Section 6.6

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1.) $\int_0^\infty e^{-x}\,dx = \lim_{b\to\infty} \int_0^b e^{-x}\,dx$

$= \lim_{b\to\infty} -e^{-x}\Big|_0^b = \lim_{b\to\infty} \left(\frac{-1}{e^b} - \frac{-1}{e^0}\right) = 0+1 = 1$

2.) $\int_{-\infty}^0 e^{2x}\,dx = \lim_{b\to-\infty} \int_b^0 e^{2x}\,dx$

$= \lim_{b\to-\infty} \frac{1}{2}e^{2x}\Big|_b^0 = \lim_{b\to-\infty} \frac{1}{2}e^0 - \frac{1}{2}e^{2b}$

$= \frac{1}{2} - 0 = \frac{1}{2}$

3.) $\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty} \int_1^b x^{-2}\,dx$

$= \lim_{b\to\infty} -x^{-1}\Big|_1^b = \lim_{b\to\infty} \frac{-1}{x}\Big|_1^b$

$= \lim_{b\to\infty} \left(\frac{-1}{b} - \frac{-1}{1}\right) = 0+1 = 1$

4.) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty} \int_1^b x^{-1/2}\,dx$

$= \lim_{b\to\infty} 2x^{1/2}\Big|_1^b = \lim_{b\to\infty} (2\sqrt{b} - 2\sqrt{1}) = +\infty$

so diverges

5.) $\int_0^\infty e^{\frac{x}{3}}\,dx = \lim_{b\to\infty} \int_0^b e^{\frac{x}{3}}\,dx$

$= \lim_{b\to\infty} 3e^{\frac{x}{3}}\Big|_0^b = \lim_{b\to\infty} (3e^{\frac{b}{3}} - 3\cdot e^0)$

$= +\infty - 3$ so diverges

6.) $\int_0^\infty \frac{5}{e^{2x}}\,dx = \lim_{b\to\infty} \int_0^b 5\cdot e^{-2x}\,dx$

$$= \lim_{b\to\infty} -\frac{5}{2} e^{-2x}\Big|_0^b = \lim_{b\to\infty}\left(\frac{-5}{2}\cdot\frac{1}{e^{2b}} - \frac{-5}{2}\cdot e^0\right)$$

$$= \frac{-5}{2}(0) + \frac{5}{2} = \frac{5}{2}$$

7.) $$\int_5^\infty \frac{x}{\sqrt{x^2-16}}\,dx = \lim_{b\to\infty}\int_5^b x(x^2-16)^{-\frac{1}{2}}dx$$

$$= \lim_{b\to\infty} (x^2-16)^{1/2}\Big|_5^b = \lim_{b\to\infty}\left(\sqrt{b^2-16} - \sqrt{9}\right)$$

$= +\infty$ so diverges

8.) $$\int_{\frac{1}{2}}^\infty \frac{1}{\sqrt{2x-1}}\,dx = \int_{\frac{1}{2}}^1 \frac{1}{\sqrt{2x-1}}\,dx + \int_1^\infty \frac{1}{\sqrt{2x-1}}\,dx$$

$$= A + B;$$

$$A = \lim_{x\to\frac{1}{2}^+}\int_b^1 \frac{1}{\sqrt{2x-1}}\,dx = \lim_{x\to\frac{1}{2}^+} (2x-1)^{1/2}\Big|_b^1$$

$$= \lim_{x\to\frac{1}{2}^+}\left(1-(2b-1)^{1/2}\right) = \underline{1} \quad \text{and}$$

$$B = \lim_{b\to\infty}\int_1^b \frac{1}{\sqrt{2x-1}}\,dx = \lim_{b\to\infty}(2x-1)^{1/2}\Big|_1^b$$

$$= \lim_{b\to\infty}\left(\sqrt{2b-1} - 1\right) = \underline{+\infty - 1} = \infty$$ so diverges

9.) $$\int_{-\infty}^0 e^{-x}\,dx = \lim_{b\to-\infty}\int_b^0 e^{-x}\,dx$$

$$= \lim_{b\to-\infty} -e^{-x}\Big|_b^0 = \lim_{b\to-\infty}\left(-e^0 - -e^{-b}\right)$$

$= -1 + \infty$ so diverges

10.) $$\int_{-\infty}^{-1}\frac{1}{x^2}\,dx = \lim_{b\to-\infty}\int_b^{-1} x^{-2}\,dx$$

$$= \lim_{b\to-\infty} -x^{-1}\Big|_b^{-1} = \lim_{b\to-\infty}\left(\frac{-1}{(-1)} - \frac{-1}{(b)}\right) = 1+0 = 1$$

12.) $$\int_{-\infty}^{0} \frac{x}{x^2+1}\,dx = \lim_{A\to-\infty}\int_A^0 \frac{x}{x^2+1}\,dx$$

$$= \lim_{A\to-\infty} \frac{1}{2}\ln(x^2+1)\Big|_A^0 = \lim_{A\to-\infty}\left(\cancel{\frac{1}{2}\ln 1}^{\,0} - \frac{1}{2}\ln(A^2+1)\right)$$

$= -\infty$ diverges

14.) $$\int_{-\infty}^{\infty} x^2 e^{-x^3}\,dx = \int_{-\infty}^{0} x^2 e^{-x^3}\,dx + \int_0^{\infty} x^2 e^{-x^3}\,dx$$

$$= A + B\ ;$$

$$B = \lim_{b\to\infty}\int_0^b x^2 e^{-x^3}\,dx = \lim_{b\to\infty} \frac{-1}{3}e^{-x^3}\Big|_0^b$$

$$= \lim_{b\to\infty}\left(\frac{-1}{3}\cdot\frac{1}{e^{b^3}} - \frac{-1}{3}e^0\right) = (0) + \frac{1}{3} = \underbrace{\frac{1}{3}}\ \text{and}$$

$$A = \lim_{b\to-\infty}\int_b^0 x^2 e^{-x^3}\,dx = \lim_{b\to-\infty} \frac{-1}{3}e^{-x^3}\Big|_b^0$$

$$= \lim_{b\to-\infty}\left(\frac{-1}{3}e^0 - \frac{-1}{3}e^{-b^3}\right) = \underbrace{\frac{-1}{3}+\infty}\ \text{so}$$

diverges

15.) $$\int_0^4 \frac{1}{\sqrt{x}}\,dx = \lim_{b\to0^+}\int_b^4 x^{-1/2}\,dx$$

$$= \lim_{b\to0^+} 2\sqrt{x}\Big|_b^4 = \lim_{b\to0^+}(4-2\sqrt{b}) = 4$$

16.) $\int_3^4 \frac{1}{\sqrt{x-3}}\,dx = \lim_{b\to 3^+} \int_b^4 (x-3)^{-1/2}\,dx$

$= \lim_{b\to 3^+} 2\sqrt{x-3}\,\Big|_b^4 = \lim_{b\to 3^+} \left(2 - 2\sqrt{b-3}\right) = 2$

18.) $\int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 (x-1)^{-2}\,dx + \int_1^2 (x-1)^{-2}\,dx$

$= A + B$;

$A = \int_0^1 (x-1)^{-2}\,dx = \lim_{b\to 1^-} \int_0^b (x-1)^{-2}\,dx$

$= \lim_{b\to 1^-} -(x-1)^{-1}\Big|_0^b = \lim_{b\to 1^-} \frac{-1}{x-1}\Big|_0^b$

$= \lim_{b\to 1^-} \frac{-1}{b-1} - \frac{-1}{-1} = +\infty - 1$ so diverges

(note that $B = +\infty$ also!)

19.) $\int_0^1 \frac{1}{1-x}\,dx = \lim_{b\to 1^-} \int_0^b \frac{1}{1-x}\,dx$

$= \lim_{b\to 1^-} -\ln|1-x|\,\Big|_0^b = \lim_{b\to 1^-} \left(-\ln|1-b| + \cancel{\ln 1}^{\,0}\right)$

$= +\infty$ diverges

22.) $\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx = \lim_{b\to 2^-} \int_0^b \frac{x}{\sqrt{4-x^2}}\,dx$

$= \lim_{b\to 2^-} -(4-x^2)^{1/2}\Big|_0^b = \lim_{b\to 2^-} \left(-\sqrt{4-b^2} + 2\right) = 2$

23.) $\int_0^1 \frac{1}{x^2}\,dx = \lim_{b\to 0^+} \int_b^1 \frac{1}{x^2}\,dx = \lim_{b\to 0^+} \frac{-1}{x}\Big|_b^1$

$= \lim_{b\to 0^+} \left(\frac{-1}{1} - \frac{-1}{b}\right) = -1 + \infty$ diverges

24.) $\int_0^1 \frac{1}{x}\,dx = \lim_{b\to 0^+} \int_b^1 \frac{1}{x}\,dx = \lim_{b\to 0^+} \ln|x| \Big|_b^1$

$= \lim_{b\to 0^+} (\ln 1 - \ln b) = \infty$ diverges

25.) $\int_0^2 (x-1)^{-1/3}\,dx = \int_0^1 (x-1)^{-1/3}\,dx + \int_1^2 (x-1)^{-1/3}\,dx$

$= \quad A \quad + \quad B \quad ;$

$A = \lim_{b\to 1^-} \int_0^b (x-1)^{-1/3}\,dx = \lim_{b\to 1^-} \frac{3}{2}(x-1)^{2/3} \Big|_0^b$

$= \lim_{b\to 1^-} \left\{ \frac{3}{2}(b-1)^{2/3} - \frac{3}{2}(-1)^{2/3} \right\} = 0 - \frac{3}{2} = -\frac{3}{2}$;

$B = \lim_{b\to 1^+} \int_b^2 (x-1)^{-1/3}\,dx = \lim_{b\to 1^+} \frac{3}{2}(x-1)^{2/3} \Big|_b^2$

$= \lim_{b\to 1^+} \left\{ \frac{3}{2}(1)^{2/3} - \frac{3}{2}(b-1)^{2/3} \right\} = \frac{3}{2} - 0 = \frac{3}{2}$; so

$\int_0^2 (x-1)^{-1/3}\,dx = A + B = -\frac{3}{2} + \frac{3}{2} = \boxed{0}$

26.) $\int_0^2 (x-1)^{-4/3}\,dx = \int_0^1 (x-1)^{-4/3}\,dx + \int_1^2 (x-1)^{-4/3}\,dx$

$= \quad A \quad + \quad B \quad ;$

$A = \lim_{b\to 1^-} \int_0^b (x-1)^{-4/3}\,dx$

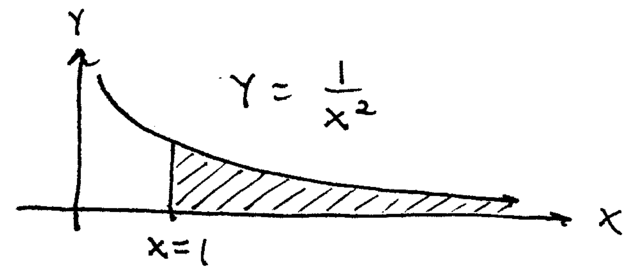
$= \lim_{b\to 1^-} -3(x-1)^{-1/3} \Big|_0^b$

$= \lim_{b\to 1^-} \left\{ \frac{-3}{(b-1)^{1/3}} - \frac{-3}{(-1)^{1/3}} \right\} = +\infty - 3$

so diverges

(Note that $B = +\infty$ also!)

29.)



a.) Area $= \int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx$

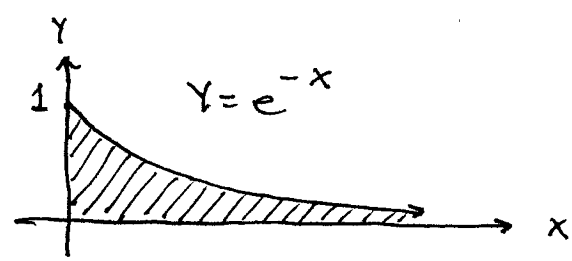
$= \lim_{b\to\infty} \left.\frac{-1}{x}\right|_1^b = \lim_{b\to\infty} \left(\frac{-1}{b} - \frac{-1}{1}\right) = 0+1 = 1$

b.) Volume $= \pi \int_1^\infty \left(\frac{1}{x^2}\right)^2 dx = \lim_{b\to\infty} \pi \int_1^b x^{-4}\,dx$

$= \lim_{b\to\infty} \pi \cdot \left.\frac{x^{-3}}{-3}\right|_1^b = \lim_{b\to\infty} -\frac{\pi}{3}\left(\frac{1}{b^3} - \frac{1}{1^3}\right)$

$= -\frac{\pi}{3}(0-1) = \frac{\pi}{3}$

30.)



a.) Area $= \int_0^\infty e^{-x}\,dx = \lim_{b\to\infty} \int_0^b e^{-x}\,dx$

$= \lim_{b\to\infty} -e^{-x}\Big|_0^b = \lim_{b\to\infty} \left(\frac{-1}{e^b} - \frac{-1}{e^0}\right) = 0+1 = 1$

b.) Volume $= \pi \int_0^\infty (e^{-x})^2\,dx = \lim_{b\to\infty} \pi \int_0^b (e^{-2x})\,dx$

$= \lim_{b\to\infty} \pi \cdot \left.\frac{e^{-2x}}{-2}\right|_0^b = \lim_{b\to\infty} -\frac{\pi}{2}\left(\frac{1}{e^{2b}} - \frac{1}{e^0}\right)$

$= -\frac{\pi}{2}(0-1) = \frac{\pi}{2}$

Math 16B
Kouba
Improper Integrals

Determine the following improper integrals.

1.) $\displaystyle\int_1^{\infty} e^{-7x}\ dx$

2.) $\displaystyle\int_{-\infty}^{0} e^{(1/4)x}\ dx$

3.) $\displaystyle\int_{-\infty}^{\infty} e^{3x}\ dx$

4.) $\displaystyle\int_4^{7} \frac{3}{4-x}\ dx$

5.) $\displaystyle\int_2^{\infty} \frac{1}{\sqrt{x-2}}\ dx$

6.) $\displaystyle\int_0^{\infty} \frac{1}{(x-1)^3}\ dx$

7.) $\displaystyle\int_5^{\infty} \frac{1}{x^2-9}\ dx$

8.) $\displaystyle\int_3^{\infty} \frac{x^3-x+1}{x^2-4}\ dx$

Improper Integrals

1.) $\int_1^{\infty} e^{-7x}\,dx = \lim\limits_{A\to\infty} \int_1^{A} e^{-7x}\,dx$

$= \lim\limits_{A\to\infty} \frac{-1}{7} e^{-7x}\Big|_1^A = \lim\limits_{A\to\infty} \left(\frac{-1}{7}e^{-7A} - \frac{-1}{7}e^{-7}\right)$

$= \lim\limits_{A\to\infty} \left(\frac{-1}{7e^{7A}} + \frac{1}{7e^{7}}\right) = \cancel{\frac{-1}{\infty}}^{\,0} + \frac{1}{7e^7}$

$= \frac{1}{7e^7}$

2.) $\int_{-\infty}^{0} e^{\frac{1}{4}x}\,dx = \lim\limits_{A\to-\infty} \int_A^0 e^{\frac{1}{4}x}\,dx$

$= \lim\limits_{A\to-\infty} 4e^{\frac{1}{4}x}\Big|_A^0 = \lim\limits_{A\to-\infty} \left(4e^0 - 4e^{\frac{1}{4}A}\right)$

$= 4 - 4e^{-\infty} = 4 - 4\cdot\cancel{\frac{1}{e^{\infty}}}^{\,0} = 4$

3.) $\int_{-\infty}^{\infty} e^{3x}\,dx = \int_{-\infty}^{0} e^{3x}\,dx + \int_0^{\infty} e^{3x}\,dx$

$= B + C$;

$B = \lim\limits_{A\to-\infty} \int^0 e^{3x}\,dx = \lim\limits_{A\to-\infty} \frac{1}{3}e^{3x}\Big|_A^0$

$= \lim\limits_{A\to-\infty} \left(\frac{1}{3}e^0 - \frac{1}{3}e^{3A}\right) = \frac{1}{3} - \frac{1}{3}e^{-\infty}$

$= \frac{1}{3} - \frac{1}{3}\cdot\cancel{\frac{1}{e^{\infty}}}^{\,0} = \frac{1}{3}$;

$$C = \lim_{A\to\infty} \int_0^A e^{3x}\,dx = \lim_{A\to\infty} \frac{1}{3}e^{3x}\Big|_0^A$$

$$= \lim_{A\to\infty}\left(\frac{1}{3}e^{3A} - \frac{1}{3}e^0\right) = \frac{1}{3}\cancel{e}^{\infty} - \frac{1}{3}$$

$$= \infty - \frac{1}{3} = \infty \quad ; \text{ then}$$

$$\int_{-\infty}^{\infty} e^{3x}\,dx = B + C = \frac{1}{3} + \infty = \infty \text{ (Diverges)}$$

4.) $$\int_4^7 \frac{3}{4-x}\,dx = \lim_{A\to 4^+} \int_A^7 \frac{3}{4-x}\,dx$$

$$= \lim_{A\to 4^+} -3\ln|4-x|\Big|_A^7$$

$$= \lim_{A\to 4^+}\left(-3\ln 3 - -3\underbrace{\ln|4-A|}_{-\infty}\right)$$

$$= -3\ln 3 - \infty = -\infty \text{ (Diverges)}$$

5.) $$\int_2^{\infty} \frac{1}{\sqrt{x-2}}\,dx = \int_2^3 \frac{1}{\sqrt{x-2}}\,dx + \int_3^{\infty} \frac{1}{\sqrt{x-2}}\,dx$$

$$= B + C \quad ;$$

$$B = \lim_{A\to 2^+} \int_A^3 \frac{1}{\sqrt{x-2}}\,dx = \lim_{A\to 2^+} 2\sqrt{x-2}\Big|_A^3$$

$$= \lim_{A\to 2^+}\left(2\sqrt{1} - 2\sqrt{A-2}\right) = 2(1) - 2(0) = 2$$

$$C = \lim_{A\to\infty} \int_3^A \frac{1}{\sqrt{x-2}}\,dx = \lim_{A\to\infty} 2\sqrt{x-2}\,\Big|_3^A$$

$$= \lim_{A\to\infty} \left(2\sqrt{A-2} - 2\sqrt{1}\right) = \infty - 2 = \infty \ ;$$

then $\displaystyle\int_2^\infty \frac{1}{\sqrt{x-2}}\,dx = B + C = 2 + \infty = \infty$

(Diverges)

6.) $$\int_0^\infty \frac{1}{(x-1)^3}\,dx = \int_0^1 \frac{1}{(x-1)^3}\,dx + \int_1^\infty \frac{1}{(x-1)^3}\,dx$$

$$= \int_0^1 \frac{1}{(x-1)^3}\,dx + \int_1^2 \frac{1}{(x-1)^3}\,dx + \int_2^\infty \frac{1}{(x-1)^3}\,dx$$

$$= \quad B \quad + \quad C \quad + \quad D \quad ;$$

$$B = \lim_{A\to 1^-} \int_0^A (x-1)^{-3}\,dx = \lim_{A\to 1^-} \frac{(x-1)^{-2}}{-2}\Big|_0^A$$

$$= \lim_{A\to 1^-} \left(\frac{(A-1)^{-2}}{-2} - \frac{(-1)^{-2}}{-2}\right)$$

$$= \lim_{A\to 1^-} \left(\frac{-1}{2}\cdot\frac{1}{(A-1)^2} + \frac{1}{2}\right)$$

$$= \frac{-1}{2}\cdot\left(\frac{1}{0^+}\right) + \frac{1}{2} = -\frac{1}{2}(\infty) + \frac{1}{2}$$

$= -\infty + 1/2 = -\infty$; since $B = -\infty$ it won't matter whether C and D are finite, $+\infty$, or $-\infty$; we conclude that

$$\int_0^\infty \frac{1}{(x-1)^3}\,dx = B+C+D \quad \text{Diverges.}$$

7) $$\int_5^\infty \frac{1}{x^2-9}\,dx = \int_5^\infty \frac{1}{(x-3)(x+3)}\,dx$$

$$= \int_5^\infty \left[\frac{A}{x-3} + \frac{B}{x+3}\right] dx$$

$$\Big(A(x+3) + B(x-3) = 1$$

$$\text{Let } \underline{x=3}:\ 6A = 1 \rightarrow A = 1/6$$

$$\text{Let } \underline{x=-3}:\ -6B = 1 \rightarrow B = -1/6 \Big)$$

$$= \lim_{c\to\infty} \int_5^c \left[\frac{1/6}{x-3} + \frac{-1/6}{x+3}\right] dx$$

$$= \lim_{c\to\infty} \left[\frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3|\right]\Bigg|_5^c$$

"$\infty - \infty$" indeterminate form

$$= \lim_{c\to\infty} \left[\left(\frac{1}{6}\ln|c-3| - \frac{1}{6}\ln|c+3|\right) - \left(\frac{1}{6}\ln 2 - \frac{1}{6}\ln 8\right)\right]$$

$$= \lim_{c\to\infty} \frac{1}{6}\ln\left|\frac{c-3}{c+3}\right| - \frac{1}{6}\ln 2 + \frac{1}{6}\ln 8$$

"$\frac{\infty}{\infty}$" indeterminate form

$$= \lim_{c\to\infty} \frac{1}{6}\ln\left|\frac{c-3}{c+3}\cdot\frac{1/c}{1/c}\right| + \frac{1}{6}(\ln 8 - \ln 2)$$

$$= \lim_{c\to\infty} \frac{1}{6}\ln\left|\frac{1-3/c}{1+3/c}\right| + \frac{1}{6}\ln\left(\frac{8}{2}\right)$$

$$= \frac{1}{6}\ln\left|\frac{1-0}{1+0}\right| + \frac{1}{6}\ln 4$$

$$= \frac{1}{6}\underbrace{\ln 1}_{0} + \frac{1}{6}\ln 4 = \frac{1}{6}\ln 4$$

8.) $\displaystyle\int_3^\infty \frac{x^3-x+1}{x^2-4}\,dx$

$$\begin{array}{r} x \\ x^2-4\,\overline{\smash{)}\,x^3-x+1} \\ -(x^3-4x) \\ \hline 3x+1 \end{array}$$

$$= \int_3^\infty\left[x + \frac{3x+1}{x^2-4}\right]dx$$

$$= \lim_{A\to\infty}\int_3^A\left[x + \frac{3x+1}{x^2-4}\right]dx$$

$$= \lim_{A\to\infty}\left(\frac{x^2}{2}\Big|_3^A + \int_3^A \frac{3x+1}{(x-2)(x+2)}\,dx\right)$$

$$= \lim_{A\to\infty}\left(\frac{A^2}{2} - \frac{9}{2} + \int_3^A\left[\frac{B}{x-2} + \frac{C}{x+2}\right]dx\right)$$

$\Big($ $B(x+2) + C(x-2) = 3x+1$

Let $\underline{x=2}$: $4B = 7 \rightarrow B = 7/4$

Let $\underline{x=-2}$: $-4C = -5 \rightarrow C = 5/4$ $\Big)$

$$= \lim_{A\to\infty}\left(\frac{A^2}{2} - \frac{9}{2} + \int_3^A\left[\frac{7/4}{x-2} + \frac{5/4}{x+2}\right]dx\right)$$

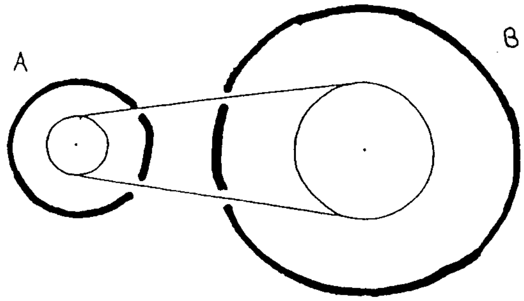
$$= \lim_{A\to\infty}\left(\frac{A^2}{2} - \frac{9}{2} + \left(\frac{7}{4}\ln|x-2| + \frac{5}{4}\ln|x+2|\right)\Big|_3^A\right)$$

$$= \lim_{A\to\infty}\left(\frac{A^2}{2} - \frac{9}{2} + \frac{7}{4}\ln|A-2| + \frac{5}{4}\ln|A+2| - \left(\frac{7}{4}\cancel{\ln 1}^{0} + \frac{5}{4}\ln 5\right)\right)$$

$$= \infty - \frac{9}{2} + \infty + \infty - \frac{5}{4}\ln 5$$

$$= \infty \quad (\text{Diverges})$$

SA13 :



a.) sprocket A: $C = 2\pi r = 2\pi(2) = 4\pi$ in.,
sprocket B: $C = 2\pi r = 2\pi(6) = 12\pi$ in., so
3 rotations of sprocket A is $3(4\pi) = 12\pi$ in.
which is 1 rotation of sprocket B.

b.) 3 rotations of sprocket B is $3(12\pi) = 36\pi$ in.
which is $\frac{36\pi}{4\pi} = 9$ rotations of sprocket A.

c.) If sprocket A rotates 3 times/sec. then
sprocket B and wheel B rotate 1 time/sec.

i.) a pt. on sprocket A is traveling
at $3(4\pi) = 12\pi \approx$ 37.7 in./sec.

or $\frac{37.7 \text{ in.}}{\text{sec.}} \times \frac{3600 \text{ sec.}}{1 \text{ hr.}} \times \frac{1 \text{ ft.}}{12 \text{ in.}} \times \frac{1 \text{ mi.}}{5280 \text{ ft.}} \approx$ 2.14 mph.

ii.) a pt. on wheel B ($C = 2\pi r = 2\pi(30) = 60\pi$ in.)
is traveling at $60\pi \approx$ 188.5 in./sec.
or $\frac{188.5 \text{ in.}}{\text{sec.}} \times \frac{3600 \text{ sec}}{\text{hr.}} \times \frac{1 \text{ ft.}}{12 \text{ in.}} \times \frac{1 \text{ mi.}}{5280 \text{ ft.}} \approx$ 10.71 mph.

d.) If sprocket B rotates 3 times/sec. then
sprocket A and wheel A rotate 9 times/sec.

i.) a pt. on sprocket B is traveling at
$3(12\pi) = 36\pi \approx$ 113.1 in./sec or 6.43 mph.

ii.) a pt. on wheel A is traveling at
$9(2\pi r) = 9(2\pi(6)) = 108\pi \approx$ 339.3 in./sec.

or 19.28 mph.