Section 4.2

Page 270

1.) a.) $(e^3)(e^4) = e^{3+4} = e^7$

b.) $(e^3)^4 = e^{3\cdot 4} = e^{12}$

c.) $(e^3)^{-2} = e^{3\cdot(-2)} = e^{-6} = \frac{1}{e^6}$

d.) $e^0 = 1$

4.) a.) $(e^{-3})^{2/3} = e^{-\cancel{3}\cdot\frac{2}{\cancel{3}}} = e^{-2} = \frac{1}{e^2}$

b.) $\frac{e^4}{e^{-1/2}} = e^{4-(-1/2)} = e^{9/2}$

c.) $(e^{-2})^{-4} = e^{(-2)\cdot(-4)} = e^8$

d.) $(e^{-4})(e^{-3/2}) = e^{-4+{-3/2}} = e^{-11/2} = \frac{1}{e^{11/2}}$

5.) $e^{-3x} = e \rightarrow e^{-3x} = e^1 \rightarrow -3x = 1 \rightarrow$
$x = -1/3$

6.) $e^x = 1 \rightarrow x = 0$

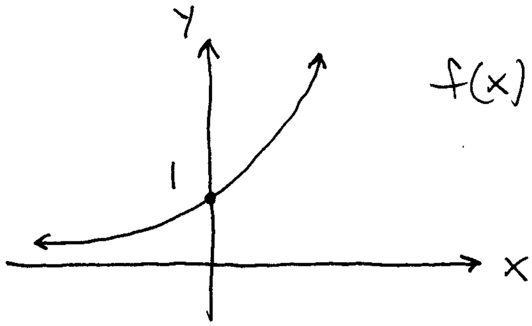
8.) $e^{-1/x} = \sqrt{e} \rightarrow e^{-1/x} = e^{1/2} \rightarrow \frac{-1}{x} = \frac{1}{2} \rightarrow$
$-x = 2 \rightarrow x = -2$

12.) $x^{-2} = \frac{2}{e^2} \rightarrow \frac{1}{x^2} = \frac{2}{e^2} \rightarrow x^2 = \frac{e^2}{2} \rightarrow$
$\sqrt{x^2} = \sqrt{\frac{e^2}{2}} \rightarrow |x| = \frac{e}{\sqrt{2}} \rightarrow x = \pm\frac{e}{\sqrt{2}}$

13.) (f) 14.) (e) 15.) (d)

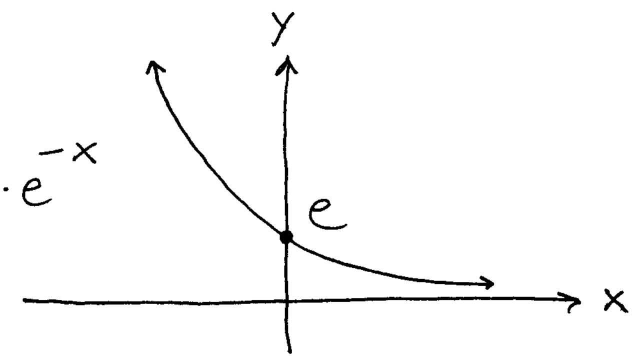
16.) (b) 17.) (c) 18.) (a)

20.)



$f(x) = e^{2x}$

21.) $y = e^{1-x} = e \cdot e^{-x}$



32.) $A = P\left(1 + \frac{r}{n}\right)^{nt} \longrightarrow A = 2500\left(1 + \frac{0.05}{n}\right)^{20n}$

n:	1	2	4
A:	\$6633.24	\$6712.66	\$6753.71
t:	12		cont.
A:	\$6781.60		
t:	365		
A:	\$6795.24		

$A = Pe^{rt} = 2500e^{(0.05)(20)}$

$\approx \$6795.70$

36.) $A = Pe^{rt} \longrightarrow P = \frac{A}{e^{rt}} = \frac{100{,}000}{e^{0.03t}}$

t:	1	10	20
P:	\$97,044.55	\$74,081.82	\$54,881.16
t:	30	40	50
P:	\$40,656.97	\$30,119.42	\$22,313.02

37.) $A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow P = \dfrac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \dfrac{100{,}000}{\left(1+\frac{0.05}{12}\right)^{12t}}$

t:	1	10	20
P:	\$95,132.82	\$60,716.10	\$36,864.45
t:	30	40	50
P:	\$22,382.66	\$13,589.88	\$8251.24

39.) $A = P\left(1+\frac{r}{n}\right)^{nt} = P\left(1+\frac{0.09}{n}\right)^{n(1)}$

a.) $n=1: (1+0.09)^1 = 1.09 \rightarrow$ 9% eff. rate

b.) $n=2: \left(1+\frac{0.09}{2}\right)^2 \approx 1.0920 \rightarrow$ 9.2% eff. rate

c.) $n=4: \left(1+\frac{0.09}{4}\right)^4 \approx 1.0931 \rightarrow$ 9.31% eff rate

d.) $n=12: \left(1+\frac{0.09}{12}\right)^{12} \approx 1.0938 \rightarrow$ 9.38% eff. rate

42.) $A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow P = \dfrac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \dfrac{21{,}154.03}{\left(1+\frac{0.078}{12}\right)^{12(4)}}$

$\approx \$15{,}500.00$

44.) $A = P\left(1+\frac{r}{n}\right)^{nt} = 6000\left(1+\frac{0.0625}{12}\right)^{12(3)}$

$\approx \$7233.86$

45.) $p = 5000\left(1 - \dfrac{4}{4+e^{-0.002x}}\right)$

a.) $x=100 \rightarrow p \approx \$849.53$

b.) $x=500 \rightarrow p \approx \$421.12$

c.) $\lim\limits_{x \to \infty} 5000\left(1 - \dfrac{4}{4+e^{-0.002x}}\right)$

$$= 5000\left(1 - \frac{4}{4+\underbrace{\cancel{e}^{-\infty}}_{0}}\right) = 5000\left(1 - \frac{4}{4}\right) = 0$$

48.) $y = 28e^{0.6 - 0.012s}$

s:	50	55	60	65	70
y:	28	26.37	24.83	23.39	22.03

53.) $p = \dfrac{0.83}{1+e^{-0.2n}}$

a.) $n = 10 \longrightarrow p = \dfrac{0.83}{1+e^{-0.2(10)}} \approx 0.731$

or 73.1 %

b.) $p = 0.75 \rightarrow 0.75 = \dfrac{0.83}{1+e^{-0.2n}} \rightarrow$

$1+e^{-0.2n} = \dfrac{0.83}{0.75} \rightarrow e^{-0.2n} = \dfrac{0.83}{0.75} - 1 \rightarrow$

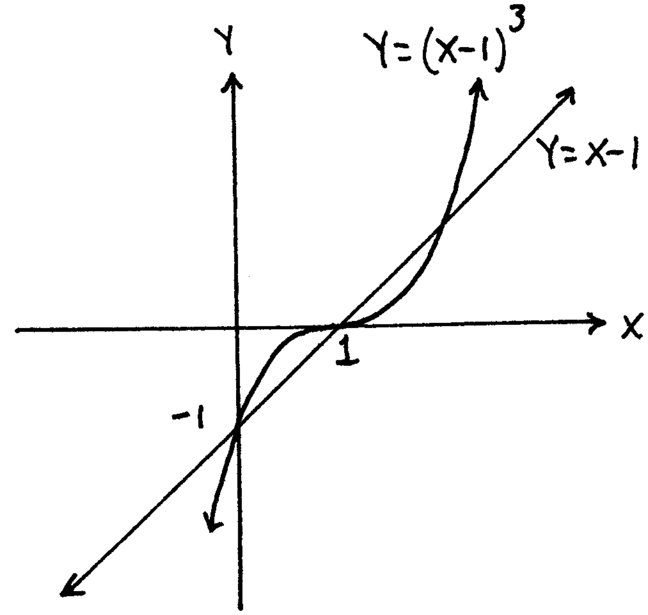
$\ln e^{-0.2n} = \ln\left(\dfrac{83}{75} - \dfrac{75}{75}\right) \longrightarrow -0.2n = \ln\left(\dfrac{8}{75}\right)$

$\rightarrow n = \dfrac{\ln(8/75)}{-0.2} \approx 11.2$ or 11 trials

c.) $\lim\limits_{n \to \infty} \dfrac{0.83}{1+e^{-0.2n}} = \dfrac{0.83}{1+\underbrace{\cancel{e}^{-\infty}}_{0}} = 0.83$

or 83 %

$\boxed{5A7}$ d.) $Y = X - 1$, $Y = (X-1)^3$



intersection:

$(X-1) = (X-1)^3 \rightarrow$

$0 = (X-1)^3 - (X-1) \rightarrow$

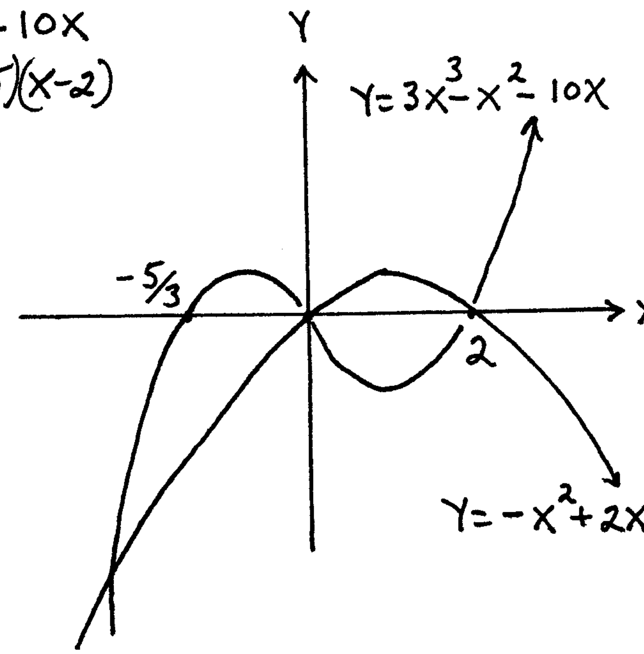
$0 = (X-1)[(X-1)^2 - 1] \rightarrow$

$0 = (X-1)[X^2 - 2X + 1 - 1] \rightarrow$

$0 = (X-1)X[X-2] \rightarrow$

$X=1, Y=0$ and $X=0, Y=-1$ and $X=2, Y=1$

$\boxed{5A7}$ e.) $Y = -X^2 + 2X$, $Y = 3X^3 - X^2 - 10X$
$= X(2-X)$ $\quad = X(3X+5)(X-2)$



intersection:

$-X^2 + 2X = 3X^3 - X^2 - 10X \rightarrow$

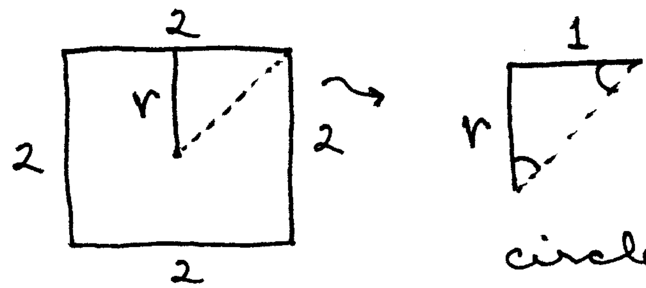
$0 = 3X^3 - 12X \rightarrow$

$0 = 3X(X-2)(X+2) \rightarrow$

$X=0, Y=0$ and $X=2, Y=0$

and $X=-2, Y=-8$

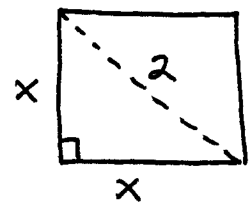
$\boxed{5A8}$



Larger square is 2 by 2 so larger circle has radius $r = 1$; thus

the diagonal of the smaller square

is 2



so by Pythagorean Theorem

$x^2 + x^2 = 2^2 \rightarrow 2x^2 = 4 \rightarrow x = \sqrt{2}$; and $x = \sqrt{2}$ is diameter of smaller circle so its radius is $\frac{\sqrt{2}}{2}$ and its area is $A = \pi r^2 = \pi\left(\frac{\sqrt{2}}{2}\right)^2 = \pi \cdot \frac{1}{2} = \frac{\pi}{2}$.

$\boxed{\text{SA17}}$ a.) $A + B = 3 \rightarrow A = 3 - B$

$2A - 3B = 4 \leftarrow$ substitute, so

$2(3-B) - 3B = 4 \rightarrow 6 - 2B - 3B = 4 \rightarrow 2 = 5B$

$\rightarrow B = 2/5$ and $A = 13/5$

b.) $\left.\begin{array}{l} 2A - 5B = 3 \\ 3A + 2B = 7 \end{array}\right\} \begin{array}{l} 6A - 15B = 9 \\ -6A - 4B = -14 \end{array}$ } add equations

$\rightarrow 0 - 19B = -5 \rightarrow B = 5/19$ and

$2A - 5\left(\frac{5}{19}\right) = 3 \rightarrow 2A = \frac{82}{19} \rightarrow A = \frac{41}{19}$