d.) $\text{Vol} = \pi \int_0^2 R^2\,dx - \pi \int_0^2 r^2\,dx$

$= \pi \int_0^2 (8 - x^3)^2\,dx - \pi \int_0^2 (8 - 4\sqrt{2x})^2\,dx$