Section 6.1

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1.) $\int (x-2)^4 dx$ (Let $u = x-2 \rightarrow du = 1\,dx$)

$= \int u^4 du = \frac{1}{5}u^5 + c = \frac{1}{5}(x-2)^5 + c$

4.) $\int \frac{4}{(1-t)^3} dt$ (Let $u = 1-t \rightarrow du = -1 \cdot dt$
$\rightarrow -du = dt$)

$= -4\int \frac{1}{u^3} du = -4\int u^{-3} du = -4 \cdot \frac{u^{-2}}{-2} + c$

$= 2 \cdot \frac{1}{u^2} + c = 2 \cdot \frac{1}{(1-t)^2} + c$

6.) $\int \frac{2y^3}{y^4+1} dy$ (Let $u = y^4+1 \rightarrow$
$du = 4y^3 dy \rightarrow$
$\frac{1}{2} du = 2y^3 dy$)

$= \frac{1}{2}\int \frac{1}{u} du$

$= \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|y^4+1| + c$

7.) $\int \sqrt{1+x}\, dx$ (Let $u = 1+x \rightarrow du = 1 \cdot dx$)

$= \int \sqrt{u}\, du = \frac{u^{3/2}}{3/2} + c = \frac{2}{3}(1+x)^{3/2} + c$

10.) $\int \frac{6x^2+2}{x^3+x} dx$ (Let $u = x^3+x \rightarrow$
$du = (3x^2+1)\, dx \rightarrow$
$2\,du = (6x^2+2)\, dx$)

$= 2\int \frac{1}{u} du = 2\ln|u| + c = 2\ln|x^3+x| + c$

11.) $\int \frac{1}{(5x+1)^3} dx$ (Let $u = 5x+1 \rightarrow du = 5\,dx$
$\rightarrow \frac{1}{5} du = dx$)

$= \frac{1}{5}\int \frac{1}{u^3}\,du = \frac{1}{5}\int u^{-3}\,du = \frac{1}{5}\cdot\frac{u^{-2}}{-2} + c$

$= \frac{-1}{10}\cdot(5x+1)^{-2} + c$

13.) $\int \frac{1}{\sqrt{x+1}}\,dx$ (Let $u = x+1 \rightarrow du = 1\cdot dx$)

$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + c$

$= 2\sqrt{x+1} + c$

16.) $\int \frac{4e^{2x}}{1+e^{2x}}\,dx$ (Let $u = 1+e^{2x} \rightarrow$
$du = 2e^{2x}\,dx \rightarrow 2du = 4e^{2x}dx$)

$= 2\int \frac{1}{u}\,du = 2\ln|u| + c = 2\ln|1+e^{2x}| + c$

17.) $\int \frac{2x}{e^{3x^2}}\,dx = \int 2xe^{-3x^2}dx$ (Let $u = -3x^2 \rightarrow$
$du = -6x\,dx \rightarrow -\frac{1}{6}\,du = x\,dx$)

$= \frac{-1}{6}\int 2\cdot e^u\,du = \frac{-1}{3}e^u + c = \frac{-1}{3}e^{-3x^2} + c$

18.) $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx$ (Let $u = \sqrt{x+1} \rightarrow$
$du = \frac{1}{2}(x+1)^{-1/2}\,dx \rightarrow$
$2\,du = \frac{1}{\sqrt{x+1}}\,dx$)

$= 2\int e^u\,du$

$= 2e^u + c = 2e^{\sqrt{x+1}} + c$

19.) $\int \frac{x^2}{x-1}\,dx$ (Let $u = x-1 \rightarrow du = 1\,dx$
and $x = u+1$)

$= \int \frac{(u+1)^2}{u}\,du = \int \frac{u^2+2u+1}{u}\,du$

$$= \int \left(\frac{u^2}{u} + \frac{2u}{u} + \frac{1}{u}\right) du = \int \left(u + 2 + \frac{1}{u}\right) du$$

$$= \frac{1}{2}u^2 + 2u + \ln|u| + c$$

$$= \frac{1}{2}(x-1)^2 + 2(x-1) + \ln|x-1| + c$$

20.) $\int \frac{2x}{x-4}\,dx$ (Let $u = x-4 \rightarrow du = 1\,dx$ and $x = u+4$)

$$= \int \frac{2(u+4)}{u}\,du = 2\int \left(\frac{u}{u} + \frac{4}{u}\right) du$$

$$= 2\int \left(1 + 4 \cdot \frac{1}{u}\right) du = 2(u + 4\ln|u|) + c$$

$$= 2((x-4) + 4\ln|x-4|) + c$$

23.) $\int e^{5x}\,dx$ (Let $u = 5x \rightarrow du = 5\,dx \rightarrow \frac{1}{5}du = dx$)

$$= \frac{1}{5}\int e^u\,dx$$

$$= \frac{1}{5}e^u + c = \frac{1}{5}e^{5x} + c$$

27.) $\int \frac{x}{(x+1)^4}\,dx$ (Let $u = x+1 \rightarrow du = 1\,dx$ and $x = u-1$)

$$= \int \frac{u-1}{u^4}\,du = \int \left(\frac{u}{u^4} - \frac{1}{u^4}\right) du = \int \left(\frac{1}{u^3} - \frac{1}{u^4}\right) du$$

$$= \int (u^{-3} - u^{-4})\,du = \frac{u^{-2}}{-2} - \frac{u^{-3}}{-3} + c$$

$$= \frac{-1}{2}(x+1)^{-2} + \frac{1}{3}(x+1)^{-3} + c$$

29.) $\int \frac{x}{(3x-1)^2}\,dx$. (Let $u = 3x-1 \to du = 3\,dx$
$\to \frac{1}{3}du = dx$ and
$u+1 = 3x \to x = \frac{1}{3}(u+1)$)

$$= \left(\frac{1}{3}\right)\int \frac{\frac{1}{3}(u+1)}{u^2}\,du = \frac{1}{9}\int\left(\frac{u}{u^2}+\frac{1}{u^2}\right)du$$

$$= \frac{1}{9}\int\left(\frac{1}{u}+u^{-2}\right)du = \frac{1}{9}\left(\ln|u| + \frac{u^{-1}}{-1}\right)+c$$

$$= \frac{1}{9}\left(\ln|3x-1| - (3x-1)^{-1}\right)+c$$

31.) $\int \frac{1}{\sqrt{t}-1}\,dt$ (Let $u = \sqrt{t}-1 \to$
$du = \frac{1}{2\sqrt{t}}\,dt$ and $\sqrt{t} = u+1$, so
$2\sqrt{t}\,du = dt \to 2(u+1)\,du = dt$)

$$= 2\int \frac{1}{u}(u+1)\,du = 2\int\left(\frac{u}{u}+\frac{1}{u}\right)du$$

$$= 2\int\left(1+\frac{1}{u}\right)du = 2(u+\ln|u|)+c$$

$$= 2\left(\sqrt{t}-1+\ln|\sqrt{t}-1|\right)+c$$

34.) $\int \frac{6x+\sqrt{x}}{x}\,dx = \int\left(\frac{6x}{x}+\frac{\sqrt{x}}{x}\right)dx$

$$= \int\left(6+x^{-1/2}\right)dx = 6x + \frac{x^{1/2}}{1/2}+c$$

35.) $\int \frac{x}{\sqrt{2x+1}}\,dx$ (Let $u = 2x+1 \to$
$du = 2\,dx \to$
$\frac{1}{2}du = dx$ and $2x = u-1 \to$
$x = \frac{1}{2}(u-1)$)

$= \frac{1}{2}\int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\,du = \frac{1}{4}\int\left(\frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\right)du$

$= \frac{1}{4}\int\left(u^{1/2} - u^{-1/2}\right)du = \frac{1}{4}\left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right) + c$

$= \frac{1}{4}\left(\frac{2}{3}(2x+1)^{3/2} - 2(2x+1)^{1/2}\right) + c$

37.) $\int t^2\sqrt{1-t}\,dt$ (Let $u = 1-t \to$
$du = (-1)dt \to -du = dt$ and
$t = 1-u$)

$= -\int (1-u)^2\sqrt{u}\,du = -\int (u^2-2u+1)u^{1/2}\,du$

$= -\int \left(u^{5/2} - 2u^{3/2} + u^{1/2}\right)du$

$= -\left(\frac{u^{7/2}}{7/2} - 2\cdot\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2}\right) + c$

$= -\left(\frac{2}{7}(1-t)^{7/2} - \frac{4}{5}(1-t)^{5/2} + \frac{2}{3}(1-t)^{3/2}\right) + c$

40.) $\int_2^4 \sqrt{4x+1}\,dx$ (Let $u = 4x+1 \to du = 4dx$
$\to \frac{1}{4}du = dx$, and $x: 2 \to 4$
so $u: 9 \to 17$)

$= \frac{1}{4}\int_9^{17}\sqrt{u}\,du = \frac{1}{4}\cdot\frac{u^{3/2}}{3/2}\Big|_9^{17} = \frac{1}{6}\left(17^{3/2} - 9^{3/2}\right)$

$= \frac{1}{6}\left(17^{3/2} - 27\right)$

41.) $\int_0^1 3xe^{x^2}dx$ (Let $u = x^2 \to du = 2x\,dx \to$
$\frac{1}{2}du = x\,dx$, and $x: 0 \to 1$ so
$u: 0 \to 1$)

$$= \frac{1}{2}\int_0^1 3e^u\,du = \frac{3}{2}e^u\Big|_0^1 = \frac{3}{2}(e^1 - e^0)$$

$$= \frac{3}{2}(e-1)$$

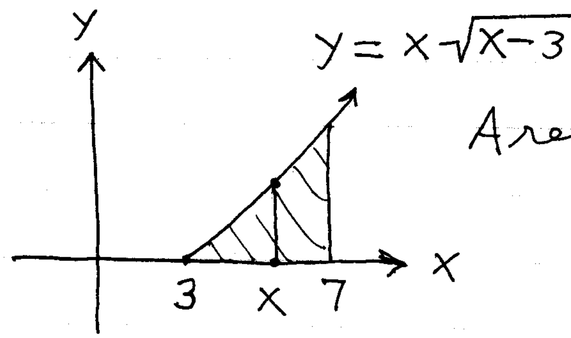
44.) $\int_0^1 x(x+5)^4\,dx$ (Let $u = x+5 \rightarrow$ $du = 1\,dx$ and $x = u-5$; $x: 0 \rightarrow 1$ so $u: 5 \rightarrow 6$)

$$= \int_5^6 (u-5)u^4\,du = \int_5^6 (u^5 - 5u^4)\,du$$

$$= \left(\frac{1}{6}u^6 - u^5\right)\Big|_5^6 = \cancelto{0}{\left(\frac{1}{6}\cdot 6^6 - 6^5\right)} - \left(\frac{1}{6}5^6 - 5^5\right)$$

$$= 3125 - \frac{15,625}{6} = \frac{3125}{6}$$

47.)



Area $= \int_3^7 x\sqrt{x-3}\,dx$

(Let $u = x-3 \rightarrow$ $du = dx$ and $x = u+3$; $x: 3 \rightarrow 7$ so $u: 0 \rightarrow 4$)

$$= \int_0^4 (u+3)\sqrt{u}\,du = \int_0^4 \left(u^{3/2} + 3u^{1/2}\right)du$$

$$= \left(\frac{2}{5}u^{5/2} + \cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2}\right)\Big|_0^4 = \frac{2}{5}\cdot 4^{5/2} + 2\cdot 4^{3/2}$$

$$= \frac{2}{5}(32) + 2(8) = \frac{64}{5} + \frac{80}{5} = \frac{144}{5}$$

58.) Area $= \int_0^4 \frac{1}{1+\sqrt{x}}\,dx$ (Let $u = 1+\sqrt{x} \rightarrow$ $du = \frac{1}{2\sqrt{x}}\,dx$ and $\sqrt{x} = u-1$ so

$du = \frac{1}{2(u-1)}\,dx \rightarrow 2(u-1)\,du = dx$;

$x: 0 \rightarrow 4$ so $u: 1 \rightarrow 3$)

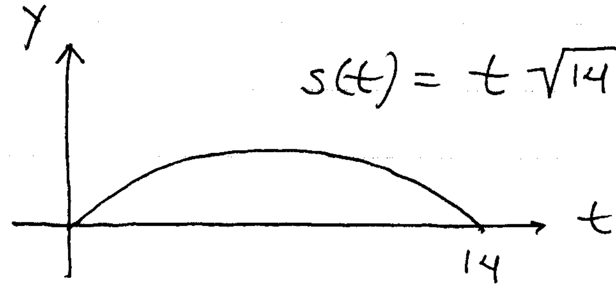
$= \int_1^3 \frac{1}{u} \cdot 2(u-1)\,du = 2\int_1^3 \left(1 - \frac{1}{u}\right) du$

$= 2(u - \ln u)\Big|_1^3 = 2(3 - \ln 3) - 2(1 - \cancelto{0}{\ln 1})$

$= 6 - 2\ln 3 - 2 = 4 - 2\ln 3$

65.) a.)

$s(t) = t\sqrt{14-t}$



b.) $\text{AVE} = \frac{1}{14-0}\int_0^{14} t\sqrt{14-t}\,dt$ (Let $u = 14 - t$

$\rightarrow du = (-1)\,dt \rightarrow -du = dt$, and

$t = 14 - u$; $t: 0 \rightarrow 14$ so $u: 14 \rightarrow 0$)

$= \frac{-1}{14}\int_{14}^{0} (14-u)\sqrt{u}\,du = \frac{-1}{14}\int_{14}^{0} \left(14u^{1/2} - u^{3/2}\right) du$

$= \frac{-1}{14}\left(14 \cdot \frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right)\Big|_{14}^{0}$

$= \frac{-1}{14}(0) - \frac{-1}{14}\left(\frac{28}{3}(14)^{3/2} - \frac{2}{5}(14)^{5/2}\right)$

$= \frac{2}{3}(14)^{3/2} - \frac{1}{35}(14)^{5/2} \approx 13.97$ in.

c.) $\text{TOTAL} = \int_0^{14} t\sqrt{14-t}\,dt$

$$= \cdots = \frac{28}{3}(14)^{3/2} - \frac{2}{5}(14)^{5/2} \approx 195.56 \text{ in.}$$

Handout 10

1.) a.) $\int \sec^2 x \cdot e^{1+\tan x} dx$ (Let $u = 1+\tan x \to du = \sec^2 x\, dx$)

$= \int e^u du = e^u + c = e^{1+\tan x} + c$

b.) $\int \sec x \tan x \cdot e^{-3\sec x} dx$ (Let $u = -3\sec x \to$ $du = -3\sec x \tan x\, dx \to \frac{-1}{3} du = \sec x \tan x\, dx$)

$= \frac{-1}{3}\int e^u du = \frac{-1}{3} e^u + c = \frac{-1}{3} e^{-3\sec x} + c$

c.) $\int e^{-x}\cos^2(e^{-x})\, dx$ (Let $u = e^{-x} \to du = -e^{-x}dx \to -du = e^{-x}dx$)

$= -\int \cos^2 u\, du = -\int \frac{1}{2}(1+\cos 2u)\, du$

$= \frac{-1}{2}(u + \frac{1}{2}\sin 2u) + c = \frac{-1}{2}(e^{-x} + \frac{1}{2}\sin(2e^{-x})) + c$

d.) $\int (e^x \cos x + e^x \cdot (-\sin x))\, dx = e^x \cos x + c$

product rule

e.) $\int \sin(e^{\tan(3+x^2)}) \cdot e^{\tan(3+x^2)} \cdot \sec^2(3+x^2) \cdot 2x\, dx$

(Let $u = e^{\tan(3+x^2)} \to du = e^{\tan(3+x^2)} \cdot \sec^2(3+x^2) \cdot 2x\, dx$)

$= \int \sin u\, du = -\cos u + c = -\cos(e^{\tan(3+x^2)}) + c$

2.) Ave. $= \frac{1}{9-4}\int_4^9 x^{-1/2} e^{x^{1/2}} dx$ (Let $u = x^{1/2} \to du = \frac{1}{2}x^{-1/2}dx$ $\to 2du = x^{-1/2}dx$; $x: 4 \to 9$ so $u: 2 \to 3$)

$= \frac{1}{5}\int_2^3 2e^u du = \frac{2}{5} e^u \Big|_2^3 = \frac{2}{5}(e^3 - e^2)$

3.) a.) $\int \frac{1}{3+\sqrt{x}}\,dx$ (Let $x=u^2 \to dx = 2u\,du$)

$$= \int \frac{1}{3+u}\,2u\,du$$

$$= \int \left[2 - \frac{6}{u+3}\right] du$$

$$= 2u - 6\ln|u+3| + C$$

$$= 2\sqrt{x} - 6\ln|\sqrt{x}+3| + C$$

$$\begin{array}{r|l} & \;2 \\ u+3 & \overline{2u} \\ & -(2u+6) \\ \hline & \;-6 \end{array}$$

b.) $\int \frac{\sqrt{x}}{4+\sqrt{x}}\,dx$ (Let $x=u^2 \to dx = 2u\,du$, $\to u=\sqrt{x}$)

$$= \int \frac{u}{4+u}\cdot 2u\,du = \int \frac{2u^2}{4+u}\,du$$

$$= \int \left[2u - 8 + \frac{32}{u+4}\right] du$$

$$= u^2 - 8u + 32\ln|u+4| + C$$

$$= x - 8\sqrt{x} + 32\ln|\sqrt{x}+4| + C$$

$$\begin{array}{r|l} & \;2u-8 \\ u+4 & \overline{2u^2} \\ & -(2u^2+8u) \\ \hline & \;-8u \\ & -(-8u-32) \\ \hline & \;32 \end{array}$$

c.) $\int \frac{1+x^{1/3}}{2+x^{1/3}}\,dx$ (Let $x=u^3 \to dx = 3u^2\,du$, $\to u = x^{1/3}$)

$$= \int \frac{1+u}{2+u}\cdot 3u^2\,du$$

$$= \int \frac{3u^3+3u^2}{u+2}\,du$$

$$= \int \left[3u^2 - 3u + 6 - \frac{12}{u+2}\right] du$$

$$\begin{array}{r|l} & \;3u^2-3u+6 \\ u+2 & \overline{3u^3+3u^2} \\ & -(3u^3+6u^2) \\ \hline & \;-3u^2 \\ & -(-3u^2-6u) \\ \hline & \;6u \\ & -(6u+12) \\ \hline & \;-12 \end{array}$$

$$= u^3 - \frac{3}{2}u^2 + 6u - 12\ln|u+2| + C$$

$$= x - \frac{3}{2}(x^{1/3})^2 + 6x^{1/3} - 12\ln|x^{1/3}+2| + C$$

d.) $$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx \quad \left(\text{Let } u = 1+\sqrt{x} \rightarrow du = \frac{1}{2}x^{-1/2}dx \rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx\right)$$

$$= 2\int \frac{1}{u}\,du = 2\ln|u| + C = 2\ln|1+\sqrt{x}| + C$$

e.) $$\int \frac{\sqrt{x}}{1-x^{1/4}}\,dx \quad \left(\text{Let } x = u^4 \rightarrow dx = 4u^3\,du \rightarrow u = x^{1/4}\right)$$

$$= \int \frac{\sqrt{u^4}}{1-(u^4)^{1/4}} \cdot 4u^3\,du$$

$$= \int \frac{u^2}{1-u} \cdot 4u^3\,du$$

$$= \int \frac{4u^5}{-u+1}\,du$$

$$\begin{array}{r}
-4u^4 - 4u^3 - 4u^2 - 4u - 4 \\
-u+1 \,\big)\, 4u^5 \qquad\qquad\qquad\qquad \\
-(4u^5 - 4u^4) \qquad\qquad\quad \\
\hline
4u^4 \qquad\qquad\quad \\
-(4u^4 - 4u^3) \qquad\quad \\
\hline
4u^3 \qquad\quad \\
-(4u^3 - 4u^2) \quad \\
\hline
4u^2 \quad \\
-(4u^2 - 4u) \\
\hline
4u \\
-(-4u - 4) \\
\hline
4
\end{array}$$

$$= \int \left[-4u^4 - 4u^3 - 4u^2 - 4u - 4 + \frac{4}{-u+1}\right] du$$

$$= -\frac{4}{5}u^5 - u^4 - \frac{4}{3}u^3 - 2u^2 - 4u - 4\ln|1-u| + C$$

$$= -\frac{4}{5}(x^{\frac{1}{4}})^5 - x - \frac{4}{3}(x^{1/4})^3 - 2(x^{1/4})^2 - 4x^{1/4} - 4\ln|1-x^{1/4}| + C$$

f.) $\int \sqrt{1-\sqrt{x}}\,dx$ (Let $u = 1-\sqrt{x} \rightarrow \sqrt{x} = 1-u \rightarrow$
$x = (1-u)^2 \rightarrow dx = 2(1-u)(-1)\,du$
$\rightarrow dx = 2(u-1)\,du$)

$= \int \sqrt{u}\cdot 2(u-1)\,du = 2\int (u^{3/2} - u^{1/2})\,du$

$= 2\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$

$= \frac{4}{5}(1-\sqrt{x})^{5/2} - \frac{4}{3}(1-\sqrt{x})^{3/2} + C$

g.) $\int \sqrt{2+\sqrt{1+\sqrt{x}}}\,dx$ (Let $u = 2+\sqrt{1+\sqrt{x}} \rightarrow$
$u-2 = \sqrt{1+\sqrt{x}} \rightarrow (u-2)^2 = 1+\sqrt{x} \rightarrow$
$u^2 - 4u + \underbrace{\cancel{4-1}}_{3} = \sqrt{x} \rightarrow (u^2-4u+3)^2 = x \rightarrow$
$dx = 2(u^2-4u+3)\cdot(2u-4)\,du$)

$= \int \sqrt{u}\cdot 2(u^2-4u+3)\cdot 2(u-2)\,du$

$= 4\int u^{1/2}(u^3-6u^2+11u-6)\,du$

$= 4\int \left[u^{7/2} - 6u^{5/2} + 11u^{3/2} - 6u^{1/2}\right]du$

$= 4\left[\frac{2}{9}u^{9/2} - 6\cdot\frac{2}{7}u^{7/2} + 11\cdot\frac{2}{5}u^{5/2} - 6\cdot\frac{2}{3}u^{3/2}\right] + C$

$= 4\left[\frac{2}{9}\left(2+\sqrt{1+\sqrt{x}}\right)^{9/2} - \frac{12}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}\right.$
$\left. + \frac{22}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2} - 4\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2}\right] + C$