Section 6.2

Page 403

2.) $\int xe^{-x}dx$ (Let $u=x$, $dv=e^{-x}dx$
$\rightarrow du=dx$, $v=-e^{-x}$)

$= -xe^{-x} - -\int e^{-x}dx$

$= -xe^{-x} + \frac{e^{-x}}{(-1)} + c = -xe^{-x} - e^{-x} + c$

5.) $\int \ln 2x\, dx$ (Let $u=\ln 2x$, $dv=dx$
$\rightarrow du = \frac{2}{2x}dx = \frac{1}{x}dx$, $v=x$)

$= x\ln 2x - \int \frac{1}{x} x\, dx = x\ln 2x - \int 1\, dx$

$= x\ln 2x - x + c$

7.) $\int e^{4x}dx = \frac{1}{4}e^{4x} + c$

9.) $\int xe^{4x}dx$ (~~Let~~ $u=x$, $dv=e^{4x}dx$
$\rightarrow du=dx$, $v=\frac{1}{4}e^{4x}$)

$= \frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x}dx$

$= \frac{1}{4}xe^{4x} - \frac{1}{4}\cdot\frac{1}{4}e^{4x} + c$

11.) $\int xe^{x^2}dx$ (Let $u=x^2 \rightarrow du=2x\,dx$
$\rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int e^{u}du = \frac{1}{2}e^{u} + c = \frac{1}{2}e^{x^2} + c$

13.) $\int x^2 e^x\,dx$ (Let $u = x^2$, $dv = e^x\,dx$
$\rightarrow du = 2x\,dx$, $v = e^x$)

$= x^2 e^x - 2\int x e^x\,dx$ (Let $u = x$, $dv = e^x\,dx$
$\rightarrow du = dx$, $v = e^x$)

$= x^2 e^x - 2[x e^x - \int e^x\,dx]$

$= x^2 e^x - 2x e^x + 2 \cdot e^x + C$

16.) $\int x^3 \ln x\,dx$ (Let $u = \ln x$, $dv = x^3\,dx$
$\rightarrow du = \frac{1}{x}\,dx$, $v = \frac{1}{4}x^4$)

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int \frac{1}{x} x^4\,dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4} \cdot \frac{1}{4}x^4 + C$

17.) $\int \frac{e^{1/t}}{t^2}\,dt$ (Let $u = \frac{1}{t} = t^{-1} \rightarrow du = -t^{-2}\,dt$
$\rightarrow -du = \frac{1}{t^2}\,dt$)

$= -\int e^u\,du = -e^u + C = -e^{1/t} + C$

18.) $\int \frac{1}{x(\ln x)^3}\,dx$ (Let $u = \ln x \rightarrow du = \frac{1}{x}\,dx$)

$= \int \frac{1}{u^3}\,du = \int u^{-3}\,du = \frac{u^{-2}}{-2} + C = \frac{(\ln x)^{-2}}{-2} + C$

19.) $\int x(\ln x)^2\,dx$ (Let $u = (\ln x)^2$, $dv = x\,dx$

$$\rightarrow du = \frac{2(\ln x)}{x}dx, \ v = \tfrac{1}{2}x^2)$$

$$= \tfrac{1}{2}x^2(\ln x)^2 - \int x \ln x \, dx$$

(Let $u = \ln x$, $dv = x\,dx$

$\rightarrow du = \frac{1}{x}dx$, $v = \frac{1}{2}x^2$)

$$= \tfrac{1}{2}x^2(\ln x)^2 - \left[\tfrac{1}{2}x^2\ln x - \tfrac{1}{2}\int x\,dx\right]$$

$$= \tfrac{1}{2}x^2(\ln x)^2 - \tfrac{1}{2}x^2\ln x + \tfrac{1}{2}\cdot\tfrac{1}{2}x^2 + c$$

21.) $\int \frac{(\ln x)^2}{x}dx$ (Let $u = \ln x \rightarrow du = \frac{1}{x}dx$)

$$= \int u^2\,du = \tfrac{1}{3}u^3 + c = \tfrac{1}{3}(\ln x)^3 + c$$

22.) $\int \frac{1}{x\ln x}dx$ (Let $u = \ln x \rightarrow du = \frac{1}{x}dx$)

$$= \int \tfrac{1}{u}\,du = \ln|u| + c = \ln|\ln x| + c$$

24.) $\int \frac{x}{\sqrt{x-1}}dx$ (Let $u = x-1 \rightarrow du = dx$ and $x = u+1$)

$$= \int \frac{u+1}{\sqrt{u}}\,du = \int\left(\frac{u}{u^{1/2}} + \frac{1}{u^{1/2}}\right)du$$

$$= \int (u^{1/2} + u^{-1/2})\,du = \tfrac{2}{3}u^{3/2} + 2u^{1/2} + c$$

$$= \tfrac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + c$$

27.) $\int \frac{xe^{2x}}{(2x+1)^2}dx$ (Let $u = xe^{2x}$, $dv = \frac{1}{(2x+1)^2}dx$

$\rightarrow du = (x \cdot 2e^{2x} + e^{2x})dx = (2x+1)e^{2x}dx,$

$v = \frac{1}{2} \cdot \frac{-1}{2x+1}\ )$

$= \frac{-xe^{2x}}{2(2x+1)} - \frac{-1}{2}\int e^{2x}dx$

$= \frac{-xe^{2x}}{2(2x+1)} + \frac{1}{2} \cdot \frac{1}{2}e^{2x} + c$

32.) $\int_1^e 2x \ln x \, dx$ (Let $u = \ln x$, $dv = 2x\,dx$

$\rightarrow du = \frac{1}{x}dx, \quad v = x^2)$

$= x^2 \ln x \Big|_1^e - \int_1^e x \, dx$

$= e^2 \cancel{\ln e}^{1} - 1^2 \cancel{\ln 1}^{0} - \frac{1}{2}x^2 \Big|_1^e$

$= e^2 - \left(\frac{1}{2}e^2 - \frac{1}{2}\right) = \frac{1}{2}e^2 + \frac{1}{2}$

33.) $\int_{-1}^0 \ln(x+2)\,dx$ (Let $u = \ln(x+2)$, $dv = dx$

$\rightarrow du = \frac{1}{x+2}dx, \quad v = x)$

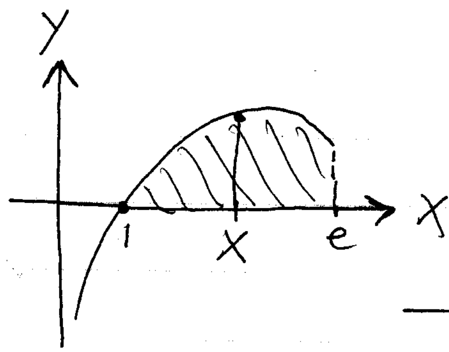
$= x\ln(x+2)\Big|_{-1}^0 - \int_{-1}^0 \frac{x}{x+2}dx$

$= (0)\ln 2 - (-1)\cancel{\ln 1}^{0} - \int_{-1}^0 \frac{x+2-2}{x+2}dx$

$= -\int_{-1}^0 \left(\frac{x+2}{x+2} - \frac{2}{x+2}\right)dx$

$= -\int_{-1}^0 \left(1 - 2 \cdot \frac{1}{x+2}\right)dx = -(x - 2\ln(x+2))\Big|_{-1}^0$

$= -(0 - 2\ln 2) - -(-1 - 2\cancel{\ln 1}^{0}) = 2\ln 2 - 1$

38.) 

$$\text{Area} = \int_1^e \frac{\ln x}{x^2}\,dx$$

(Let $u = \ln x$, $dv = \frac{1}{x^2}dx$

$\rightarrow du = \frac{1}{x}dx$, $v = \frac{-1}{x}$)

$$= -\frac{\ln x}{x}\Big|_1^e - -\int_1^e \frac{1}{x^2}\,dx$$

$$= \left(-\frac{\ln e}{e} - -\frac{\ln 1}{1}\right) + \frac{-1}{x}\Big|_1^e$$

$$= \frac{-1}{e} - \left(\frac{1}{e} - 1\right) = 1 - \frac{2}{e}$$

40.) a.) $\int x\sqrt{4+x}\,dx$ (Let $u = x$, $dv = \sqrt{4+x}\,dx$

$\rightarrow du = dx$, $v = \frac{2}{3}(4+x)^{3/2}$)

$$= \frac{2}{3}x(4+x)^{2/3} - \frac{2}{3}\int (4+x)^{3/2}\,dx$$

$$= \frac{2}{3}x(4+x)^{2/3} - \frac{2}{3}\cdot\frac{2}{5}(4+x)^{5/2} + C$$

b.) $\int x\sqrt{4+x}\,dx$ (Let $u = \sqrt{4+x} \rightarrow$

$du = \frac{1}{2}(4+x)^{-1/2}\,dx = \frac{1}{2u}dx \rightarrow$

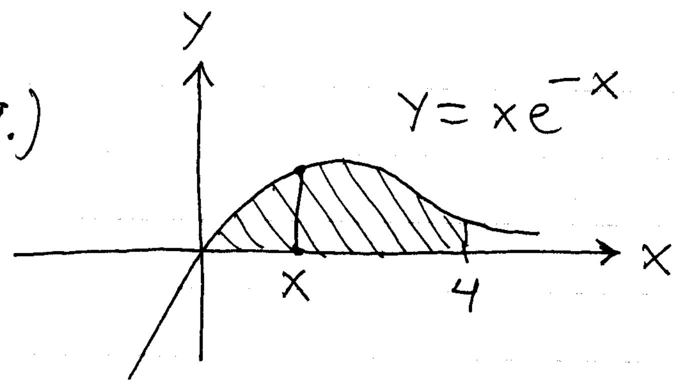
$2u\,du = dx$, and $u^2 = 4+x \rightarrow$

$x = u^2 - 4$)

$$= \int (u^2-4)\cdot u\cdot 2u\,du = \int (2u^4 - 8u^2)\,du$$

$$= \frac{2}{5}u^5 - \frac{8}{3}u^3 + C = \frac{2}{5}\left(\sqrt{4+x}\right)^5 - \frac{8}{3}\left(\sqrt{4+x}\right)^3 + C$$

49.)



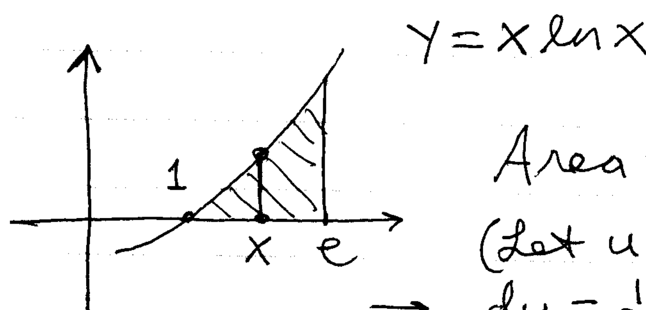
$$\text{Area} = \int_0^4 xe^{-x}\,dx \quad (\text{Let } u = x,\ dv = e^{-x}dx \;\rightarrow\; du = dx,\ v = -e^{-x})$$

$$= -xe^{-x}\Big|_0^4 - -\int_0^4 e^{-x}\,dx$$

$$= -4e^{-4} - 0 + \frac{e^{-x}}{(-1)}\Big|_0^4 = \frac{-4}{e^4} + (-e^{-4} - -e^0)$$

$$= \frac{-4}{e^4} + \frac{-1}{e^4} + 1 = 1 - \frac{5}{e^4}$$

51.)



$$\text{Area} = \int_1^e x\ln x\,dx$$

$$(\text{Let } u = \ln x,\ dv = x\,dx \;\rightarrow\; du = \frac{1}{x}dx,\ v = \frac{1}{2}x^2)$$

$$= \frac{1}{2}x^2\ln x\Big|_1^e - \frac{1}{2}\int_1^e x\,dx$$

$$= \frac{1}{2}e^2\cancel{\ln e}^{\,1} - \frac{1}{2}1^2\cancel{\ln 1}^{\,0} - \frac{1}{2}\cdot\frac{1}{2}x^2\Big|_1^e$$

$$= \frac{1}{2}e^2 - \frac{1}{4}(e^2 - 1) = \frac{1}{4}e^2 + \frac{1}{4}$$

54.) b.) $\text{Vol.} = \pi\int_0^1 r^2\,dx = \pi\int_0^1 (xe^x)^2\,dx$

$= \pi \int_0^1 x^2 e^{2x}\,dx$ (Let $u = x^2$, $dv = e^{2x}\,dx$

$\rightarrow du = 2x\,dx$, $v = \frac{1}{2}e^{2x}$)

$= \pi\left[\frac{1}{2}x^2 e^{2x}\Big|_0^1 - \int_0^1 x e^{2x}\,dx\right]$

(Let $u = x$, $dv = e^{2x}\,dx$

$\rightarrow du = dx$, $v = \frac{1}{2}e^{2x}$)

$= \pi\left[\frac{1}{2}e^2 - \left\{\frac{1}{2}xe^{2x}\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x}\,dx\right\}\right]$

$= \pi\left[\frac{1}{2}e^2 - \left\{\frac{1}{2}e^2 - \frac{1}{2}\cdot\frac{1}{2}e^{2x}\Big|_0^1\right\}\right]$

$= \pi\left[\cancel{\frac{1}{2}e^2} - \cancel{\frac{1}{2}e^2} + \frac{1}{4}(e^2-1)\right] = \frac{\pi}{4}(e^2-1)$

59.) SEE NEXT PAGE

61.) a.) $AVE = \frac{1}{2-1}\int_1^2 (1 + 1.6\,t\ln t)\,dt$

$= t\Big|_1^2 + 1.6\int_1^2 t\ln t\,dt$ (Let $u = \ln t$, $dv = t\,dt$

$\rightarrow du = \frac{1}{t}\,dt$, $v = \frac{1}{2}t^2$)

$= (2-1) + 1.6\left[\frac{1}{2}t^2\ln t\Big|_1^2 - \frac{1}{2}\int_1^2 t\,dt\right]$

$= 1 + 1.6\left[(2\ln 2 - \frac{1}{2}\cancel{\ln 1}^{0}) - \frac{1}{2}\cdot\frac{1}{2}t^2\Big|_1^2\right]$

$= 1 + 1.6\left[2\ln 2 - \frac{1}{4}(4-1)\right]$

$= 1 + (3.2)\ln 2 - 1.2$

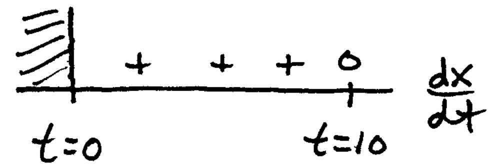
$= (3.2)\ln 2 - 0.2 \approx 2.018$

59.) x: # of units (demand) t: # yrs.

$x = 500(20 + te^{-0.1t}) \longrightarrow$

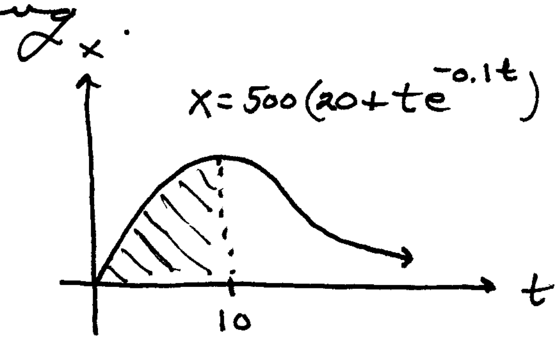
$$\frac{dx}{dt} = 500 \cdot \left[t e^{-0.1t}(-0.1) + e^{-0.1t} \right]$$

$$= 500 \cdot e^{-0.1t}\left(1 - \frac{1}{10}t\right) = 0$$



a.) Since $\frac{dx}{dt} \geq 0$ for $0 \leq t \leq 10$ the demand is increasing.

b.) Total Demand is



$$TOT = \int_0^{10} 500\left(20 + te^{-\frac{1}{10}t}\right) dt$$

$$= \int_0^{10} 10{,}000\, dt + 500 \int_0^{10} te^{-\frac{1}{10}t}\, dt$$

(Let $u = t$, $dv = e^{-\frac{1}{10}t}\, dt$
$du = dt$, $v = -10e^{-\frac{1}{10}t}$)

$$= 100{,}000 + 500\left\{ -10te^{-\frac{1}{10}t}\Big|_0^{10} + 10\int_0^{10} e^{-\frac{1}{10}t}\, dt \right\}$$

$$= 100{,}000 + 500\left\{ \frac{-100}{e} - 100e^{-\frac{1}{10}t}\Big|_0^{10} \right\}$$

$$= 100{,}000 + 500\left\{ \frac{-100}{e} - 100\left(\frac{1}{e} - 1\right) \right\} \approx 113{,}212 \text{ units}$$

c.) Average Demand is

$$AVE = \frac{1}{10-0} \int_0^{100} 500\left(20 + te^{-\frac{1}{10}t}\right) dt$$

$\approx 11{,}321$ units / yr.