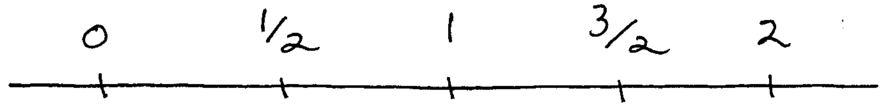
Section 6.5

Page 433

1.) $\int_0^2 x^2\,dx$ with $f(x)=x^2$ and $n=4$:



a.) $T_4 = \frac{2-0}{2(4)}\left[f(0)+2f\left(\frac{1}{2}\right)+2f(1)+2f\left(\frac{3}{2}\right)+f(2)\right]$

$= \frac{1}{4}\left[(0)^2+2\left(\frac{1}{2}\right)^2+2(1)^2+2\cdot\left(\frac{3}{2}\right)^2+(2)^2\right]$

$= \frac{1}{4}\left[0+\frac{1}{2}+2+\frac{9}{2}+4\right] = \frac{11}{4}$

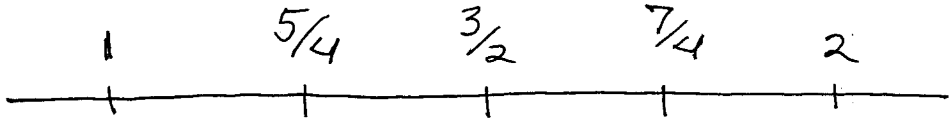
b.) $S_4 = \frac{2-0}{3(4)}\left[f(0)+4f\left(\frac{1}{2}\right)+2f(1)+4f\left(\frac{3}{2}\right)+f(2)\right]$

$= \frac{1}{6}\left[(0)^2+4\left(\frac{1}{2}\right)^2+2(1)^2+4\left(\frac{3}{2}\right)^2+(2)^2\right]$

$= \frac{1}{6}\left[0+1+2+9+4\right] = \frac{16}{6} = \frac{8}{3}$

c.) Exact: $\int_0^2 x^2\,dx = \frac{1}{3}x^3\Big|_0^2 = \frac{8}{3}$

4.) $\int_1^2 \frac{1}{x}\,dx$ with $f(x)=\frac{1}{x}$ and $n=4$:



a.) $T_4 = \frac{2-1}{2(4)}\cdot\left[f(1)+2f\left(\frac{5}{4}\right)+2f\left(\frac{3}{2}\right)+2f\left(\frac{7}{4}\right)+f(2)\right]$

$= \frac{1}{8}\left[1+2\left(\frac{4}{5}\right)+2\left(\frac{2}{3}\right)+2\left(\frac{4}{7}\right)+\frac{1}{2}\right] \approx 0.6970$

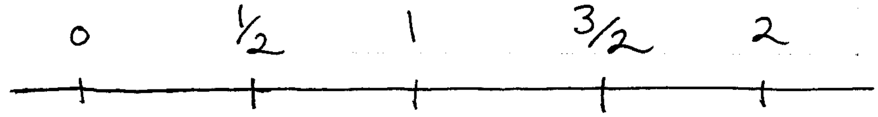
b.) $S_4 = \frac{2-1}{3(4)} \cdot \left[f(1) + 4f\left(\frac{5}{4}\right) + 2f\left(\frac{3}{2}\right) + 4f\left(\frac{7}{4}\right) + f(2) \right]$

$= \frac{1}{12} \cdot \left[1 + 4\left(\frac{4}{5}\right) + 2\left(\frac{2}{3}\right) + 4\left(\frac{4}{7}\right) + \frac{1}{2} \right] \approx 0.6932$

c.) Exact : $\int_1^2 \frac{1}{x}\,dx = \ln|x| \Big|_1^2 = \ln 2 - \cancelto{0}{\ln 1}$

≈ 0.6931

10.) $\int_0^2 \sqrt{1+x}\,dx$ with $f(x) = \sqrt{1+x}$ and $n = 4$:



a.) $T_4 = \frac{2-0}{2(4)} \cdot \left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2) \right]$

$= \frac{1}{4} \left[1 + 2\sqrt{\frac{3}{2}} + 2\sqrt{2} + 2\sqrt{\frac{5}{2}} + \sqrt{3} \right] \approx 2.7931$

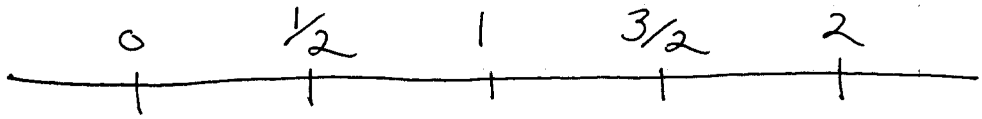
b.) $S_4 = \frac{2-0}{3(4)} \cdot \left[f(0) + 4f\left(\frac{1}{2}\right) + 2f(1) + 4f\left(\frac{3}{2}\right) + f(2) \right]$

$= \frac{1}{6} \left[1 + 4\sqrt{\frac{3}{2}} + 2\sqrt{2} + 4\sqrt{\frac{5}{2}} + \sqrt{3} \right] \approx 2.7973$

c.) Exact: $\int_0^2 \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2} \Big|_0^2$

$= \frac{2}{3}(3)^{3/2} - \frac{2}{3}(1)^{3/2} \approx 2.7974$

14.) $\int_0^2 \frac{1}{\sqrt{1+x^3}}\,dx$ with $f(x) = \frac{1}{\sqrt{1+x^3}}$ and $n = 4$:



a.) $T_4 = \frac{2-0}{2(4)} \cdot \left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2)\right]$

$= \frac{1}{4}\left[1 + \frac{2}{\sqrt{\frac{9}{8}}} + \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{\frac{35}{8}}} + \frac{1}{3}\right] \approx 1.397$

b.) $S_4 = \frac{2-0}{3(4)} \cdot \left[f(0) + 4f\left(\frac{1}{2}\right) + 2f(1) + 4f\left(\frac{3}{2}\right) + f(2)\right]$

$= \frac{1}{6} \cdot \left[1 + \frac{4}{\sqrt{9/8}} + \frac{2}{\sqrt{2}} + \frac{4}{\sqrt{\frac{35}{8}}} + \frac{1}{3}\right] \approx 1.405$

18.) $\int_0^2 e^{-x^2}\,dx$ with $f(x) = e^{-x^2}$ and $n = 4$:

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2$

a.) $T_4 = \frac{2-0}{2(4)} \cdot \left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2)\right]$

$= \frac{1}{4} \cdot \left[1 + 2e^{-1/4} + 2e^{-1} + 2e^{-9/4} + e^{-4}\right] \approx 0.881$

b.) $S_4 = \frac{2-0}{3(4)}\left[f(0) + 4f\left(\frac{1}{2}\right) + 2f(1) + 4f\left(\frac{3}{2}\right) + f(2)\right]$

$= \frac{1}{6}\left[1 + 4e^{-1/4} + 2e^{-1} + 4e^{-9/4} + e^{-4}\right] \approx 0.882$

20.) $\int_0^3 \frac{x}{2+x+x^2}\,dx$ with $f(x) = \frac{x}{2+x+x^2}$

$n = 6$ $\quad 0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2 \quad \frac{5}{2} \quad 3$

a.) $T_6 = \frac{3-0}{2(6)}\left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + 2f(2)\right.$

$\left. + 2f\left(\frac{5}{2}\right) + f(3)\right]$

$$= \frac{1}{4}\left[0+2\left(\frac{2}{11}\right)+2\left(\frac{1}{4}\right)+2\left(\frac{6}{23}\right)+2\left(\frac{1}{4}\right)+2\left(\frac{10}{43}\right)+\frac{3}{14}\right] \approx 0.641$$

b.) $$S_6 = \frac{3-0}{3(6)}\left[f(0)+4f\left(\tfrac{1}{2}\right)+2f(1)+4f\left(\tfrac{3}{2}\right)+2f(2) + 4f\left(\tfrac{5}{2}\right)+f(3)\right]$$

$$= \frac{1}{6}\left[0+4\left(\frac{2}{11}\right)+2\left(\frac{1}{4}\right)+4\left(\frac{6}{23}\right)+2\left(\frac{1}{4}\right)+4\left(\frac{10}{43}\right)+\frac{3}{14}\right] \approx 0.653$$

29.) $$S_{10} = \frac{1000-0}{3(10)}\{f(0)+4f(100)+2f(200)+4f(300) + 2f(400)+4f(500)+2f(600)+4f(700)+2f(800) + 4f(900)+f(1000)\}$$

$$= \frac{100}{3}\left[125+4(125)+2(120)+4(112)+2(90) + 4(90)+2(95)+4(88)+2(75)+4(35)+0\right]$$

$$= 89{,}500 \text{ ft.}^2$$

31.) $\int_0^2 x^4\,dx$ so $f(x)=x^4 \xrightarrow{D} f'(x)=4x^3 \xrightarrow{D} f''(x)=12x^2 \xrightarrow{D} f'''(x)=24x \xrightarrow{D} f^{(4)}(x)=24$;

$$\max_{0\le x\le 2}|f''(x)| = \max_{0\le x\le 2}|12x^2| = 12(2)^2 = \boxed{48}$$ and

$$\max_{0\le x\le 2}|f^{(4)}(x)| = \max_{0\le x\le 2}|24| = \boxed{24}$$;

a.) $$|E_4| \le \frac{(2-0)}{12}\cdot\left(\frac{2-0}{4}\right)^2 \cdot \max_{0\le x\le 2}|f''(x)|$$

$$= \frac{1}{24}\cdot(48) = 2$$

b.) $|E_4| \le \frac{(2-0)}{180} \cdot \left(\frac{2-0}{4}\right)^4 \cdot \max_{0 \le x \le 2} |f^{(4)}(x)|$

$= \frac{2^5}{180 \cdot 4^4} (24) \approx 0.0167$

32.) $\int_0^1 \frac{1}{x+1}\,dx$ so $f(x) = (x+1)^{-1} \xrightarrow{D}$

$f'(x) = -(x+1)^{-2} \xrightarrow{D} f''(x) = 2(x+1)^{-3} \xrightarrow{D}$

$f'''(x) = -6(x+1)^{-4} \xrightarrow{D} f^{(4)}(x) = 24(x+1)^{-5}$;

$\max_{0 \le x \le 1} |f''(x)| = \max_{0 \le x \le 1} \left|\frac{2}{(x+1)^3}\right| \le \frac{2}{(0+1)^3} = \boxed{2}$ and

$\max_{0 \le x \le 1} |f^{(4)}(x)| = \max_{0 \le x \le 1} \left|\frac{24}{(x+1)^5}\right| \le \frac{24}{(0+1)^5} = \boxed{24}$;

a.) $|E_4| \le \frac{(1-0)}{12} \left(\frac{1-0}{4}\right)^2 \cdot \max_{0 \le x \le 1} |f''(x)|$

$= \frac{1}{192}(2) = \frac{1}{96} \approx 0.0104$

b.) $|E_4| \le \frac{(1-0)}{180} \cdot \left(\frac{1-0}{4}\right)^4 \cdot \max_{0 \le x \le 1} |f^{(4)}(x)|$

$= \frac{1}{180 \cdot 4^4}(24) \approx 0.0005$

36.) $\int_1^3 \frac{1}{x}\,dx$ so $f(x) = x^{-1} \rightarrow$

$f'(x) = -x^{-2}$, $f''(x) = 2x^{-3}$, $f'''(x) = -6x^{-4}$, and $f^{(4)}(x) = 24x^{-5}$;

$$\max_{1 \le x \le 3} |f''(x)| = \max_{1 \le x \le 3} \left|\frac{2}{x^3}\right| = \frac{2}{(1)^3} = \boxed{2} \ ;$$

$$\max_{1 \le x \le 3} |f^{(4)}(x)| = \max_{1 \le x \le 3} \left|\frac{24}{x^5}\right| = \frac{24}{(1)^5} = \boxed{24} \ ;$$

a.) $|E_n| \le \frac{(3-1)^3}{12n^2} \cdot \max_{1 \le x \le 3} |f''(x)|$

$= \frac{8}{12n^2} \cdot (2)$

$= \underline{\frac{4}{3n^2} < 0.0001}$ then

$\frac{4}{3(0.0001)} < n^2 \rightarrow \frac{40{,}000}{3} < n^2 \rightarrow$

$\sqrt{\frac{40{,}000}{3}} < n \rightarrow n > 115.5$ so

$\boxed{n = 116}$ works ;

b.) $|E_n| \le \frac{(3-1)^5}{180n^4} \cdot \max_{1 \le x \le 3} |f^{(4)}(x)|$

$= \frac{32}{180n^4} \cdot (24)$

$= \underline{\frac{64}{15n^4} < 0.0001}$ then

$\frac{64}{15(0.0001)} < n^4 \rightarrow \frac{640{,}000}{15} < n^4 \rightarrow$

$\left(\frac{640{,}000}{15}\right)^{\frac{1}{4}} < n \rightarrow n > 14.4$ so $\boxed{n = 16}$ works.

n must be even!

38.) $\int_3^5 \ln x \, dx$ so $f(x) = \ln x \rightarrow$

$f'(x) = \frac{1}{x}$, $f''(x) = -x^{-2}$, $f'''(x) = 2x^{-3}$, and

$f^{(4)}(x) = -6x^{-4}$;

$$\max_{3 \le x \le 5} |f''(x)| = \max_{3 \le x \le 5} \left|\frac{1}{x^2}\right| = \frac{1}{(3)^2} = \frac{1}{9} \ ;$$

$$\max_{3 \le x \le 5} |f^{(4)}(x)| = \max_{3 \le x \le 5} \left|\frac{6}{x^4}\right| = \frac{6}{(3)^4} = \frac{2}{27} \ ;$$

a.) $$|E_n| \le \frac{(5-3)^3}{12n^2} \cdot \max_{3 \le x \le 5} |f''(x)|$$

$$= \frac{8}{12n^2} \cdot \frac{1}{9}$$

$$= \underline{\frac{2}{27n^2} < 0.0001} \quad \text{then}$$

$$\frac{2}{27(0.0001)} < n^2 \rightarrow n > \sqrt{\frac{2}{27(0.0001)}} \approx 27.2$$

so $\boxed{n = 28}$ works ;

b.) $$|E_n| \le \frac{(5-3)^5}{180n^4} \cdot \max_{3 \le x \le 5} |f^{(4)}(x)|$$

$$= \frac{32}{180n^4} \cdot \frac{2}{27}$$

$$= \underline{\frac{16}{1215n^4} < 0.0001} \quad \text{then}$$

$$\frac{16}{1215(0.0001)} < n^4 \rightarrow \left(\frac{16}{1215(0.0001)}\right)^{1/4} < n \rightarrow$$

$n > 3.8$ so $\boxed{n = 4}$ works.

48.) Estimate the average value of $f(t) = 12 - 4\ln(t^2 - 4t + 6)$ for $0 \le t \le 6$, i.e., estimate $AVE = \frac{1}{6-0}\int_0^6 f(t)\,dt$:

Use Simpson's Rule with $n = 6$ $\longrightarrow$

0 1 2 3 4 5 6

$$S_6 = \frac{6-0}{3(6)}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + 2f(4) + 4f(5) + f(6)\right]$$

$$= \frac{1}{3}\Big[(12 - 4\ln 6) + 4(12 - 4\ln 3) + 2(12 - 4\ln 2)$$

$$+ 4(12 - 4\ln 3) + 2(12 - 4\ln 6) + 4(12 - 4\ln 11)$$

$$+ (12 - 4\ln 18)\Big]$$

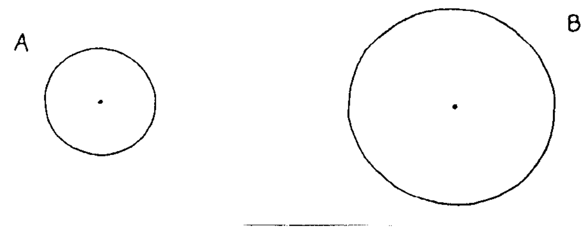
$$= \frac{1}{3}\left[216 - 8\ln 2 - 32\ln 3 - 12\ln 6 - 16\ln 11 - 4\ln 18\right]$$

≈ 34.62 so

$$AVE = \frac{1}{6}\int_0^6 f(t)\,dt \approx \frac{1}{6}(34.62)$$

≈ 5.77 gm./l.

SA12 :



a.) wheel A : circumference is

$C = 2\pi r = 2\pi(2) = 4\pi$ in. so

10 rotations is a total distance of

$10(4\pi) = 40\pi$ in. ;

wheel B : circumference is

$C = 2\pi r = 2\pi(6) = 12\pi$ in. so

10 rotations is a total distance of

$10(12\pi) = 120\pi$ in.

b.) 10 ft. = 10(12) in. = 120 in.

wheel A : $C = 4\pi$ in. so 120 in. is

$$\frac{120}{4\pi} = \frac{30}{\pi} \approx 9.55 \text{ rotations} \ ;$$

wheel B : $C = 12\pi$ in. so 120 in. is

$$\frac{120}{12\pi} = \frac{10}{\pi} \approx 3.18 \text{ rotations}$$