Section 6.4

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14.) $\int \frac{\sqrt{x^2-9}}{x^2}\,dx$ {Use 24.) with $a=3, u=x$.}

$= -\frac{\sqrt{x^2-9}}{x} + \ln\left|x+\sqrt{x^2-9}\right| + c$

16.) $\int x^2 (\ln x^3)^2\,dx$ (Let $u = x^3 \rightarrow$ $du = 3x^2\,dx \rightarrow \frac{1}{3}du = x^2\,dx$)

$= \int (\ln u)^2 \cdot \frac{1}{3}du = \frac{1}{3}\int (\ln u)^2\,du$ {Use 42.)}

$= \frac{1}{3}u\left[2 - 2\ln u + (\ln u)^2\right] + c$

$= \frac{1}{3}x^3\left[2 - 2\ln x^3 + (\ln x^3)^2\right] + c$

20.) $\int \frac{x}{x^4-9}\,dx = \int \frac{x}{(x^2)^2-9}\,dx$

(Let $u = x^2 \rightarrow du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int \frac{1}{u^2-3^2}\,du$ {Use 29.) with $a=3$.}

$= \frac{1}{2}\cdot\frac{1}{2(3)}\cdot\ln\left|\frac{u-3}{u+3}\right| + c$

$= \frac{1}{12}\ln\left|\frac{x^2-3}{x^2+3}\right| + c$

22.) $\int \frac{\sqrt{3+4t}}{t}\,dt$ {Use 17.) with $a=3, b=4, u=t$.}

$= 2\sqrt{3+4t} + 3\int \frac{1}{t\sqrt{3+4t}}\,dt$ {Use 15.) with $a=3, b=4, u=t$}

$$= 2\sqrt{3+4t} + 3 \cdot \frac{1}{\sqrt{3}} \cdot \ln \left| \frac{\sqrt{3+4t} - \sqrt{3}}{\sqrt{3+4t} + \sqrt{3}} \right| + c$$

34.) $\int (\ln x)^3 \, dx$ {Use 43.) with $n=3$, $u=x$.}

$= x(\ln x)^3 - 3\int (\ln x)^2 dx$ {Use 43.) with $n=2$.}

$= x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int (\ln x)^1 dx\right]$

{Use 43.) with $n=1$.}

$= x(\ln x)^3 - 3x(\ln x)^2 + 6\left[x\ln x - 1\int \underbrace{(\ln x)^0}_{1} dx\right]$

$= x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + c$.

54.) $\int \frac{1}{x^2+4x-5} dx = \int \frac{1}{(x^2+4x+4)-9} dx$

$= \int \frac{1}{(x+2)^2 - 3^2} dx$ (Let $u = x+2 \rightarrow du = dx$)

$= \int \frac{1}{u^2-3^2} du$ (Use 29.) with $a=3$)

$= \frac{1}{6} \ln\left|\frac{u-3}{u+3}\right| + c = \frac{1}{6}\ln\left|\frac{(x+2)-3}{(x+2)+3}\right| + c = \frac{1}{6}\ln\left|\frac{x-1}{x+5}\right| + c$

56.) $\int \sqrt{x^2-6x}\, dx = \int \sqrt{(x^2-6x+9)-9}\, dx$

$= \int \sqrt{(x-3)^2 - 3^2}\, dx$ (Let $u = x-3 \rightarrow du = dx$)

$= \int \sqrt{u^2-3^2}\, du$ {Use 21.) with $a=3$}

$= \frac{1}{2}\left[u\sqrt{u^2-3^2} - (3)^2 \ln\left|u + \sqrt{u^2-3^2}\right|\right] + c$

$= \frac{1}{2}\left[(x-3)\sqrt{(x-3)^2-3^2} - 9 \ln\left|(x-3) + \sqrt{(x-3)^2-3^2}\right|\right] + c$

58.) $$\int \frac{\sqrt{7-6x-x^2}}{x+3}\,dx = \int \frac{\sqrt{7-(x^2+6x)}}{x+3}\,dx$$

$$= \int \frac{\sqrt{16-(x^2+6x+9)}}{x+3}\,dx = \int \frac{\sqrt{4^2-(x+3)^2}}{x+3}\,dx$$

(Let $u = x+3 \rightarrow du = dx$)

$$= \int \frac{\sqrt{4^2-u^2}}{u}\,du$$ (Use 31.) with $a=4$.)

$$= \sqrt{4^2-u^2} - 4\cdot \ln\left|\frac{4+\sqrt{4^2-u^2}}{u}\right| + C$$

$$= \sqrt{16-(x+3)^2} - 4\ln\left|\frac{4+\sqrt{16-(x+3)^2}}{x+3}\right| + C$$

60.) $$\int \frac{x\sqrt{x^4+4x^2+5}}{x^2+2}\,dx = \int \frac{x\sqrt{(x^2)^2+4(x^2)+4+1}}{x^2+2}\,dx$$

$$= \int \frac{x\sqrt{(x^2+2)^2+1}}{x^2+2}\,dx$$ (Let $u = x^2+2 \rightarrow du = 2x\,dx$
$\rightarrow \frac{1}{2}du = x\,dx$)

$$= \frac{1}{2}\int \frac{\sqrt{u^2+1^2}}{u}\,du$$ (Use 23.) with $a=1$.)

$$= \frac{1}{2}\sqrt{u^2+1^2} - \frac{1}{2}\cdot 1\cdot \ln\left|\frac{1+\sqrt{u^2+1}}{u}\right| + C$$

$$= \frac{1}{2}\cdot\sqrt{(x^2+2)^2+1} - \frac{1}{2}\cdot\ln\left|\frac{1+\sqrt{(x^2+2)^2+1}}{x^2+2}\right| + C$$

Handout 12 Solutions

1.) $\int \cos^2 7x \, dx = \int \frac{1}{2}(1+\cos 14x)\, dx$
$= \frac{1}{2}(x + \frac{1}{14}\sin 14x) + c$

2.) $\int \sin^3 3x \cdot \cos 3x \, dx = \frac{1}{3}\cdot\frac{1}{4}\sin^4 3x + c$
(Let $u = \sin 3x \rightarrow du = 3\cos 3x \, dx \rightarrow \ldots$)

3.) $\int \cos^5 9x \cdot \sin 9x \, dx = \frac{-1}{9}\cdot\frac{1}{6}\cos^6 9x + c$
(Let $u = \cos 9x \rightarrow du = -9\sin 9x \, dx \rightarrow \ldots$)

4.) $\int \sin^3 x \, dx = \int \sin x \cdot \sin^2 x \, dx$
$= \int \sin x \cdot (1-\cos^2 x)\, dx = \int [\sin x - \cos^2 x \sin x]\, dx$
$= -\cos x + \frac{1}{3}\cos^3 x + c$

5.) $\int \sin^3 x \cos^3 x \, dx = \int \sin^3 x \cdot \cos^2 x \cdot \cos x \, dx$
$= \int \sin^3 x \cdot (1-\sin^2 x)\cos x \, dx$
$= \int [\sin^3 x \cos x - \sin^5 x \cos x]\, dx$
$= \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c$

6.) $\int \tan 4x \cdot \sec^2 4x \, dx = \frac{1}{4}\cdot\frac{1}{2}\tan^2 4x + c$
(Let $u = \tan 4x \rightarrow du = 4\sec^2 4x \, dx \rightarrow \ldots$)

7.) $\int (\sec x \tan x)^2 \sqrt{1+\tan x}\, dx$
$= \int \sec^2 x \cdot \tan^2 x \cdot (1+\tan x)^{1/2}\, dx$
(Let $u = 1+\tan x \rightarrow du = \sec^2 x \, dx$
and $\tan x = u-1$)

$= \int (u-1)^2 u^{1/2}\, du = \int (u^2 - 2u + 1)\, u^{1/2}\, du$
$= \int [u^{5/2} - 2u^{3/2} + u^{1/2}]\, du = \frac{2}{7} u^{7/2} - 2 \cdot \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + c$
$= \frac{2}{7}(1+\tan x)^{7/2} - \frac{4}{5}(1+\tan x)^{5/2} + \frac{2}{3}(1+\tan x)^{3/2} + c$

8.) $\int \tan^2 x \cdot \sec^4 x\, dx = \int \tan^2 x \cdot \sec^2 x \cdot \sec^2 x\, dx$
$= \int \tan^2 x \cdot (1+\tan^2 x) \cdot \sec^2 x\, dx$
$= \int [\tan^2 x \sec^2 x + \tan^4 x \sec^2 x]\, dx$
$= \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + c$

9.) $\int \tan^3 x\, dx = \int \tan x \cdot \tan^2 x\, dx$
$= \int \tan x \cdot (\sec^2 x - 1)\, dx$
$= \int [\tan x \sec^2 x - \tan x]\, dx$
$= \frac{1}{2}(\tan x)^2 - \ln|\sec x| + c$

10.) $\int \tan^3 x \sec x\, dx = \int \tan^2 x \cdot \tan x \cdot \sec x\, dx$
$= \int (\sec^2 x - 1) \sec x \tan x\, dx$
$= \int [\sec^2 x (\sec x \tan x) - \sec x \tan x]\, dx$
$= \frac{1}{3}\sec^3 x - \sec x + c$

11.) $\int \csc^4 5x \cot 5x\, dx = \int \csc^3 5x \cdot \csc 5x \cot 5x\, dx$
$= \frac{-1}{5} \cdot \frac{1}{4} \csc^4 5x + c$
(Let $u = \csc 5x \rightarrow du = -5 \csc 5x \cot 5x\, dx \rightarrow \cdots$)

12.) $\int \cot^3 10x\, dx = \int \cot 10x \cdot \cot^2 10x\, dx$
$= \int \cot 10x \cdot (\csc^2 10x - 1)\, dx$

$$= \int \left[\cot 10x \cdot \csc^2 10x - \cot 10x\right] dx$$

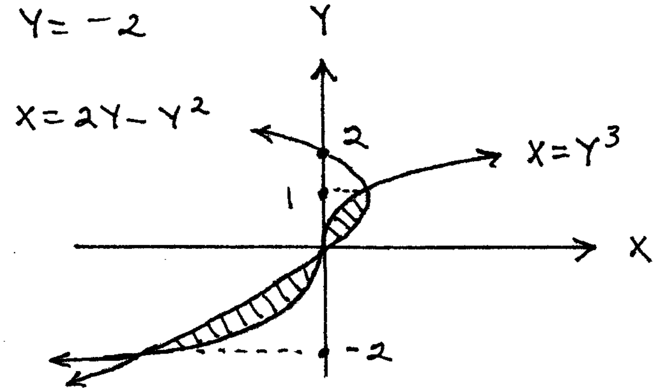
$$= \frac{-1}{10} \cdot \frac{1}{2} \cot^2 10x - \frac{1}{10} \ln|\sin 10x| + C$$

13.) $x = 2y - y^2$ and $x = y^3 \rightarrow$

$y^3 = 2y - y^2 \rightarrow y^3 + y^2 - 2y = 0 \rightarrow$

$y(y^2 + y - 2) = y(y-1)(y+2) = 0 \rightarrow$

$y = 0 \quad y = 1 \quad y = -2$



$$\text{Area} = \int_{-2}^{0} \left[y^3 - (2y - y^2)\right] dy$$

$$+ \int_0^1 \left[(2y - y^2) - y^3\right] dy$$

$$= \left(\frac{1}{4}y^4 - y^2 + \frac{1}{3}y^3\right)\Big|_{-2}^{0}$$

$$+ \left(y^2 - \frac{1}{3}y^3 - \frac{1}{4}y^4\right)\Big|_0^1$$

$$= (0) - \left(4 - 4 - \frac{8}{3}\right) + \left(1 - \frac{1}{3} - \frac{1}{4}\right) - (0)$$

$$= \frac{8}{3} + \frac{5}{12}$$

$$= \frac{37}{12}.$$