Section 6.2

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3.) $\int x^2 e^{-x}\,dx$ (Let $u = x^2$, $dv = e^{-x}dx$

$\rightarrow du = 2x\,dx$, $v = -e^{-x}$)

$= -x^2 e^{-x} - -2\int x e^{-x}\,dx$

(Let $u = x$, $dv = e^{-x}dx$

$\rightarrow du = 1\,dx$, $v = -e^{-x}$)

$= -x^2 e^{-x} + 2\left[-xe^{-x} - -\int e^{-x}dx\right]$

$= -x^2 e^{-x} - 2xe^{-x} + 2\cdot(-e^{-x}) + c$

4.) $\int x^2 e^{2x}\,dx$ (Let $u = x^2$, $dv = e^{2x}dx$

$\rightarrow du = 2x\,dx$, $v = \frac{1}{2}e^{2x}$)

$= \frac{1}{2}x^2 e^{2x} - \int x e^{2x}\,dx$

(Let $u = x$, $dv = e^{2x}dx$

$\rightarrow du = dx$, $v = \frac{1}{2}e^{2x}$)

$= \frac{1}{2}x^2e^{2x} - \left[\frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx\right]$

$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{2}\cdot\frac{1}{2}e^{2x} + c$

19.) $\int x(\ln x)^2\,dx$ (Let $u = (\ln x)^2$, $dv = x\,dx$

$\rightarrow du = \frac{2\ln x}{x}\,dx$, $v = \frac{1}{2}x^2$)

$= \frac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx$

(Let $u = \ln x$, $dv = x\,dx$

$\rightarrow du = \frac{1}{x}\,dx$, $v = \frac{1}{2}x^2$)

$= \frac{1}{2}x^2(\ln x)^2 - \left[\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx\right]$

$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{2}\cdot\frac{1}{2}x^2 + c$

28.) $\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx = \int \frac{x}{(x^2+1)^2} \cdot x^2 e^{x^2}\,dx$

(Let $u = x^2 e^{x^2}$, $dv = \frac{x}{(x^2+1)^2}\,dx$,

$\rightarrow du = (x^2 \cdot 2x e^{x^2} + 2x e^{x^2})\,dx$

$= 2x e^{x^2}(x^2+1)\,dx$, $v = \frac{-1/2}{x^2+1}$)

$= \frac{-\frac{1}{2} x^2 e^{x^2}}{x^2+1} - -\int x e^{x^2}\,dx$ (Let $u = x^2 \rightarrow \ldots$)

$= \frac{-\frac{1}{2} x^2 e^{x^2}}{x^2+1} + \frac{1}{2} e^{x^2} + C$

29.) $\int_0^1 x^2 e^x\,dx$ (Let $u = x^2$, $dv = e^x\,dx$

$\rightarrow du = 2x\,dx$, $v = e^x$)

$= x^2 e^x \Big|_0^1 - 2\int_0^1 x e^x\,dx$ (Let $u = x$, $dv = e^x\,dx$

$\rightarrow du = dx$, $v = e^x$)

$= e - 0 - 2\left[x e^x \Big|_0^1 - \int_0^1 e^x\,dx\right]$

$= e - 2\left[e - 0 - e^x \Big|_0^1\right]$

$= e - 2\left[e - (e-1)\right]$

$= e - 2[1]$

$= e - 2$

Handout 13

1.) a.) $\int_1^e \ln x \, dx$ (Let $u = \ln x$, $dv = dx$
$\rightarrow$ $du = \frac{1}{x} dx$, $v = x$)

$= x \ln x \Big|_1^e - \int_1^e 1 \, dx$

$= e \cancel{\ln} e - 1 \cancel{\ln} 1 - x \Big|_1^e$

$= e - (e-1) = 1$

b.) $\int (\ln x)^2 dx$ (Let $u = (\ln x)^2$, $dv = dx$
$\rightarrow du = 2 \ln x \cdot \frac{1}{x} dx$, $v = x$)

$= x(\ln x)^2 - 2 \int \ln x \, dx$ (Let $u = \ln x$, $dv = dx$
$\rightarrow$ $du = \frac{1}{x} dx$, $v = x$)

$= x(\ln x)^2 - 2 \left[x \ln x - \int 1 \, dx \right]$

$= x(\ln x)^2 - 2 \cdot x \ln x + 2(x) + C$

c.) $\int x^3 e^x dx$ (Let $u = x^3$, $dv = e^x dx$
$\rightarrow$ $du = 3x^2 dx$, $v = e^x$)

$= x^3 e^x - 3 \int x^2 e^x dx$ (Let $u = x^2$, $dv = e^x dx$
$\rightarrow$ $du = 2x \, dx$, $v = e^x$)

$= x^3 e^x - 3 \left[x^2 e^x - 2 \int x e^x dx \right]$

$= x^3 e^x - 3x^2 e^x + 6 \int x e^x dx$

$$\left(\text{Let } u = x,\ dv = e^x\,dx \right.$$
$$\left. \rightarrow du = dx,\ v = e^x \right)$$
$$= x^3e^x - 3x^2e^x + 6\left[xe^x - \int e^x\,dx\right]$$
$$= x^3e^x - 3x^2e^x + 6xe^x - 6\cdot e^x + c$$

d.) $\int e^x \sin e^x\,dx$ (Let $u = e^x \rightarrow du = e^x\,dx$)
$$= \int \sin u\,du = -\cos u + c$$
$$= -\cos e^x + c$$

e.) $A = \int e^x \sin x\,dx$ (Let $u = e^x$, $dv = \sin x\,dx$
$\rightarrow du = e^x\,dx$, $v = -\cos x$)
$$= -e^x\cos x - -\int e^x\cos x\,dx$$
(Let $u = e^x$, $dv = \cos x\,dx$
$\rightarrow du = e^x\,dx$, $v = \sin x$)
$$= -e^x\cos x + \left[e^x\sin x - \int e^x\sin x\,dx\right]$$
$$= -e^x\cos x + e^x\sin x - \underbrace{\int e^x\sin x\,dx}_{A} \rightarrow$$
$$2A = 2\int e^x\sin x\,dx$$
$$= -e^x\cos x + e^x\sin x + C \rightarrow$$
$$A = \int e^x\sin x\,dx = \frac{-1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + C$$

f.) $A = \int \sin(\ln x)\,dx$ (Let $u = \sin(\ln x)$, $dv = dx$
$\rightarrow du = \cos(\ln x)\cdot\frac{1}{x}\,dx$, $v = x$)

$= x\sin(\ln x) - \int \cos(\ln x)\,dx$
(Let $u = \cos(\ln x)$, $dv = dx$
$\rightarrow du = -\sin(\ln x)\cdot\frac{1}{x}\,dx$, $v = x$)

$= x\sin(\ln x) - \left[x\cos(\ln x) - {}^{-}\int \sin(\ln x)\,dx\right]$

$= x\sin(\ln x) - x\cos(\ln x) - \underbrace{\int \sin(\ln x)\,dx}_{A} \rightarrow$

$2A = 2\int \sin(\ln x)\,dx$
$= x\sin(\ln x) - x\cos(\ln x) + C \rightarrow$

$A = \int \sin(\ln x)\,dx = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$

g.) $\int \frac{(\ln x)^2}{x}\,dx$ (Let $u = \ln x \rightarrow du = \frac{1}{x}\,dx$)

$= \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$

h.) $\int x\ln x\,dx$ (Let $u = \ln x$, $dv = x\,dx$
$\rightarrow du = \frac{1}{x}\,dx$, $v = \frac{1}{2}x^2$)

i.) $\int x\sin 3x\,dx$ (Let $u = x$, $dv = \sin 3x\,dx$
$\rightarrow du = dx$, $v = -\frac{1}{3}\cos 3x$)

$$= \frac{-1}{3}x\cos 3x - \frac{-1}{3}\int \cos 3x\,dx$$
$$= \frac{-1}{3}x\cos 3x + \frac{1}{3}\left(\frac{1}{3}\sin 3x\right) + C$$

j.) $\int x^2\cos 2x\,dx$ (Let $u = x^2$, $dv = \cos 2x\,dx$
$\rightarrow du = 2x\,dx$, $v = \frac{1}{2}\sin 2x$)

$$= \frac{1}{2}x^2\sin 2x - \int x\sin 2x\,dx$$

(Let $u = x$, $dv = \sin 2x\,dx$
$\rightarrow du = dx$, $v = \frac{-1}{2}\cos 2x$)

$$= \frac{1}{2}x^2\sin 2x - \left[\frac{-1}{2}x\cos 2x - \frac{-1}{2}\int \cos 2x\,dx\right]$$
$$= \frac{1}{2}x^2\sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{2}\left(\frac{1}{2}\sin 2x\right) + C$$

k.) $A = \int e^{-x}\cos x\,dx$ (Let $u = e^{-x}$, $dv = \cos x\,dx$
$\rightarrow du = -e^{-x}dx$, $v = \sin x$)

$$= e^{-x}\sin x - -\int e^{-x}\sin x\,dx$$

(Let $u = e^{-x}$, $dv = \sin x\,dx$
$\rightarrow du = -e^{-x}$, $v = -\cos x$)

$$= e^{-x}\sin x + \left[-e^{-x}\cos x - \int e^{-x}\cos x\,dx\right]$$
$$= e^{-x}\sin x - e^{-x}\cos x - \underbrace{\int e^{-x}\cos x\,dx}_{A}, \text{ then}$$

$$2A = 2\int e^{-x}\cos x\,dx$$
$$= e^{-x}\sin x - e^{-x}\cos x + C \longrightarrow$$

$$A = \int e^{-x} \cos x \, dx = \frac{1}{2} e^{-x} \sin x - \frac{1}{2} e^{-x} \cos x + c$$

l.) $\int x^3 \sin x \, dx$ (Let $u = x^3$, $dv = \sin x \, dx$
$\rightarrow du = 3x^2 \, dx$, $v = -\cos x$)

$= -x^3 \cos x - -3 \int x^2 \cos x \, dx$

(Let $u = x^2$, $dv = \cos x \, dx$
$\rightarrow du = 2x \, dx$, $v = \sin x$)

$= -x^3 \cos x + 3 [x^2 \sin x - 2 \int x \sin x \, dx]$

$= -x^3 \cos x + 3x^2 \sin x - 6 \int x \sin x \, dx$

(Let $u = x$, $dv = \sin x \, dx$
$\rightarrow du = dx$, $v = -\cos x$)

$= -x^3 \cos x + 3x^2 \sin x$
$\quad - 6 [-x \cos x - - \int \cos x \, dx]$

$= -x^3 \cos x + 3x^2 \sin x$
$\quad + 6x \cos x - 6 \sin x + c$